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The Golden Ratio in Music: Compositional Practice or the Result of Genius?

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Abstract: *The Golden Ratio, present in almost everything that exists in nature, has fascinated and inspired scientists, philosophers, artists, and architects since Antiquity. Throughout history, the Golden Ratio played an important role in the compositional structure of various artworks, but the mystery of this divine proportion has remained unexplained. From an aesthetic point of view, this enigmatic number is considered the expression of perfect proportion. Throughout history, one of the main concerns of composers from all periods was the creation of balanced and harmonious structures in their musical compositions. Numerous musical works employ already familiar rules, while in others, the emergence of new principles may be a sign of originality.*
Certainly, in the few studies that deal with the Golden Ratio in music, convincing or at least plausible arguments have been advanced regarding the importance of the golden number. Yet, exploring the musical literature, there are still relevant aspects to be found and formulated concerning this subject. In this sense, the aim of the current study is to present several methods of using this compositional technique, and implicitly the positioning of the Golden Ratio in correlation with its effects and auditory perception, exemplified through several compositional landmarks from the works of various composers.

Key-words: *Golden Ratio, Fibonacci sequence, divine proportion, music*

1. Introduction

Throughout the history of mankind, the Golden Ratio, also known as the Golden Section or Golden Number, has led to extraordinary revelations in our awareness and perception regarding the everyday and spiritual dimensions of physical existence, constituting a valuable tool and not a set of rules for composition. Through the identification of divine proportion, several rules that govern the Universe have been founded, from a philosophical and mathematic point of view.

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German mathematician Martin Ohm (1792-1872) was the first to introduce the term *Golden Section* in 1835, in the second edition of *Die Reine Elementar-Mathematik*, referring to the ratio as *goldener Schnitt* (Livio 2007).

The Golden Ratio refers to the unequal division of a line into two parts, in such a way that the ratio of the whole to the longer part is equal to the ratio of the longer part to the shorter section. This mathematical proportion has a value of approximately 1.618033 and is often denoted by the Greek letter *Phi* Φ (Livio, 2007).

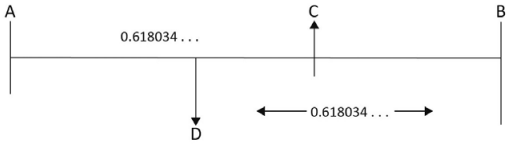

Fig.1. *Representation of the Golden Number – Mathematical Proportions*

The Golden Ratio is considered to represent the ideal proportions from an aesthetic point of view and throughout history it has given rise to numerous discussions, detailed in studies regarding nature (Allen, 2004; Cooke & Bliss, 2006), engineering (Zhu, Jing, & Cheng, 2012), philosophy, architecture, painting, sculpture, music, mathematics, chemistry, physics (Huntley 1970; Livio 2002; Olsen 2006; Akhtaruzzaman 2011).

It has been revealed that Φ can be identified in the anatomy and physiology of the human body, in the configuration and functions of various organs (Petekkaya, Ulusoy, Bagheri, et al. 2021; Persaud-Sharma and O'Leary 2015; Tamargo and Pindrik 2019; Yalta et al. 2016; Yamagishi and Shimabukuro 2008; Purnell, et al. 2019).

Unlike visual arts, where adherence to certain proportions is immediately noticeable at the first glance of a painting, in music the entire composition must be heard, in order to have a complete perception of it. This explains the significance of aural perception, as well as the importance of theoretically understanding the Golden Ratio within a composition. According to Lendvai, the Golden Ratio can only fulfil its purpose when perceived – whether by imbuing the work with the power to evoke emotions in the listener, or by emphasizing the contrast between different parts of the composition (Lendvai 1984, 257).

Gifted composers conceive their music in such a manner that the various parts are not only perfectly balance, but each provides the ideal setting for the musical unfolding.

Certainly, in the few studies dedicated to the Golden Ratio in music convincing, or at least plausible arguments have been advanced regarding the

importance of proportion and the golden number. However, by analyzing the vast body of musical literature, relevant aspects of the subject still remain open to identification and discussion. Thus, through several musical examples from the works of various composers, the current study aims to present various means of employing the concept of proportion within the compositional process, and implicitly, to reveal the correlation between the positioning of the Golden Ratio, its effect, and aural perception.

However, it is of utmost importance to mention that not all valuable compositions were devised according to the principles of the Golden Ratio, which leads to the question whether perfect balance and proportion in music must necessarily be the products of the Golden Ratio and whether composers truly sought to respect mathematical proportions within their works?

2. Proportion and the Golden Ratio – Landmarks

The first written description of the geometric methods for obtaining the Golden Ratio belongs to the Greek mathematician Euclid of Alexandria (about 325 BC – 260 BC) (Meisner 2018), who published around the year 300 BC his work *Stoicheia* ("Elements"), in the sixth book of which he offered the following definition: *"A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less"* (Euclid 2017).

The Golden Number was used by the Ancient Egyptians in the construction of the pyramids, where the ratio between the height and the apothem of the base is equal to the square root of the Golden Ratio, and the angle formed by a lateral face with the base plane, in the case of the Great Pyramid of Giza, is given by arctan (inverse tangent function) √φ = 51°49'.

The Golden Ratio found in Greek vessels and the poetry of Virgil was apparently derived from the Fibonacci numbers, as were certain proportions in Minoan, Greek, and Gothic architecture. The mathematician Fibonacci, also known as Leonardo Pisano, Leonardo Fibonacci, or Leonardo Bonacci (1170- 1250), published in 1202 the work *Liber Abacci*, which introduced and popularized throughout Europe the Hindu-Arabic numeral system and the decimal numeral system (Moyon 2016). In the 12th chapter of this work, Fibonacci offers a rule regarding the formation of a sequence of numbers: Fn = Fn-2 + Fn-1. According to this, a series of infinite numbers that start with 0 and 1 follows a set pattern, where each number in the sequence is equal to the sum of the two numbers before it: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, etc. The ratio between any two consecutive numbers approximates the Golden Ratio (Tattersall 2005).

The concept of Golden Ratio was studied and popularized throughout the Renaissance by artists who employed this concept in their works, in order to obtain balanced and aesthetically pleasing proportions. In 1509, Fra Luca Bartolomeo de Pacioli (about 1447-1517), important figure of the period and renown mathematician, dedicated an entire book to the Golden Ratio, bearing the title *De Divina Proportione.* Leonardo da Vinci (1452–1519), student and friend of Pacioli, created the geometric drawings of the polyhedron in this book, demonstrating the application of the Golden Ratio in three-dimensional form. Da Vinci used the Golden Ratio in his painting, beginning with 1472, when he painted the *Annunciation* (1472-1476). After the mathematician's death, the concept of Golden Ratio continued to influence artists and mathematicians alike, becoming a symbol of harmony and beauty in the arts.

The reciprocal of the Golden Ratio, (1/Φ) — approximately 0.6180340 — was calculated by the German astronomer and mathematician Michael Mästlin (1550–1631), who included this number in a letter addressed to Johannes Kepler in 1597 (Meisner 2018).

The Fibonacci sequence can be recognized everywhere in nature, from the arrangement of leaves and the petals of a flower to the phalanges of the human hand and the construction of the pyramids. Architecture provides numerous remarkable examples, such as the CN Tower in Canada, the Notre-Dame Cathedral in Paris (where divine proportion is strictly followed), or the Guggenheim Museum in New York, inspired by the structure of a shell, which in turn follows a pattern governed by the Golden Ratio.

Regarding the human body, for example, if one measures the length of the finger bones, it appears that the ratio between the longest bone and the middle bone, as well as the ratio between the middle bone and the shortest bone at the tip, represents the Golden Ratio Phi. Jean-Claude Perez has suggested since 1991 that there is a strong connection between DNA and the Golden Ratio (Perez 2013). The Golden Ratio can also be discovered in the activity of the heart, in the ratio between systolic and diastolic blood pressure, which is close to 1.61. Dental structure follows the Golden Ratio as well, generally observed in the ratio between the width of the central incisor and the lateral incisor. Additionally, the rectangle surrounding the two central incisors forms a Golden Rectangle. In recent years, Fibonacci numbers have been used in cancer research, where they are employed to construct a mathematical model for the movement of malignant cells.

Taken as a norm, the Golden Ratio is ambivalent, like any canon, in the sense that what complies with this norm possesses alluring traits of harmonious regularity, while what deviates from it gains striking qualities of pathos and expressiveness. The secret of beauty in art has not yet been established: is it the

concordance and constant use of certain norms, or the deviation from them? In music, there are countless examples which prove that breaking away from strict rules does not result in something unpleasant or lacking in harmony.

3. The Golden Ratio in Music

The Golden Ratio can be found in nature, visual arts, and architecture, but also in musical compositions. Throughout history, composers strived to obtain balanced and harmonious structures, incorporating this ratio into their works, even if not always consciously. The Golden Ratio can be used to structure musical forms, highlight important phrases or sections, or create an aesthetic balance that resonates with the listener. The solid foundation of a musical composition may assure the success of the work, which explains the multitude of structural possibilities in music composition, through which logical coherence and expressiveness is sought. The Golden Ratio is one of the most interesting features, deliberately used in music. Musicologists and researchers, fascinated by the existence of the Golden Ratio, have identified numerous scores in which the Golden Number is expressed through proportion, as well as other musical instances which are based on this principle (Kramer 1988).

The first example is that of the luthier Antonio Stradivarius, who followed the Golden Ratio when designing the specific parts of his violins. Later, the study regarding the Golden Ratio was applied to the crafting of other stringed instruments, in order to obtain a pleasant sound.

The Golden Ratio can be applied to various elements of the musical discourse, at the level of intonation and melody, harmony, rhythm, or formal structure. In terms of intonation, if the half step (semitone) = 1 is considered the basic unit of measurement and is then correlated with the values in the Fibonacci sequence, the following sequence is obtained: 1=1 half step (semitone); 2=1 whole step (tone); 3=minor third; 5=perfect fourth; 8=minor sixth, 13=minor ninth etc.

In the pentatonic scale, commonly encountered in the traditional music of various cultures, a sequence of music intervals similar to the one previously mentioned can be encountered: 2, 3, 5, 8. (Figure 2).

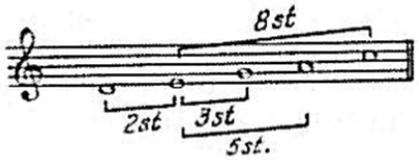

Fig. 2. *The pentatonic scale*

In terms of harmony, the major sixth chord (chord in first inversion), with a 3-5 structure, and the second inversion of a chord, with a 5-3 structure, can be incorporated within the Fibonacci sequence (Figure 3).

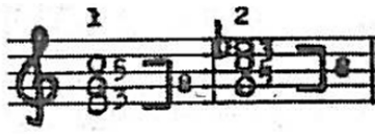

Fig. 3. *The major chord: 1- chord in first inversion; 2- chord in the second inversion*

Upon examining the proportional divisions of the 12-tone equal temperament, Swiss aesthetician and musicologist Denéréaz observed that all the notes within a tempered scale could be derived using the Golden Ratio (starting with the perfect chord Do-Mi-Sol), because the equal temperament introduces the Golden Ratio into the proportions of the scale: *if the major chord Do-Mi-Sol feels harmonious spontaneously, this is not due to a simple ration between the string segments that express it, but because the minor third Mi-Sol relates to the major third Do-Mi in the same manner that the latter relates to the fifth Do-Sol – the precise definition of the Golden Ratio* (Dénéréaz 1937).

The Golden Ratio can be found in compositions across all periods of music history. In the works of certain composers, this ratio can be identified, for example, by examining the numerical correlations between the various sections of the musical work. Thus, by aligning the length of a piece to the mathematical principles of proportion, the culminating point of the composition is revealed at the precise location of the Golden Ratio (Livio 2007).

The Golden Ratio influenced Baroque art and music, contributing to the creation of a visual and aural balance, while during the Classical period it continued to play an important role, although integrated in a more subtle and rational manner, corresponding to the aesthetic principles of the period. The Classical period placed great importance on order, symmetry, proportion, and clarity, and the use of the Golden Ratio may have been aimed at achieving balance and harmony. Some studies emphasize that the Golden Ratio was used to organize the structure of such compositions as the symphony, sonata, or concert. The influence of proportion can be observed in the works of some of the most important composers of the period, such as Mozart or Beethoven, in the structures of their melodies or themes. Moreover, in the pursuit of a natural and harmonious flow, composers may have been influenced by the Golden Ratio in such areas as rhythm, phrasing, or thematic development.

In his work dedicated to the analysis of the Golden Section in the Piano Sonatas of Mozart, Putz claims that even though the music Mozart composed is divine, nonetheless divine proportion is not an intrinsic part of the compositional process (Putz 1995, 275-282).

Composers of the Romantic period may have also employed the Golden Ratio, as demonstrated in Schubert's song *Du bist die Ruh* (1823). Excluding the first seven bars of the piano introduction, the song consists of a total of 75 bars (75 x 0.618 = 46.35) and can be divided into two parts: the first part has 46 bars (bars 8-53), while the second part contains 29 bars (54-82).

The Golden Ratio can also be exemplified at the level of the musical form through works from Bartók's oeuvre, where, for instance, an 8-bar phrase is structured in groups of 5+3 bars, as opposed to the classical Viennese square phrase. According to Lendvai, in the first movement of Béla Bartók's *Sonata for Two Pianos and Percussion* the movement comprises 443 bars, so its Golden Section is 443 x 0.618 = 274, which indicates the gravitational centre of the movement; the recapitulation starts precisely at the 274th bar. In the trio from the third movement of *Contrasts*, out of a total of 93 bars, the Golden Section falls in the middle of the 58th bar, which marks the beginning of the reprise – precisely where the value of the Golden Ratio is obtained: 1.618 (Lendvai 1971).

In a time when AI intervention in music is gaining importance and music is often composed through the merging of sound databases, the Golden Ratio within a score can be calculated using the software employed in music composition (for example, Sibelius is able to make these calculations with the assistance of a plug-in). However, AI has not yet succeeded in developing an analogical method for stimulating emotions and ideas, or for transferring them, nor for the generation of creative ideas in music based on the Golden Ratio.

4. Conclusions

The Fibonacci sequence is seen everywhere in nature because it acts as a guide for growth. It has been scientifically proven that the Golden Ratio exists in nature, in the human body, it governs the functioning of various cells or organs. Thus, it can be suggested that proportion and the Golden Ratio lie at the core of human existence, therefore being mirrored in the artistic creations of humans.

Proportion, balance and harmony between parts are concepts that have governed music composition throughout history. But did composers obtain these through mathematical formulas, or is the presence of the Golden Ratio within their works the result of pure genius? Although research on the study of proportion in

musical works shows that many compositions display proportions and temporal relationships that approximate the Golden Ratio, there are also studies that argue this is mere coincidence—pure speculation about the existence of the Golden Ratio—since there are no clear confessions from composers who deliberately used this approach in their compositions.

In visual arts, painting, sculpture, and architecture, the use of mathematical calculations is implicit, but is this the case with music? To prove that a particular composer has devised his works according to mathematical principles and proportions, his entire compositional output should be analyzed.

The idea of associating music with mathematical calculations or series of numbers was of utmost interest especially to composers from the 20th century, many of whom have based their compositions on such principles. However, is it safe to assume that the Golden Number was of interest for the composers of the Baroque or Classical eras as well, and that they aimed to obtain perfect proportion and emotion through the Golden Ratio? Which again leads to the question: did composers really follow mathematical calculations when composing their works, or the presence of the Golden Ratio, perfect balance between the parts and proportion are byproducts of genius?

As in any field, there have been voices questioning how the Golden Ratio is identified in nature, the human form, or in art. In music, it is essential for specialists who wish to explore this topic to conduct a thorough analysis of a composer's works in order to accurately determine whether mathematical characteristics have been deliberately employed in their compositions. In conclusion, by valuing the knowledge that has been passed down for over two thousand years, we can assert that humans have been fascinated by this number from ancient times to the present day. And even if it has been considered an illusion, we might ask: what illusion could have persisted for so long if it weren't based on the intuition of a great truth?

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