Twentieth Century Compositional Techniques Applied to Jazz:
Pitch-Class Sets in Jazz Composition and Improvisation

Paulo PERFEITO¹

Abstract: This paper presents a speculative method to coalesce traditional jazz improvisation with techniques and elements of post-tonal composition. Throughout the 20th century, erudite composers came up with several aesthetic and conceptual developments but jazz, until very recently, did not move in the same direction. To maintain a fruitful collaboration between composers and improvisers, if the pallet of compositional techniques available to the jazz composer expands, I would argue that a similar development must occur in the field of improvisation. For composers, this paper proposes creative processes supported by pitch-class set theory and, in the interest of coherence and unity with pre-composed material, it offers a complementary approach to soloists.

Key-words: Pitch-Class Sets, Jazz, Improvisation, Composition, Creative Processes

1. Introduction

In general, jazz repertoire and its improvisational vocabulary conform to pitch-centricity following western tonal and/or modal systems. However, jazz is all but confined to a single aesthetic approach and it has always combined a multitude of styles and idioms including, after the advent of the Free Jazz, a progressive tendency towards atonality. The objective of this research is to take the first steps towards a “grand unified theory” to intelligibly systematize composition and improvisation, coalescing these influences.

The contextual application of pitch-class sets seems to be a fitting tool that fulfills these criteria. I am interested in finding relationships between pitch class set theory and common practice jazz language. Melodic and harmonic intervallic coherence is an important consideration in this study which prompts the

¹ Author’s affiliation: ESMAE – Polytechnic University of Porto, Portugal
CEIS20 – Universidade de Coimbra, Portugal; pauloperfeito@esmae.ipp.pt
development of a system to organize pitch-class sets according to parsimonious and tonally contextual voice leading. In this respect, a model that organizes pitch-class sets according to voice leading by perfect fifths is adequate and emphasizes fundamental properties of the tonal system while highlighting scalar patterns. Finding patterns within the scales we are working with, helps uncover which pitch-class sets are more effective to convey the tonal effect we have in mind at any moment. Understanding the tonal relation between a pitch-class set and the harmonic context in which it is being used is essential for its successful application.

The idea to classify trichords using the cycle of fifths was inspired by the work of Daniel Moreira’s Master dissertation (Moreira 2010) and further developed in subsequent work (Moreira 2014). Moreira’s methodological approach makes use of all trichodal types and creates a “structural hierarchization with relation to reference points of relative stability, associated to the harmonic strength of the interval of the perfect fifth” (Moreira 2014). While akin to Joseph Strauss’ graph of trichordal set-class voice-leading relations (Strauss 2004) this model systematizes voice leading relations between trichords not via semitones but via perfect fifths. As result, trichords are organized in anhemitonic/pentatonic, diatonic and chromatic levels. For the present research project, I relied on Moreira’s proposed model and advanced a similar model for tetrachords. Tetrachords are idiomatic harmonic structures in jazz, since with four distinct pitch classes it is possible to imply most vernacular chords/harmonies.

1.1. Voice leading space by perfect fifths

The following two graphs – Figures 2 and 3 - represent the trichord and tetrachord space hierarchized by voice leading steps of perfect fifth. The pentatonic/anhemitonic, diatonic and chromatic levels are emphasized using colors and relative placement in the graph expresses the distance between sets in terms of voice leading steps of perfect fifth. A set mutates into an adjacent one by means of moving one of its pitches a perfect fifth. In Figure 2 to get from trichord (027) to (013) a series of three voice leading steps by perfect fifth is necessary. Let’s represent (027) in absolute pitch space as [2,4,9]. If we take pitch-class 2 and move it down a fifth we get [4,7,9], whose prime form is (025). Moving pitch class 4 up a fifth originates [7,9,e] with prime form (024) and finally moving 7 down to 0 gets [9,e,0], a (013) trichord three steps away from the initial (027).
Fig. 1. Trichord voice-leading by perfect fifths

Fig. 1. Tetrachord voice-leading by perfect fifths
2. The Pentatonic Anhemitonic Pitch Collection

The pentatonic collection, resulting from four consecutive perfect fifths, is one of the most pervasive elements in Jazz vernacular.

The minor pentatonic scale – the fourth mode of the major pentatonic scale – with an added pitch on the s4 / f5 scale degree, commonly identified as blues scale, has been a significant melodic shaping element since the dawn of jazz. Overlapping a minor pentatonic scale and the I, IV and V chords used to frame a prototypical Blues progression produces the bluesy dominant-type harmonies and the polymodal\(^2\) effect which is characteristic of the Blues and has been a keystone of jazz vernacular. As Persechetti explains, ‘polymodality involves two or more different modes on the same or different tonal centers. The modal strands may be melodic or harmonic’ (Persichetti 1961).

The pentatonic scale may also be applied in alternative contexts, some of which like major, minor and altered dominant-type chords will be studied in detail ahead. For all these reasons the pentatonic scale is an extremely valuable resource for jazz composers as well as improvisers.

2.1. (027)

According to our model, (027) is the tightest of all trichords with an amplitude of two consecutive fifths. Within a pentatonic span this set has, in addition to its prime form, two transpositions – T\(_7\) and T\(_2\) – and because it is symmetrical, its inversions – T\(_2I\), T\(_9I\) and T\(_4I\) – coincide with P and its respective transpositions. As a result, a pentatonic scale and its modes contain only three possible (027) tetrachords and the entire pentatonic gamut is encompassed by P and its respective T\(_2\).

2.2. (025)

In our classification by fifths, (025) results from a simple linear transformation of (027), moving only one of the outlying pitches a perfect fifth away. Thus, the amplitude of this set is three perfect fifths. Within a pentatonic context this set has one possible transposition – T\(_7\) – and two inversions – T\(_9I\) and T\(_4I\).

The pentatonic scale and its modes contain four possible (025) tetrachords. The pentatonic gamut may be fulfilled by the following trichord combinations:

\[ P + T\_7; \ T\_7 + T\_9I; \ T\_9I + T\_4I \]

\(^2\) expressed by “simultaneous” major and enharmonic minor thirds in chords like I7(s9)
2.3. (037)

The (037) set results from a transformation of (025), by further moving the detached pitch another perfect fifth away. Thus, the amplitude of this set becomes four perfect fifths, the same as the pentatonic gamut itself. Therefore, in the pentatonic context, this set has no transpositions and only one inversion – T₄I. The pentatonic scale and its modes contain two possible (037) tetrachords which do not cover the full pentatonic gamut. (037) represents the major and minor triads, since one is the transposition equivalent of the other, and an anhemitonic collection contains only one of each. This scarcity makes (037) less prone to motivic development all by itself in a pentatonic setting but useful as a pivotal element.

2.4. (024)

If we move the middle pitch of a (037) a perfect fifth up (or down if it’s an inversion), the (024) symmetric tetrachord is produced. The amplitude of this set is also four consecutive fifths, again the same as the pentatonic collection itself. Therefore, in a pentatonic context, this set has no transpositions and its inversion T₄I is invariant with its Prime form. The pentatonic scale and its modes contain only one possible (024) tetrachord which does not cover the full pentatonic gamut. Like (037) this set is not ideal for individual melodic development. However, both (024) and (037) can be excellent links for parsimonious linear development between other non-adjacent sets.

2.5. (0257)

According to our model, (0257) is the tightest of all tetrachords with an amplitude of three consecutive perfect fifths. Within a pentatonic context this set has, in addition to its Prime form, only one transposition – T₇ – and because it is symmetrical, its diatonic inversions – T₉I and T₄I – coincide with P and T₇ respectively. The pentatonic scale and its modes contain two possible (0257) tetrachords, and the entire pentatonic gamut is encompassed by the sum of P and T₇.

Because of the three consecutive fifths (0257) is, like (027), very ambiguous and can also have several tonal implications. As an illustration, we can interpret pitch class a as fundamental, c as minor third, g as minor seventh and d as perfect eleventh of Am11. Analogously, other chord-voicings can be deduced: A7sus⁹, Gsus⁹, D7sus, and some rootless voicings like F6/9, BfM13, EfM13₉, or E7sus¹₃, s⁹.
2.6. (0247)
Moving one of the outlying pitches of a (0257) a fifth away generates a (0247) tetrachord. The amplitude of this new set is four consecutive fifths, the same as the pentatonic collection itself. Therefore, in a pentatonic context, this set has no transpositions and only one inversion – T₄₋₁. The full pentatonic gamut is contained in the combination of P and T₄₋₁. (0247) implies a major triad with an added ninth or a minor triad with an added eleventh. This set, I concluded in a supplementary analysis, is a John Coltrane favorite particularly in the 1959 *Giant Steps* recording session³.

2.7. (0358)
The amplitude of (0358) is four consecutive fifths but because its inversion T₄₋₁ is invariant with P, there is only one possibility to generate this set within an anhemitonic pentatonic collection. This set implies both a minor seventh chord (Am7) or a major triad with an added sixth (C₆).

3. The Major Scale and its Modes
Expanding our universe of study to encompass a diatonic pitch collection raises some questions because pitch-class sets need to be understood in the framework of functional jazz tonality. As Terefenko so well put it: “functional tonality in jazz has different properties than that of common-practice classical music. These properties are represented by a unique set of rules dictating the unfolding of harmonic function, voice-leading conventions, and the overall behavior of chord tones and chordal extensions”. (Terefenko 2014, 128)

Harmonic progressions obey a functional logic whose goal is the transmutation of the dissonant tritone into an agreeable third. From this point on, I will refer to this structural interval as the *key-defining tritone*.

A cycle of six perfect fifths establishes the *key-defining tritone* as its diatonic boundary. Additionally, any tritone works as a pivot between two keys, which are themselves a tritone apart. This is the main idea behind the reharmonization technique known as *tritone substitution* of dominant seventh chords, very prevalent in the jazz idiom. The fact that within a diatonic set there are only two possible triads that don’t contain either note of the *key – defining tritone*, gives you something to think about, especially because these are I and vi, the tonic chords of our own dual Major – minor system. This last property lays the foundation for my

idea of a Funktionsharmonische concerning the classification of pitch-class sets within jazz tonality. In other words, a catalog of pitch-class sets that informs us about the type of harmonic function a given set most accurately conveys.

3.1. A pitch class set “fFunktionsharmonische”

Let us take (015) in the key of C major and two of its variations \([b, c, e]\) and \([c, e, f]\). Although equivalent, these two sets embody rather distinct tonal meanings. While \([b c e]\) suggests a CM7 chord and therefore a tonic function, \([c e f]\) suggests an FM7 or perhaps a rootless Dm9 and in any case a pre-dominant function.

It is thus very useful to compile sets according to their tonal jazz function. Although some exceptions may apply, the guidelines for assembling pitch-class sets into functional families are:

- a primarily dominant pitch-class sets is one that includes both pitches of the key-defining tritone.
- a primarily pre-dominant pitch-class set is one that includes the subdominant pitch, but certainly not the leading tone.
- a primarily tonic pitch-class sets will generally contain the tonic and the mediant pitches. Additionally, it may also contain the leading tone, but it never contains the subdominant pitch.

Nonetheless, if a pitch-class set includes both the leading tone and the tonic, it makes sense to include it as a member of the tonic family as the leading tone may be interpreted as the tonic chord’s major seventh. However, \((016)\) as \([0, 5, e]\) is an exception since, containing the tonic pitch and both pitches of the key defining tritone, lacks tonal stability and bears an urgency towards resolution.

We must keep in mind that these definitions should be taken as guidelines. Pitch-class set affiliation with functional families is contextual and contingent with its own rhythm and metric placement. As professor Terefenko explains, “chords from each functional family create certain expectations and display behavioral patterns that largely depend on their metric position and duration within harmonic progressions. In addition, each functional family is defined by a specific musical affect: the tonic with stability, the predominant with forward motion, and the dominant with tension seeking resolution.” (Terefenko 2014, 128)

3.2. Diatonic assessment of previously analyzed pentatonic pitch-class sets

3.2.1. \((027)\)

Within this new diatonic context \((027)\) expanded its possibilities and, in addition to its prime form, the diatonic scale and its modes contain five possible
(027) tetrachords and we need at least three trichords to cover the entire diatonic gamut. Two of these are consistently T₅ and T₉ with the third one being among P, T₇ or T₂. Except for T₅, which shows pre-dominant characteristics, all (027) variants are tonic-type.

Because of the two consecutive fifths, (027) is tonally ambiguous. This set can be interpreted in a variety of ways and have different chord-voicing implications. For instance, we can interpret pitch-class c as minor third, g as minor seventh and d as perfect eleventh of a rootless Am11. These same pitch classes can also be interpreted, as the major third, major seventh and sharp eleventh of a rootless AfM7⁽¹⁾. Similar processes will yield Gsus, D7sus or a rootless EfM13.

3.2.2. (025)

In a diatonic context (025) has, beyond its prime form, three possible transpositions – T₅, T₇ and T₂ – and four inversions – T₉I, T₄I, T₆I and T₉I.

The diatonic scale and its modes contain eight possible (025) tetrachords. The diatonic gamut may be fulfilled by one of the following combinations of three (025) sets:

<table>
<thead>
<tr>
<th>T₅ + T₇ + T₂</th>
<th>T₅ + T₄I + T₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₅ + P + T₆I</td>
<td>T₅ + T₄I + T₆I</td>
</tr>
<tr>
<td>T₆I + T₉I + T₆I</td>
<td>T₆I + P + T₆I</td>
</tr>
</tbody>
</table>

3.2.3. (037)

As we have seen (037) tetrachords are representations of major and minor triads. Consequently, the major scale and its respective modes have a total of six of these trichords. Three of these – T₅, P and T₇ – correspond to IV, I and V major triads. The remaining three are inversions – T₉I, T₄I and T₆I – and correspond to ii, vi and iii which are the diatonic minor triads. To complete the diatonic gamut, we need to combine a minimum of three (037) pitch-class sets:

<table>
<thead>
<tr>
<th>P + T₅I + T₆I</th>
<th>I ii iii</th>
<th>T₉I + T₆I + T₄I</th>
<th>ii iii iv</th>
</tr>
</thead>
<tbody>
<tr>
<td>P + T₉I + T₇</td>
<td>I ii V</td>
<td>T₉I + T₇ + T₄I</td>
<td>ii V vi</td>
</tr>
<tr>
<td>P + T₅ + T₇</td>
<td>I IV V</td>
<td>T₆I + T₅ + T₇</td>
<td>iii IV V</td>
</tr>
<tr>
<td>T₉I + T₆I + T₅</td>
<td>ii iii IV</td>
<td>T₅ + T₇ + T₄I</td>
<td>IV V vi</td>
</tr>
</tbody>
</table>

The concept of Triad pairs is a widespread technique used by contemporary jazz practitioners. Several authors, (Campbell 2001; Bergonzi 2006; Weiskopf 2010), presented interesting variations of this model in their treatises and
methods. The method consists in using two triads, typically with the same quality, to imply certain *chord scales*/modes.

In my opinion, a few of the most convincing combinations include:

| Dorian – fIII + IV | Dorian f2 – vo + vi° |
| Phrygian – fII + fIII | Lydian Augmented – II + III |
| Lydian – vi + vii | Lydian Dominant – iii° + siv° |
| Mixolydian – v + vi | Mixolydian f13, 9 – ii° + iii° |
| Aeolian – fVI + fVII | Locrian n9 – ii° + fiii |
| Locrian – fiii + iv | Super – Locrian – fii + fiii, fvi° + i° |
| Augmented scale – I s5 + II s5 | Mixolydian f9, f13 – vo + fVI s5 |

3.2.4. (024)

The (024) symmetric tetrachord has, in addition to its prime form, two transposition possibilities within the diatonic scale – T5 and T7. These three sets are invariant in relation to its inversions – T5l, T4l and T6l. The only possibility to cover the full diatonic gamut is a combination of the three (024) trichords.

3.2.5. (0257)

In addition to the prime form, (0257) has three transpositions – T5, T7 and T2 – and because it is a symmetrical set, its inversions are invariant in relation to the normal forms. The entire diatonic gamut is encompassed by T5 and T2.

We have seen in the previous chapter dealing with (0257), that a collection of chords may be deduced from this set.

| Am11 | EbM13(s11) | BbM13 | A7sus4(s 9) | D7sus4 | Gsus4(add 9) | E7sus4(f13, s9) |

3.2.5. (0247)

In a diatonic context, set (0247) has its prime form, two transpositions – T5 and T7 – and three inversions – T5l, T4l and T6l. The full diatonic gamut is contained in the combination of:

- T5 + P + T7
- T5 + T4l + T7
- T5 + T6l + T7
- T5 + P + T6l
- T5 + T7 + T6l

Pitch-class set (0247) implies a major triad with an added ninth or a minor triad with an added eleventh.
3.2.2. (0358)

For a jazz musician, pitch-class set (0358) is an example of a very suggestive tetrachord. This set can imply a minor seventh chord (Am7), the upper structure of a major ninth chord (rootless FM9) or a major triad with an added sixth (C6). An inversional invariant set, (0358) has three possible diatonic variations – T5, P and T7. This is a very intuitive result since the diatonic scale only contains three minor seventh chords placed on ii, iii and vi. The diatonic collection is covered by the sum of T5 and T7.

4. Diatonic Pitch class Sets

Purely diatonic pitch class sets correspond to the green sections on Figures 2 and 3. Some of these sets tend to cluster according to their tonal function. Ensuing the previously described “Funktionsharmonische”, it’s easy to distinguish a group of dominant-type trichords comprised of (016), (026) and (036). In this group, all possible five pitch-class set variations – transpositions and inversions – include both elements of the key-defining tritone and consequently these trichords fit the dominant family categorically. Instead, diatonic variations of (015) and (013) are split equally among the pre-dominant and tonic functions.

In the field of tetrachords (0135) and (0235) express most possible combinations of successive whole and/or half steps which are the building blocks of most scales and lastly, I’ll catalogue a supercluster of dominant-type tetrachords.

4.1. (015) and (013)

In a diatonic context, pitch class set (015) has two normal forms – P and T5 – and two inversions – TeI and T4I. Pitch-class sets (015) are very effective expressing major seventh harmonies. TeI as I [b, c, e] and T4I as IV [e, f, a] are in fact what jazz musicians refer to as shell voicings.

Pitch class set (013) is similar to (015) in the fact that it has two normal forms – P and T5 – and two inversions – TeI and T4I. The full diatonic scale is covered by the following (013) pitch-class sets combinations:

\[ P + T_5 + T_{eI} \quad T_5 + T_{eI} + T_{dI} \]

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4 Purely diatonic pitch class sets are pitch-class sets that span between 4 and 6 steps of perfect fifth. They include at least one element of the pentatonic collection and at least one element of the key defining tritone.
Twentieth Century Compositional Techniques Applied to Jazz:

Pitch-class sets (013) can simply be interpreted as scale fragment containing both a semitone and a whole-step. However, in jazz vernacular it has a more idiomatic implication as a double diatonic approach also known as melodic enclosure.

4.2. (016), (026) and (036)

This group of pitch-class sets shares a very important tonal characteristic. All five pitch-class set variations include both elements of the key defining tritone, which makes it easy to unequivocally classify them as dominant trichords. Due to this peculiarity, none of these trichords has a diatonic prime form – built from the tonic – within the major scale.

It is impossible to complete the diatonic collection with only one type of trichord and in fact, to do so we would need to overlap all five trichord variations.

4.3. (0158)

Pitch-class sets (0158) represents the major seventh chord and it is very intuitive to conclude that there are only two diatonic variations of this trichord – P and T₅I corresponding to IM7 and IVM7. It is impossible to meet the diatonic gamut with this pitch-class set. Analogously to (0358), (0158) can also be interpreted as the upper structure of a minor ninth chord, or in other words a rootless vi m9.

4.4. (0135) and (0235)

These two tetrachords are adjacent in our voice-leading by fifths scheme, and both express stepwise combinations of two whole steps and one half-step. Also, in terms of voice leading possibilities, (0135) is a privileged tetrachord because in a diatonic context, it has more connections – 6 – than any other trichord. There are four diatonic variations of (0135) – T₅, P, T₄I, TₑI – and two diatonic variations of (0235) – P and T₅.

Most diatonic modes, except for Lydian and Locrian that need a (0246), are a combined juxtaposition of (0135) and (0235). Most four-note diatonic stepwise fragments fall on one of these two tetrachords and the ones who don’t are certainly (0246).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Pitch-Class Set</th>
<th>Ionian</th>
<th>Dorian</th>
<th>Phrygian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>c d e f / 1 2 3 4</td>
<td>d e f g / 1 2 f 3 4</td>
<td>e f g a / 1 f 2 f 3 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>g a b c / 5 6 7 1</td>
<td>a b c d / 5 6 f 7 1</td>
<td>b c d e / 5 f 6 f 7 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0135)</td>
<td>(0235)</td>
<td>(0135)</td>
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<td></td>
<td></td>
<td>(0135)</td>
<td>(0235)</td>
<td>(0135)</td>
</tr>
</tbody>
</table>
Although it goes beyond the scope of this research, it is noteworthy to point that (0235) is a fundamental building block of the symmetric diminished scale.

4.5.(0237)

Pitch class set (0237) is the minor equivalent of (0247).

In a diatonic context, this set has its prime form, one transposition –T₅– and two inversions – T₄ I and T₆ I. The full pentatonic gamut is contained in P + T₄ I.

Pitch-class set (0237) implies a major triad with an added eleventh or a minor triad with an added ninth. Although major triads with an added eleventh are uncommon, minor triads with an added ninth are frequent. The first part of John Coltrane’s solo mentioned early, uses a few instances of (0237).

4.6. The constellation of diatonic dominant-type tetrachords: (0258), (0246), (0157), (0136), (0137) and (0156)

As before, with diatonic dominant trichords, the ten pitch-class set variations in this group include both elements of the key-defining tritone, which makes it easy to classify them as dominant-type tetrachords.

From a compositional standpoint, these tetrachords are so invariant that for diversity sake it is sometimes necessary to alternate between them. For instance, (0246) and (0156) only have one form and in relation to (0136), T₅ and T₆ I share all but one pitch class.

Pitch-class set (0258) has only two variations and a very interesting characteristic. Its normal form coincides with a dominant seventh chord – g, b, d, f – while its inversion reveals a half-diminished – b, d, f, a – or a minor sixth – d, f, a, b – chord.

Comparable to (0135) and (0235), (0246) is also a stepwise fragment of a diatonic scale. It connects both pitch classes of the key defining tritone and has a clear affinity to the hexatonic mode.
The normal form of (0157) can be interpreted as a dominant chord with an added eleventh. Because of the extra dissonance created between the leading-tone and tonic pitches, which arguably deters from the key defining tritone as the main functional dissonance requiring resolution, the normal form of (0157) is not common, especially as a harmonic simultaneity. The same effect is recognizable in the normal forms of (0136) and (0137), which makes them more appropriate to suggest Dorian and Lydian sound colors respectively. (0156) is another example of what we just described. This does not mean these trichords are inadequate to express dominant-type harmonies. It means that aware of these tonal consequences, they are most effectively used as melodic/horizontal material. The rhythmic placement of the dissonant tonic pitch as a structural tone or alternatively as an ornament will yield very distinct tonal effects.

The inversion of (0157) is very suitable for implying a dominant chord with a ninth and a thirteenth or alternatively, a major seventh chord with a sharp eleventh, or in other words a Lydian voicing. The inversion of (0136) implies a rootless dominant chord with a thirteenth or a minor sixth chord with a ninth. The inversion of (0137) also implies a dominant chord with a thirteenth or alternatively a minor triad with an added Phrygian flat ninth.

5. Conclusion

In this research, important steps were given in the direction of generating a method based on pitch class set theory to analyze, compose and improvise common practice jazz. The hierarchic model by fifths, proposed by Professor Daniel Moreira propelled a limitless string of creative possibilities, some of which presented in this paper.

Two important and extensive topics relating to this research are left to be developed on subsequent chapters. Although they were alluded in this text, they deserve a more careful exploration.

The first is the pursuit for idiomatic melodic shapes within the dominion of purely chromatic pitch class sets. The main objective is identifying approaches, enclosures and other types of ornamental figures.

The second, far more complex, pertains to the expansion of the hierarchic model by fifths to include minor keys – melodic and harmonic scales – and other non-diatonic pitch collections commonly used by jazz musicians.

Once absorbed by composers and improvisers, this method will enable the use of one “grand unified theory” encompassing all the aesthetic galaxies of the
western equal-temperament universe. A single creative process valid for modal, tonal and atonal jazz composition and improvisation.

References