Theme with variations as
Mathematical representation of Fuzzy sets

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Abstract: *Artistic representations follow, at a closer look, formal and structural archetypes, which often have a physical and mathematical basis, even if the inspiration frequently transcends the scientific dimension of the construction of the musical discourse. Aware or not, the composers personify in their art archetypal structures which are general, and embody their ideals of their expression one way or another. The organization of musical forms constitutes no exception from this phenomenon which has never been investigated in the Romanian musicology: the compatibility between physical and mathematical concepts and significant musical realities. Among the great musical genres and forms, the theme and variations is the closest to one of the elements present in the mathematics of the 20th century: the logic of fuzzy sets (or systems).*

Key-words: *fuzzy, variations, mathematics, logic, composition.*

1. Introduction

Artistic representations follow, at a closer look, formal and structural archetypes, which often have a physical and mathematical basis, even if the inspiration frequently transcends the scientific dimension of the construction of the musical discourse. Aware or not, the composers personify in their art archetypal structures which are general, and embody their ideals of their expression one way or another. The organization of musical forms constitutes no exception from this phenomenon which has never been investigated in the Romanian musicology: the compatibility between physical and mathematical concepts and significant musical realities. Among the great musical genres and forms, the theme and variations is the closest to one of the elements present in the mathematics of the 20th century: the logic of fuzzy sets (or systems).

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2.a. Fuzzy logic – mathematical-philosophical concept

"Fuzzy logic is defined as a concept of conventional logic, which was extended to encompass partial truth, values of truth ranging from "completely true" to "completely false" (Negoiță, Ralescu 1974, 36). Defined in 1965 by the American professor Lotfi Zadeh, the fuzzy logic operates with a multitude of POSSIBLE logical values ranging from the two exact traditional values (0 for false and 1 for true), taking into account the degree of membership of the object to a certain set, which can take values from 0 to 1. The most important aspect concerns the VARIABLE values which can be found in a range of POSSIBLE representations among the two extremes. Based on the fuzzy logic, the fuzzy systems are considered a particular case of certain systems which offer a flexible method to treat UNCERTAINTY.

Despite its well-supported exactness, mathematics has well-incorporated dark areas in which artistic laws operate: in mathematics, the vague sets are sets whose elements have different degrees of membership, being in fact extensions of the classical notion of set. Unlike statistics and probabilities, which deal with objective random uncertainty, vague sets deal with subjective random uncertainty – thus, with typical elements of artistic expressivity, particularized in the musical composition.

The fuzzy logic is present in all fields of life, suggesting the idea of unclear, vague, blurry, contours, able of approximating an uncertain reality through variables. The fuzzy logic is found in many fields, from linguistics to art, from mathematics to life sciences. The fuzzy logic offers the instruments necessary for the representation in intelligent systems of imprecise concepts which are big, hot, cheap, etc., concepts called linguistic variables or fuzzy variables: to represent them we use fuzzy sets, which capture from a quantitative point of view the qualitative interpretation of terms.

2.b. Theme with variations as a musical embodiment of fuzzy logic

Theme and variations is, from this avant-garde perspective, a phenomenon related to this fuzzy thinking because, once the theme is announced, each variation has a certain degree of closeness, of membership to the theme. In vivid and creative music, each variation can become closer to or further from the theme, according to very complex criteria, which musical analyses can depict up to a point: beyond this, the ineffable of music tells its entire story nonchalantly as only a work of art can do in history; there is nothing closer to the composer's attitude that the author uses to unfold the initial expressivity of the theme in various fascicles which show, just like

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The field of vague sets (fuzzy sets) is related to cross-disciplinary areas ranging from mathematics to cybernetics, and represents one of the most interesting fields in applied mathematics (the automata theory, the systems theory and decisions theory, informatics), in LINGUISTICS and in SEMANTICS: "after mathematical models were implemented in the humanities and social sciences, the refinement of mathematical methods became essential so that they could SHAPE STRUCTURES which are increasingly nuanced; modern mathematics can shape structures and processes which quintessentially belong to the field under discussion (…) a delicate manifestation of the spirit" (Solomon 1974, 17).

The concept emerged out of the need to shape, to QUANTITATIVELY MEASURE what is fuzzy, unclear, imprecise. Mathematics supplies much older elements related to these concepts than the fuzzy sets theory, "which deals with the study of RANDOM PROCESSES: probability theory (the concept of measure), mathematical statistics, information theory and other" (Negoiță 1974, 7). Fuzzy mathematics applies to the humanities (social and behavioural sciences, in matters concerning the systems control - The musical discourse is also a reality to which the concept can be applied), "reflecting the degree of IMPRECISION OF SUBJECTIVITY of our knowledge" (Negoiță 1974, 9).

"For a fuzzy set there is no transition from membership to non-membership for an element in this set. (…) The model of the fuzzy set theory is the CONTINUOUS LOGIC, unlike the classical theory (which is based on BIVALENT LOGIC). Basically, any imprecise statement (in the classical sense) can be associated with a fuzzy set" (Negoiță 1974, 8). Fuzzy sets (vague, imprecise, diffuse, weakly defined) can be the terminological roots for the fuzzy language, logic, decision, grammar, fuzzy variables; and the other famous mathematician who dealt with this topic, Y. Gentilhomme, launched the concept of "ensemble flou" (flou set), based

on the valence logic. All of these are connected to the ideas of PROBABILITY and ENTROPY, referring to the generalization of bivalent logic in a universe with multiple valences.

Shaping vague sets AND CONCEPTS is based on MANY-VALUE LOGIC, on the LOGIC OF NUANCED THINKING or on PROBABILISTIC ASPECTS – thus on semantic and artistic concepts that are directly connected to the musical process (the interpretative creator or the understanding of the significance of a work of art). The form/genre (musical structure) most deeply connected to the idea of nuance, probabilistic value or value is THEME AND VARIATIONS, in its older or more modern forms. This aspect can also be analyzed from the point of view of measuring the quantity of the information that is transmitted/modified through variations with respect to the theme. In mathematics, random phenomena (a feature of modern music) and fuzzy ones occur simultaneously within the same problem.

In applied mathematics and in music, we can apply concepts that involve a certain degree of subjectivity, necessary for the manipulation of vague concepts – here is one of the essential qualities of the human as compared to the industrial. Random music uses vague concepts, imbued by subjectivity, and the concepts of RANDOM and VAGUE do not overlap in philosophy or in mathematics: "the random phenomenon derives from the uncertainty concerning the membership or non-membership of an object to a class, while in the case of a fuzzy phenomenon, there are DEGREES OF INTERMEDIATE MEMBERSHIP ranging from full membership to non-membership" (Negoiță, 1974, 9) – this is very similar to the types of variations which exist within the similar traditional genre (ornamental homophone variations have a higher degree of membership to the theme, while character homophone variations have a lower degree of membership to the theme). The theme and variations proposes a multi-variable logic, with multiple valences, in which the variations replicate the various modified versions of the theme from different perspectives.

There are multiple applications of fuzzy sets: in systems theory, the concept is very important from the point of view of systems control. In music, we could draw a parallel with the various stylistic algorithms that suffer changes in the variations belonging to the genre under discussion. Another important idea that can relate to the archetype through which a composer creates variations of a theme refers to the fuzzy automata, which can generate A SERIES OF EVENTS requiring a closure, a rhythm, a final punctuation. In decision theory the concepts of fuzzy or random apply to "making decisions through decision processes in imprecise conditions" (Negoiță 1974, 12) – elements that can be managed only through the control of physical systems.

3. Aurel Stroe - Oresteia II Opera (7. Variazioni)

Semantics proposes, from the point of view of the terms analyzed in this paper, a type of quantitative analysis of something that we think cannot, apparently, have such a dimension: the sense, the meaning. One of the great examples in contemporary Romanian music is Theme and variations (actually, only VARIAZIONI) from Opera Oresteia II (Choephori) by Aurel Stroe. Representative for the 80s, the work is part of the "Oresteia" trilogy by Greek playwright Aeschylus, which consists of three connected plays: "Agamemnon", "Choephori" and "Eumenides". Set in the 5th century BC, Oresteia brings to the fore the vicissitudes of the House of Atreus, from Agamemnon being killed by his wife (Clytemnestra) to the subsequent revenge planned by their son, Orestes, and its consequences.

In the second part of the trilogy, Aurel Stroe proposes for number 7 - VARIAZIONI, a variation idea without a theme (very rare in the history of classical music): within this segment the choir shouts revenge in a musical piece played molto p, sotto voce, with the accompaniment of two percussion instruments, on a untampered six-note scale. The variational element is subtly outlined in this situation, the genre not being approached on a traditional level. But we can't help but notice that the theme is missing, so the variational control is much more modernly exercised.

Aurel Stroe, Oresteia II, Variazioni, I

The choir chanting (variations 1, 3, 5, 7) is interrupted three times by Electra's brutal intervention (variations 2, 4, 6: voix de poitrine, parlando rubato, subito ff, intensive). In variation 8 Orestes intervenes (libero, drammatico: invoking the father). Variation 8 also includes the trombone (playing a special soloist quasi-vocal role in Oresteia), which is present throughout variation 9 (the final one) as well.

Aurel Stroe, Oresteia II, Variazioni, VIII

4. Conclusions

Fuzzy mathematics represents "the degree of limitation, vagueness of our knowledge" (Negoiță 1974, 12), and themes with variations have great adherence to the stylistics of modern discourse, precisely because of the freedom it offers the composer to model different realities starting from at a fixed point, a landmark. Precisely the ambiguity (Iandoli 2009, 37) assumed by the unfolding of variations (especially those of character) is likely to bring notions closer to the field of fuzzy thinking. I have chosen the example taken from the major creation of the 20th century precisely to illustrate the freer conception of the genre.

The mathematics of fuzzy sets is a field that could provide multiple possibilities to investigate musical discourse from a modern perspective, based on technical-scientific concepts. Thus, musical analysis gains in objectivity, in a scientific, rationally regulated spirit.

References

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