

Anatol Vieru and his theory. A continuum of growing up in musical structuring

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The composer Anatol Vieru intuited the extraordinary force of the concept of symmetry. This attracted him to mathematics. A mathematician, Dan Vuza, articulated the form in which the domain deepened by Vieru can turn into a theoretical development shaped by structural mathematics. This study tries to catch what a contemporary theory can do/undo by its forms, its rule of elaboration. The Book of Modes is the compendium, in which Vieru develops his theory. Touching upon sensitive points which we might sometimes miss from the referential frames through a clock-type lecture – or through a reflex-wisely maintained lecture with things familiar to a musician – ,but which (often even just themselves) can convey meaning, solidly arrange the aggregation elements of that respective theory is the main reason that justifies this study. The merit of Vieru’s theory is that it operates organically between the streaks of the empirical and scientific (with a pronounced tendency towards the scientific). It will show along the way that this theory is neither a commonplace understanding, nor does it educate a musician; it “initiates” him.

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1. Introduction

Anatol Vieru’s theory on the modes can be added to the revolutionary music theories of the 20th century about a specific, carefully delimited domain. If we consider it (rightfully so...!) a macro-theory – as it involves a set of sub-theories outside music– , we will see that what it postulates develops into a variety worth considering; we might represent its surface as a series of reference points, as a graph, for instance, where the nodes are the targets (results) it will have to reach, while the arches indicate the route (the process from the operation of which correlations arise, the results are configured up to the moment of reaching them). The organization system of European music never carefully focuses the parameters, of which it should result that the sound-interval unit involves a bi-functionality; it mirrors a modal medium, of course, but not a systematics of construction units for a

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modal scale. Perhaps this organisation is similar to the bi-functionality of the signification in mathematics: equipped with a certain type of operations and relations, *computational* (traditional) mathematics comes with its function; the *structural* (more recent) one has an entirely different function. Structural mathematics highlights the abstract properties of the structures of numerical sets; it is a model for describing any structure, “any non-empty set, equipped with relations and operations, that complies with the axioms” (Vieru, 1978). The terms through which the concept of *symmetry* is attributed a multitude of meanings are, I think, one of the best examples this modern mathematics suggests; its *implicit* definition makes it go on a new circuit – even if this remains conformed on the same law of existence. Due to axiomatic, the representations of symmetry are parted from the eye, the landscape, their natural framework, and move completely to the human mind, into the abstract, in order to be unfolded, *naturalized* there, as well. Continuing the analogies, we notice that the theory of groups of Evariste Galois unfolds the manifestation of the extensions of the symmetry notion in the abstract in all its plenitude, without sacrificing the experience of Euclidian geometry in which it is born. This *formal* axiomatisation can only emerge at the endeavor altitude of advanced, *structural* mathematics, with fruitful endeavours in extended algebraic applications, like the group ones. Even if it seemed of less practical interest as compared to arithmetic, that elementary mathematics (computational, of numbers), structural mathematics has gained and is still gaining ground through the force of its modelling, simulation capacity. Omnipresent, the cohabitation of the two types of mathematics is present not only in the sciences, but also in the arts, in music. From structural mathematics, relations and structures are received, with the connections specific to them, it creates the conditions for passing from one theory to another; from computational mathematics, the authority of figure/numerical relations is received, as it exploits those primary levels of abstractisation. Vieru appeals to structural mathematics (including its specifications which pertain to axiomatic), even if a certain type of calculation, of protocol pointed at the game of figures/operations detects logical activities from the sphere of computational mathematics. The modal dimension articulated by the Romanian composer does not substitute the old morphologies to the new ones; thus, one lingers sufficiently in a certain topos, in the theory of invariants and structural auto-morphisms. We will probably succeed in successfully unveil a comprehensive class of theories which reverberate in the advanced composing practice; it is a special class, with loose edges, especially assembled in order to articulate new concepts and working tools; practically, we are dealing with a technology of reflecting on something...

This study fragmentarily synthetizes what a theory can do and undo through its systemic forms of elaboration. Touching upon sensitive points which we might sometimes miss from the referential frames through a *clock*-type or a reflex-wisely

maintained lecture with things familiar to a musician (or emerged as a consequence of the empirical relationship music-mathematics/ geometry/ architecture), but which (often even just themselves) can convey meaning, solidly arrange the aggregation elements of that respective theory, is also a challenge. It will show along the way that this theory is neither a commonplace understanding (as it will allow no square conduct even for the most naïve aspects it assembles in order to make them known *in a different manner*); at the same time, it is not intended to educate a musician, but *initiate* him. Generally, musical education tells one *how* to do things, how to take an operation scheme for granted (see the habits of the school institution), while the *initiation* sets one's *why*, un-limits one's perspective. Music has been and still will be lived in without initiation, as well, so also without this theory. Precisely because it is not meant for the education system, for seasonal clothing, this theory has great expectations; it lights other lamps in the reflecting conscience. Speaking to *that one* who acquires human understanding as an action of continuously growing up, individual modelling in a universe of perception refinement and abstractions, it opens the gate to *new attentions* and new habits of outlining the reality of sound.

2. The Discourse of the theory of modes as a suggestion for a higher understanding of music

To be able to understand the variety and axiological co-ordinates this theory develops, it is good to know that this is an intertwining of imagination, structuring and axiomatisation. Just like a scientific paradigm is articulated and imposes itself through the collaboration of several researchers, Vieru's theoretical thinking could also gauge its consistence through a nucleus aired by (also other) awesome scientists (see further on); gaining its canonical form through massive structuring efforts, his theory can be introduced into the mosaic of the script edifices which distinctive personalities in the history of 20th century music, like Heinrich Schenker, Arnold Schönberg, Olivier Messiaen, Paul Hindemith and others, developed in the European culture. When we say the theory of modes signed "Anatol Vieru", we actually refer to the group of scientists Vieru-Vuza-Marcus. It is highly important that we holistically understand the dynamic relationship that generated the real stimulant of the principles presented in the compendium *Book of Modes*. An outstanding personality in the world of contemporary music, Vieru is the composer with a respectable mathematical culture; he offered the first fruit to researching the modal world. Dan Tudor Vuza is a distinguished mathematician with a solid musical culture; he articulated the foundation of Vieru's theory of modes in order to raise it

to a high degree of scientific approach; and the mathematician Solomon Marcus, a member of the Academy – an encyclopaedic figure in the Romanian scientific environment, a reference-name that thrones through competence today (at his venerable age of 90) in order to formulate even *more* the complex design of the relationship art-science, mathematics-poetics-linguistics, mathematics and arts (music) – ,designed a large set of conclusions, a space of *results* (very coherent in itself), of which we deduct what exceptional investigative power in the music-mathematics modelling Vieru's theory has and which is Dan Vuza's massive contribution to broadening the concept of *mode*, of proving the valences of modal structuring, and putting it in a special perspective (Marcus, 1986). The feature of a hastily presented theory (also signalled by me further on in order to explain some hiatuses at the level of its enunciation) is explainable from a few points of view:

1. Through co-operation of the mathematical model with the musical ones, Vieru, as a composer, only gave a functioning form to the theory, but without focusing on systematisation, on the levers through which it can be naturally received; instead of contributing to the clarification process of his painting's strokes, he gave it an *impressionist* touch by de-naturalising it.
2. Even if Vuza attached a more unitary expression to it– when Vieru presented him the first version of the book (with the results of his findings) – , we will notice that it remains perceivably addressed and oriented towards the mathematician-musician, not the regular musician. Nobody thought of the musician who has never come across the *Calculus with finite differences in Modulo 12*, *Newton's binomial*, the operations of *passing to the complementary* or *Golomb's formula*, nor the *Modal Rows*, in any situation of his experience. So that, instead of becoming a means of habituation, this theory becomes a form of *de-familiarisation*.
3. Solomon Marcus carefully extracts the frames specific to the theory and offers us a far more didactic discourse as compared to what we would have expected from a mathematician. Without neglecting the connections between grammar/ linguistics and music, he finds congruent applications (therefore, analogies, too) for them, bringing many of the abstruse theorems manipulated by Vieru back to normal. Even if he does not get entirely close to the concreteness of the *Book of Modes* (Marcus, 1986), he analyses the relations *mode-set* and *group-structure* by connecting them to structural mathematics and showing what renders the property of the interval structure to work as an algebraic *group*. Thus, Marcus persuasively outlines the idea of Vuza according to which the entire ramification of the theory can be understood and organised based on the *symmetry* concept.

2.1. The Intricate relationship mode-modal structure

The right of the discussed theory to *gain ground* was won when the compact medium of sound called *mode* (as a scale/succession of music tones) was understood as a bi-functional reality: firstly, as a set of sounds with its well defined function within the scale (1), and secondly, as a distance-fixed order of these tones, as an organisation given by the structuring on intervals with its well defined function, too(2). Analysing the behaviour of each of the two different functions of the *mode* as a scale – both synchronically and diachronically (by combining forms, their intricate features) – , Vieru dedicates them separate theories; highlights their particular features, presents their differences, re-unites them through links distinct from those of the modal experience experimented by his predecessors (via Olivier Messiaen). Incorporating the two signification stripes after each of these was well understood, bringing close what had (intentionally) been decomposed into a *modal* theoretic nucleus, the author directs our concerns to the medium of the new modelling trends, forming our eye for a higher understanding; he puts in front of us the perfect graph of a model for establishing sounds both as a deductive operating system in the midst of the subject-matter of music, and as a variety of morphological distributions/flexions re-united in this language.

Perhaps the most important element of the endeavour is the inspired manner this decomposing was made in. The reality we come in contact with is the intricate relationship music tone-interval, intricacy that does not let the symmetry structure, induced by the action of the group of intervals on the set of sounds, shine through. The cut Vieru perform in the junction tone-interval is especially prolific, as it allows us to understand the potentialities of the complex modal structure by recomposing the tones with the intervals; more precisely, this action of the interval group on the tone set takes place by transporting the group structure of the intervals on the tones.

The characteristics of the scales called *modes* – seen as stigmatisations of the infinite of potential melodies – acquired extra constructive valences with each historical epoch, and their bordering character was ever more suppressed (in this sense we will see the morphism produced from octave-based to non-octave-based structures). In the entire alteration process of its initial construction scheme, the modal mechanism could either adjust to an empirical conditioning (I think of its gregarious use) or to an assumedly scientific experience of sound (in which its identity is shaped by mathematics, by the terms of logically reasoned relations and functions). The merit of Vieru's theory is that it operates between the streaks of the empirical and scientific, with a more pronounced tendency towards the scientific. The attempt to place the modal mechanism even more firmly on the block of mathematic vocabulary – in order to ensure its mobility and a credible interface of formalisation – is linked to Vuza's competence. He broadens its potential,

strengthens its conditions of existence by concluding that „1). The structures of the modal world highlighted by A. Vieru’s theory can be well shaped by mathematical structures. 2). The modal world phenomena stressed by this theory can be explained by mathematical methods. 3). Certain laws governing the phenomena of the modal world can acquire, if they are transposed into an adequate mathematical framework, an aspect of theorem and can therefore allow for demonstrations in the mathematical sense.”(Vuza, 1985).

The shift of interest from the use of modes as mummified historical contingencies (indifferent to any other pattern of description), therefore only fixed on constant relations between their component links (see the aspect of modulation – in its Orthodox sense), could induce new testimonies on their nature, which also suggested the potential of fresh conditions for morphological association in the subject-matter of music.

2.2. The Relationship of modes and structural numbers with the 12 numbering base

A factor that can be perceived as *common* to the two aforementioned interpretations (empirical and scientific) regarding the modal world is represented by considering a numbering base in which all scales cycled (by repetitiveness), their material being (totally) protected from absolute diversity. The 10 numbering base probably was the first version, with the most general character (having the age of that *homo ludens* – who kept counting the fingers of his own hands). Over time, the numbering bases evolved and acquired new loops (base 4 = the cycle of the seasons, base 7 = the cycle of the days of the week, base 365 = the conjugation cycle of calendar years), and the frequency of natural phenomena (that had to be counted and organised) imposed new out (markings) between them or their components. I am not sure whether the model of planets was the symbolic base for the numbering system in the base 8 of the old natural music scales, but the number 12 certainly had an aura of significations which the cogito undoubtedly absorbed, making room for the numbering base <12>. The European macro-idiom repeated the tones of the octave-based scale every 12 tones ever since the time of Johann Sebastian Bach (1685-1750); when one reached by counting the double of the frequency of the starting tone, one marked the input of a new sequence on the same tones (I refer to keeping their names regardless of the octave), but on doubled frequencies. The ample support of the octave (Modulo 12) – dominating the scale as a whole – is divided in a reductionist fashion, into equidistant intervals (of equal semitones). The division of the European octave (a division of efficiency) appeals to a tempered, unambiguous, and constantly repeatable chromatic; it is the one that could generate a culture of the noted sign, of the score. Another octave support, penetrating into the musical culture by un-tempered divisions with different cardinals, will propose other modules

between its borders (than that predestined to the European octave); this allows for scales formed on the principle of duality, awakening the curiosity of reception through their well delimited semantics. It registers other rows of units of other, unequally tempered interval types (by commenting through sound musical cultures far from the European space). This second octave support controls the musical perception from within; even if it constantly maintains the identity of its borders, it remains attached to an octave of introspection, marking the path of a spoken culture. Therefore, the octave support is defining in both cultures (of efficiency and introspection); its elasticisation and the deformation of the units which impose its morphology push it to dilate its framework; thus, the extremities of the octave-based support of the scale x (with the input frequency $\langle \text{start} = a, \text{and end limit} = 2a \rangle$) are portions of sound that roll their material in musical cultures everywhere. The point of attraction “octave” remains the place where all imagined and imaginable sound “terminations” start and end. That is why Chapter 1 of the *Book of Modes* dedicates a significant perimeter to the concept of whole number connected to the octave (Z_{12}). Then, in this chapter the special number $\langle 12 \rangle$ is reached, as a module generator in both spheres of the modal environment: both for mode/set, and for modal structure/intervals; through the numbering regime focusing on the classes of rests in modulo 12, the efficiency of (additive and multiplicative) operations between scales is maximised, these defining its *limit* function in the equally tempered system.

3. The “Elasticity” of Vieru’s exposé – Generating misunderstandings/ambiguities

We also have some reserves related to the strict meaning of the notions which aggregate said theory step by step. As we will see further on, there are elements in its corpus, which emanate clumsiness in their comments, gliding into ambiguity. However, these can be assimilated, if the terms of the author’s/ authors’ language are taken word for word (to study them very carefully). Of course, we are dealing with a musician’s language, first of all – that of composer Vieru. If we read his writings, we notice that we stand before a qualified craftsman of words. *Words about Sounds* is another book giftedly written by Anatol Vieru, with deep thinking. The form he projects through the logos in the *Book of Modes* copes (even if with complications or elisions in the flow of transmitting the *definendum*) with the theory’s complexity. However, we must say here that the approach of transmitting, organising its deployment levels, induces the sensation of *haste* in explaining the categories of notions; this very state of facts (sometimes) makes the reflection of convention in forms of understanding unavailable (reflection that would give the receptor the opportunity to “produce the theory again” (Popescu, 1978). More than 200 pages long, his theory seems like a logbook-theory (a part from *another* whole)

or a transcribed plea of a public conference; we may admit it is special, but the putting of its data on the pages has blanks left; be this mechanism a consciously assumed application in order to create an *open* space for its continuation?...This question is difficult to answer. Dan Vuza did not specify either why the theoretical exposés were integrated without (truly) ensuring the resistance of the material through which they subsist... The question is asked by me, a musician's voice, who considers that for ensuring the optimum intelligibility framework, the rapport for defining the starting terms must be (slightly) modified. In the following I bring a few examples hereto.

3.1. *Composition versus Transposition*

One of the most extensive parts of the *Book of Modes*, Chapter 4, treats the transposition as a basic technique in assembling the modal material. When Vieru attributes the features of the *composition* operation in mathematics (of a modal structure *with a sound*, of a modal structure *with a mode* or of *two* modal structures (Vieru, 1980) to the transposition procedure (so *handy* to musicians through their daily instrumental practice), defining the *composition* of these objects – at least as he presents it – is directed towards a partly complete discernment, at least at first sight. After he establishes the surjective correspondence between a mode and its modal structure – visible due to the mentioned de-coupling of the row of sound-elements (as a set) from its associated row of intervals (structure of distances) –, the author defines (starting from chapter 4.21.) the totality of transpositions <as a set of compositions> in the mathematical sense. As a basic consequence of composition, the author also introduces the term *position* of a modal structure (see BM, chap. 4.22.), term that can be validated in the referential system of musicians as a transposition (translation, movement) on any tone (in our case, from the set Z_{12}) of a mode $\{M\}$ starting from a reference-tone $\langle x \rangle$.

I will further discuss – considering the operation of *composition* in mathematics – the proof of the real relation for defining the *transposition* through the operation of composition. From a reference-tone $\langle C \rangle$ and the mode $M = \{0, 3, 6, 7, 11\}$, the transposition will start successively, and the starting tone of each transposition $\langle t \rangle$ is fixed for the structure (SM) in $\{M\}$. The application design starting from t_0 – which has the tone $\langle C \rangle$ as a reference – has the form $t_0 \rightarrow t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4 \rightarrow t_5 \rightarrow t_6 \rightarrow t_7 \rightarrow t_8 \rightarrow t_9 \rightarrow t_{10} \rightarrow t_{11}$ (Example 1).

Example 1:

t_0 M = {0 3 6 7 11}	t_1 M = {1 4 7 8 0}	t_2 M = {2 5 8 9 1}
t_3 M = {3 6 9 10 2}	t_4 M = {4 7 10 11 3}	t_5 M = {5 8 11 0 4}
t_6 M = {6 9 0 1 5}	t_7 M = {7 10 1 2 6}	t_8 M = {8 11 2 3 7}
t_9 M = {9 0 3 4 8}	t_{10} M = {10 1 4 5 9}	t_{11} M = {11 2 5 6 10}

We find above the entire transposition plan of the mode $M = \{0, 3, 6, 7, 11\}$. The invariant that permanently accompanies the transposition plan is the modal structure MS, which we define a little later. In mathematics, *composition* is governed by a unique condition: relationing; we speak of the existence of the relation established between two functions f and g (for instance), the property of which is that the co-domain of function f is included in the domain of function g ; the result of this composition launches a new function, called <the *composition* of functions f and g >. In such a *composition*, the property that to any element x in the domain of function f , an element (and only one) from the co-domain of function g is associated, will be understood as obligatory; the mathematical encoding of this relationing will have the following form:

$$f(x) = y \text{ and } g(y) = z, \text{ so } g(f(x)) = g(y) = z; \text{ so } (g \square f)(x) = z;$$

where $\langle \square \rangle$ is the symbol of the *composition* function

The actual framework in which *composition* occurs is (as we can see) rigorously prescribed; we will also note at a certain moment in our explanation that the element x (from the domain of function f) actually represents the first tone of each transposition; and the element in the co-domain of function g is the first tone of the modal structure the application is made on (MS = invariant structure); however, I believe that what the musician understands by deduction and can think of as a transposition can be explained only in part through the composition of the functions

$f(x) = y$ and $g(y) = z$. Let us see if the logic of this (particular) function creates a plausible framework for the musical morphologic medium and, likewise, at what obvious aspects is Vieru's reasoning directed when he speaks of transposition in relation to *composition*. For now, we notice that the author represents transposition through a formal frame, trying to find, through the mathematical procedure, a comprehensibility that is "left aside (somewhere)", *abandoned* by musicians in general.

Actually, we must distinguish here the features of a mode once again. The mode (as the elementary tradition of music theory defines it) is a scale; but – at a closer look of sonorous determination – it is also the result of putting a set of tones into a structure (intervallic model). Until they are compelled to the order of a scale/mode, the tones only exist as an amorphous set. According to the action property of the *algebraic group* structure on the set of sounds, "[...] even a primitive entity, as the mode is, is no longer seen numerically, but structurally, because it is represented by a set of classes of rests, so as the result of a relation of equivalence, and is integrated into a corresponding group structure." (Marthe, 1988). The group of intervals is the one that can constraint the tones – by impressing them a fixed order – to become elements in a structure: the *structure* of the mode; (we have access to identify the group of intervals through what we call *structural number* a little later). Here we deal with that transgression from inferior to superior, from the ludicrous set of tones to structuring. In the example above, $M = \{0, 3, 6, 7, 11\}$ has the interval structure $MS = (3, 3, 1, 4, 1)$; without the structure $(3, 3, 1, 4, 1)$ the tones of set M cannot become a mode; the structure dictates a priori *what might be* a mode. Given this condition, the mode, as a tank of elements, will be vested with the properties of the interval structure in order to be able to be given a norm within a profile: in *modulo* 12. Therefore, starting from any tone, a mode can be built on the scaffolding of structure $(MS)_{(x)}$. We shall agree on this: the structure itself does not "copy" the scaffolding of intervals by "looking" at the mode's tones (at any aspect through which these would suggest a priori any melodic reality), but the mode is configured as to its design by attaching the set of tones on a predestined scaffolding (MS) . The attribution function $f(x) = y$, that is the correspondence of an interval with the music tone, will generate a mode $\{M\}$ from the invariant pattern (MS) . We will see (by further investigating the mathematical formalisation) that the respective function is no function, but a functor; and that Vieru uses, I think, the term of composing a modal structure with a mode in a partly relevant manner; as, in the collaboration interval-tone that is established at the first step of transposition, there is not only the attribution relation present; but the discussion on this aspect will follow shortly. When put in a *temporal succession* (placed in the rolling process of music), $\{M\}$ is taken as a start material [see(t_0)] for the future (possible) translations (transpositions). Moving $\{M\}$ firstly on the first, then on other starting points (tones) (different from those of t_0) – on the same scaffolding (MS) – is a first (and among

the simplest) transformation of $\{M\}$, and musicians call it (almost viscerally, without thinking too much about it) <the transposition x_n of $\{M\}$ >. It is a transformation by *symmetry*, as each element of the structuring in t_0 is reflected a corresponding point (tone) on the scale obtained by transposition (t_n). Only the logic of form, accepted a priori, and the articulation time of music pretends a refreshment of t_0 , involving its transposition/translation; the refreshment implies the (partial) separation from the tones of t_0 in order to encounter others: in t_1 or in t_2 or in ... t_{11} – yet on one and the same modal structure (see in the next example, (2), defining the *modal class*). If a mode and its modal structure involve a bi-univocal association (resulting t_0), in the transposition as a procedure, both a t_0 (with its specific association between the elements of the set and their structure) and t_n (which wholly mirrors the way of *being*, the structuring of t_0) are simultaneously involved. Displacing the structure with the start from any t_0 to another t (a new transposition) – in the case of maintaining some common elements (no matter how many) – will make room for the *modulation* concept (likewise a commonplace of musical practice). The respective displacement of a modal nucleus (read: *modulation*) cannot be imagined outside the process of a composing strategy (of creating space for the forms of music), that is a collaboration process of modal zones/ areas, which are, in their turn, the segments that fulfil a certain modal plan with several stations (transpositions of t) – a far more general plan, that is. Usually, transpositions convey the musical configuration a cumulative character, being also applied by leaps and bounds in its space.

In example 2 one can notice how a mode $\{M\}$ is generated on the scaffolding of an invariant structure (MS). By generating the entire package of modes (starting canonically with each tone of the set Z_{12}), a *modal class* (that is, the complete package of successive transpositions of a reference-mode (t_0) on the structure x , considered a priori) will be obtained. So that $M=\{0, 5, 6, 7\}$ results from the structure $MS=(5, 1, 1, 5)$; $M(t_0)$ is the starting mode of the package. The other expressions follow: $M(t_1), M(t_2), \dots, M(t_{11})$ (Example 2).

Example 2:

$M(t_0)=\{0,5,6,7\} \square (5,1,1,5);$ $M(t_1)=\{1,6,7,8\} \square (5,1,1,5);$
 $M(t_2)=\{2,7,8,9\} \square (5,1,1,5);$
 $M(t_3)=\{3,8,9,10\} \square (5,1,1,5);$ $M(t_4)=\{4,9,10,11\} \square (5,1,1,5);$
 $M(t_5)=\{5,10,11,0\} \square (5,1,1,5);$
 $M(t_6)=\{6,11,0,1\} \square (5,1,1,5);$ $M(t_7)=\{7,0,1,2\} \square (5,1,1,5);$
 $M(t_8)=\{8,1,2,3\} \square (5,1,1,5);$
 $M(t_9)=\{9,2,3,4\} \square (5,1,1,5);$ $M(t_{10})=\{10,3,4,5\} \square (5,1,1,5);$
 $M(t_{11})=\{11,4,5,6\} \square (5,1,1,5);$

At this nodal point the author does not make us understand that he would notice and that, in order to be able to build the mode (= amorphous set) on a structure (until the end), the mechanism of *transposition* (read: *composition*) must be succeeded by yet another operation. Transposition = actually two successive operations (see the approach further on). We are summoned to intervene here in order to conclude that, without observing *ad litteram* the steps required by the chain of the transposition applications, we will not succeed in distinguishing between a *proper* transposition (well explained) and a *suggested* one (incompletely built). By first applying the function of composition on the structure (without coming back to the actual procedure in detail), Vieru does not proceed to complete the *composition* with the next link in the chain of steps involved by the transposition. We will realise now that it is not the composition function, but a new operation, the *functorial generation* (not expressly defined in the *Book of Modi*), the one that (fully) closes the chain of the procedure the explanation of which is intended. To render the types of operations easy to understand for mathematicians (and hard to discover for musicians) more transparent, it is necessary to go back one step and start a digression (of course, controlled) on the sense of transposition identified in the work of A. Vieru.

It has been said earlier that the *function of composition* generates a third structural object (mathematically speaking) by setting two other structural objects in relation to each other. The two objects that may be envisaged can only be the set $M(t_0)$ of elements/tones (which will generate the mode— by conjugation/composition with its *invariant* structure), and the starting tone (every time a different one, respectively $t_0, t_1, t_2, \dots, t_{11}$) in Z_{12} . Therefore, what will be transformed through the transposition procedure are the structural objects called *mode* respectively *starting tone*, and not at all the *structure*— as no transpositions can be imagined that escalate Z_{12} on a “non-structure”. *Functorial generation* is, therefore, the formula that introduces change for the transposition; *generation* is that through which the mode is built on the scaffolding of the structure. The composition will only occur in the first moment of transposition, when the starting tone of the structure MS will be combined (associated) with the starting tone of the transposition t . It is only now, I think, that our opinion can make sense: the transposition (transposing t_0 on t_1) does not (really) appeal to a function, but to a functor. The mode built on the modal structure is created in a functorial manner (the functor being the application that maintains the order), and carefully analysing the structure will clarify the way in which the transposition operation is done (in steps) (and from which point the functorial generation starts).

Regarding the modal structure both from the perspective of the intervals that make it up (which still maintain it within the perimeter of music) and from a perspective of structural mathematics, we deduct that not only the distances between the elements of the set of (initially amorphous) tones define it; it is expressed by a

structural number, an index number which exists unconditionally, without a numbering base; as it only indicates distances, we will call it the *structural number*(n) of the modal structure, as it represents the semitone distances between the elements of the set (tones of the scale). The distances (noted between round parentheses, with Arabic figures) describe 4 properties (which create the conditions of existence for the modal structure). The first property of a structure is that it is expressed by a *structural number* (n).2). All figures of the *structural number* are different than <0>. 3). The sum of the figures of n is obligatorily 12. 4).The *structural number* n is, in some way, a *special number*; we also name it *special* as it refers to a particular rapport: the one with the domain<music>, with those *distances* between the elements that produce music on a grammatical foundation. Here we are not after infirming, nor confirming the hypothesis (and conclusion at the same time) that music is the species, and the modes, its subspecies. Yet, this *special number* n defines the structures/modes of music (that is, the way in which the space is normed in the field of the set Z_{12}). It becomes clear that the strict intervallic rapports within the structure make the structural number to be unique. If the structure is permuted circularly or one of its elements is altered, a different structural number is obtained (Example 3).

Example 3:



[Vieru, CM, p. 129]

Likewise, any tone is unique; the structure, starting from that tone, will generate the mode, which is also unique. The start is made by composing the starting tone with the structure. Only this combination is a composition. After the respective composition, the other elements of the scale are generated (as specified before). Precisely due to the operation of [composition + generation], a unique structure involves several modes (the modes belong to the modal class, individualised by an invariant structure); in Z_{12} the transposition implies 12 modes, as each tone represents a starting point [$t_0 \rightarrow \dots t_{11}$]).

Concluding, M_{t_0} also is “a composition of a modal structure (MS) with a sound $\{m\}$ ” (Vieru); yet, the author mentions in chapter 4 of his book only the first step to *transposition* (namely to $\dots \rightarrow t_1 \rightarrow \dots t_{11}$). Actually, the *functor* alone maintains the order between the intervals of the structure that will involve the mode. I am speaking of the functor as a vector, through which generation starts beginning with the first tone of the scale— a reference without which there can be no structuring. The logic of the transposition complexes gauged like this: the functorial generation applies the scaffolding in order to build the mode, interval by interval (scaffolding=intervals connected to the set of tones by the *structural /special*

number). To the extent to which a reasoning alternative to that of Anatol Vieru could also be imagined, we may wonder: if the modal structure is a functor and not a function, is the transposition a *composition*? At its start, it is a *composition*, too (when the first instruction of the algorithm intervenes: <combination>), but further on it is only a functorial generation. The function is (and remains) that application which, based on an operator, transforms a numerical value into a corresponding value ($f(x) = y$ is one of the clearest examples).

Discovering the procedure of transposition required a road along which we could conclude that the composition ([*com=together*] and [*ponere=to put*] according to its etymology) works *stricto sensu* for a formation/ combination of two structural objects, rendering a third structural object as a result. Therefore, the composition is, in the case considered, a simple *putting together*; the logic of this putting<together>is only validated when combining the starting tone (**for** $t_0 \rightarrow t_1 \rightarrow \dots t_{11}$) with the first interval of the structure. Then it disappears to make room for generation. If, for the cause-effect interaction in structuring the modal medium, the function is *quantitative*, the functor is *qualitative* due to the fact that it is a generator (it triggers the quality to *generate* on the structure).

Regarding this conceptual medium – called transposition – closer up, which Vieru explains through the pattern of the composition of functions, we see what can be maintained and what can be rejected by the logic of demonstration. We note that – beyond the fact that the function (in itself) is *no* operation, but only an *address* (understanding by *address* a composition of elements that localise a certain *position*: of an element in a set, of a set of objects, of an operation in a computer, of a street in a town) – , the ensemble of concerns for explaining a notion in music through the pattern of mathematics, formal logic, becomes an extension that is perhaps too precious. In the end of a demonstration spread on entire pages in his compendium, Vieru named these transpositions (of a mode) “positions of the modal structure”, and the genesis of defining apposition as an address (see what t_0 means compared to t_6 or to t_8) had to pass through the argument of the composition of functions. In the wish to absorb, assimilate a concept, do we not lose the “natural” dimension of its existence? Vieru imposes a unity of explanations, and the level on which these are related to each other surpasses the dry constructivism, arouses the associative capacities of the mathematical pattern with the reality of sounds. The composer’s notes are maintained on the ground of facts, which triggers the action of reflecting on the structure, on structuring at every step.

4. Conclusions

Rimbaud’s words: “we have to be modern at all costs” (Marthe, 1988) only suggest an extension of significations for the gallery of values that already exist in the spirit;

applying the fruit occurs *in slow motion* in any domain and for this reason it is difficult to communicate *from the outside* if the rigid solidity of tradition or the fluid modernity have more to say in a culture. The theories, too, rest on the confrontation (in a more or less vehement form) between the levers of tradition and modern ideological movements. Composer Vieru has taken a huge step when, without contesting the outlines of modernity, he treated the history of modes from a broad, permissive perspective of confrontations. The conceptual apparatus chosen by Anatol Vieru gave life to an outstanding theory, and his “leader’s” courage – to formalise some aspects which the musician regards rather “from behind the fence” – only allows for the growth of new axiological links in the tandem of sonorous reality and modelling.

Anatol Vieru registers the avatars of sonorous conventions comprised in *decorating* the historical continuum in a higher manner, conceiving modalism not as an element of stylistic comment of the epoch x, y or z, but as a continuum of growing up in musical structuring. Thus, regarded as an ahistorical reality – which is not *lost* in fractionings of idiomatic time – , the “modal, tonal, serial and again modal” continuum reflects continuously, extensively on the subject-matter of music. Regarding modalism from this angle, it remains an invariant to the transformations of languages – and does not take the appearances (interior decorations) into account, but only the essence. Mathematical principles are the peak from where one regards the essence, the invariance of language in an abstract and very general manner; the language of this continuum of growing up is closely examined: it is a process of alliances, of evolution of scales/structures to horizons of sound. In the compendium *Book of Modes* the mathematical principles become more than ever servants to the object *music*, maintaining a way of thinking which exerts modelling effects up to their maximum action. *Transposition* as an issue appears in Vieru’s work (in chapter 4 of the *Book of Modes*) as a training ground, constantly stirred by an integrating vector (see the extended space for classifications and subordinations given by chapters 1, 2, 3: with numbers/ numbering systems, modes, structures, operations between the elements of a set). The transposition technique does not appear at Vieru as a part of an austere landscape, which is not dealt with anywhere. Transposition comes to be added; it is the node of confluences that satisfies the framework of the composing project, and is meant to maintain the subsequent resources from which the *opus* shall spring. The analysis Vieru undertakes in re-dimensioning this procedure gives a glimpse of new points of attraction and debate in the domain of modes as objects of sound with more referents. Structural mathematics, the theory of groups of symmetry, issues related to time-space and the reception of sonorous data are only some of the notional imperatives Vieru involves in the theory of modes, beyond his conceptual investment regarding the transposition. Dan Vuza’s idea that the Romanian composer’s theory of modes is maintained within the limits of the *symmetry* concept, and all correlated conceptual

ramifications only determine the relation *close-far* from this omnipresent law, will impress to our understanding an orientation on the subtle intellectual performance in the composing strategy. The model of mathematics – as we well know – has led to canons of maturation in many scientific domains; let us not forget its immediate implications in the world of physics and biology. A mathematician, like René Thom was, determined his theory by carefully watching the model of biology, and the risk conditions of demonstrating this theory were nil. It remains that the mathematical model Vieru was concerned with be soon taken into account by *other* musician users, too, as a proof of becoming more responsible and of optimisation; to be part of the live composing process, which operates at its full capacity. I do not believe at all that the theory of music is in a crisis. Vieru's theory is waiting for its supporters, as the trans-linguistic modal pattern remains a possible mark of growing up and carefully reflecting on the sonorous mediality.

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