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Systematic Mathematical thinking reflected in the Semiology of the Musical Language

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Abstract: *Music, the art that expresses the sentiments with the help of sounds, the feelings and mental states, is based on the musical sounds produced by the regular vibrations of the elastic bodies. Mathematics, captures quantitatively or structurally all natural phenomena in precise and elegant formulas and is the "golden key" of all experimental sciences. It is an intellectual discipline, which, by the precision of its formulas and expressions, offers balance, rigor and artistic sensitivity to the beauties of this world. The attempt to learn to decipher the semantics of musical language is based on understanding some concepts of rational thinking. Mathematics uses critical thinking and develops analytical and reasoning skills, processes encountered in approaching musical studies, in order to render the music message as accurately as possible. In turn, the thorough process you have to go through to play a musical creation or learn how to play a musical instrument can improve your mathematical thinking*

Key-words**:** *languages of communication, music, mathematics, intercultural education*

1. **Introduction**

There is a common misconception of the world about mathematics. It is considered cold, it is feared, because they have failed to penetrate the harmony of this discipline, a harmony comparable to that of the temples of the old Ellás.

Mathematics is the music of reason. But what do these two have in common? Both are universal languages of human communication, having the ability to communicate abstract ideas (mathematics) or artistic, emotional (music).

Mathematics is the science of numbers and forms, a science that arose from people's desire to understand and express the world around them. And since sound

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is part of this world, it is no wonder that mathematics can be used to build this harmony of sounds called music.

Great composers intuitively master mathematical rules, be it the duplication or suppression of sounds in agreements, agreement reversals, delays or suspensions, ornaments, cadences, alterations, modulations, imitations, harmonic progressions ... All these are done according to well established rules and meticulously respected by the composers; but these rules mean mathematical calculation.

The fugue in music, that is to say the polyphonic work in which the imitative repetition of one or two subjects takes place after a certain tonal-harmonic plane, so common to Bach and Handel, is also drawn up according to mathematical rules, which the composers must master almost intuitively.

And conversely, mathematicians intuitively master a special hearing. There are many mathematicians who have proven to have very good musical ears. The ones that studied the history of mathematics found that Romanian mathematicians, Gheorghe Ţiţeica, Dimitrie Pompeiu, Traian Lalescu and Petre Sergescu, played the violin, Victor Vâlcovici on flute, Mihail Ghermănescu on cello. All of them were not just amateurs, but very good performers of classical music compositions.

That is why the musical language of the 9th Symphony by Ludwig van Beethoven for a music lover or the mathematical demonstrations of Henri Poincaré, Emile Picard or Gheorghe Ţiţeica for a mathematician deeply moves the human soul. In the artful structure of the successions of the agreements of a musical phrase, with their internal beauty and dynamics, or in the structure of the same nature of the mathematical demonstration, lies all the secret that can provoke the thrill of a great artistic emotion.

2. The mathematical foundations of music and musical creation

During the Greek philosopher Plato (429 - 347 BC), the disciple of Socrates (? - 399 BC) and Aristotle's teacher (384 BC - 322 BC), the word mathematics was referred to the objects taught by the philosophers, amongst which, music was also found.

Pythagoras, 2500 years ago, using an instrument called a single chord - a single vibrating chord, analogous to the sound level meter used today to study chord vibrations – has observed that musical (or spoken) sound is the result of regular vibrations of elastic bodies. He also defined the diatonic sound scale, consisting of perfect words and quartets, the preferred scale in the instrumental music of the Middle Ages, and whose sounds (notes) were subsequently called do, re, mi, fa, sol, la, and, (do 2).

The English, Dutch, Germans and Hungarians designate the eight sounds of the octave, through the letters of the alphabet (literal notation):

do	**re**	**mi**	**fa**	**sol**	**la**	**si**	**do2**
C	**D**	**E**	**F**	**G**	**A**	**H**	**C**

Examining the successive lengths of the single agreement, Pythagoras found that if we consider equal to the unit length of the single-string that produces the sound of the do, the lengths for the other notes are smaller than one, but always expressed by rational numbers, as reports of integers, with the following correspondence:

Sounds	**do 1**	**re 1**	**mi 1**	**fa 1**	**sol 1**	**la 1**	**si 1**	**do 2**
String Lengths	**1**	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

Similarly, it was subsequently demonstrated that the frequencies of sounds are between them all in relation to integer, inverse numbers of reports that give the length of the single-agreement. Thus, if we consider equal the unit number of vibrations per second for the do of Octave I, we will have:

Sounds	**do 1**	**re 1**	**mi 1**	**fa 1**	**sol 1**	**la 1**	**si 1**	**do 2**
Number of vibrations on the string	**1**	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	**2**

A musical range – the height distance between two sounds or two musical notes – can be represented by the amount of the higher musical sound frequency and the frequency of lower sound. This means, expressing itself mathematically, that the logarithm of an interval is equal to the logarithm of the higher note frequency minus the lower frequency logarithm. But as a logarithm can be expressed as an amount of logarithms, so the intervals can be determined as a sum of component ranges.

The unison has this ratio equal to 1 and the octave is characterized by report 2/1, meaning *do 2* and *do 1* from the same octave, have their frequencies in the report 2/1; Thus, between the frequencies of two musical sounds there are reports of integers.

In the natural range, the Hz frequency values of the do sounds in the music ladder are as follows: Do_2 (from Sub-counter-octave) has a frequency of 16.5 Hz, Do_1 (Counter-octave) has a frequency of 33 Hz, Do (large octave) has 66 Hz, do (Small Octave) has the frequency of 132 Hz, and do 1 (from Octave 1) has 264 Hz.

Major third = $\frac{5}{4}$; if *do 1* has 264Hz then *mi 1* has $264\times\frac{5}{4}=$ 330 Hz

The Perfect fourth = $\frac{4}{3}$ and is the vice versa of a perfect fifth *;*

$$\frac{4}{3}=\frac{2}{1}:\frac{3}{2}=\frac{2}{1}\times\frac{2}{3}$$

Therefore, a perfect fourth and a perfect fifth gives us a perfect octave:

$$\frac{4}{3}\times\frac{3}{2}=\frac{2}{1}\Leftrightarrow \quad \lg\frac{4}{3}+\lg\frac{3}{2}=\lg\frac{2}{1}\Leftrightarrow\lg\frac{4}{3}=\lg\frac{2}{1}-\lg\frac{3}{2}$$

The logarithms were discovered by John Neper (1550-1617), so in the school of Pythagoras (sec. VI – V b.C. was not known by the logarithmic link between two intervals, instead they knew that in arithmetic there was the identity $\frac{n+1}{n}=\frac{2n+2}{2n+1}\times\frac{2n+1}{2n}$

This identity applies perfectly also in music. If to *n* we will give the value 1 we obtain $\frac{2}{1}=\frac{4}{3}\times\frac{3}{2}\Leftrightarrow\lg\frac{2}{1}=\lg\frac{4}{3}+\lg\frac{3}{2}$

This means that the perfect octave is composed by a perfect fourth and a perfect fifth. (Andonie, 1969)

The notion of symmetry is defined and theorized since antiquity, where it had a great spread in Greek architecture, and later in the Gothic one. The principles of symmetry can also be met in the creative techniques of all composers.

A typical and convincing example of the Goldberg variations is given by Johann Sebastian Bach (1685-1750).

At the ancient Greeks the significance of the notion of "symmetry" was very different from that of today's notion. So, it was not about a repetition of identical elements on one side and another of an axis, but a "co-modulation" adjusted by a proportion of the whole and the whole.

Repeating a fundamental formula, according to certain rules, gives rise to various types of symmetry.

The first musical incursion of the proportion is applied, it is known by Pythagoras, a vibration string, from which the well-known intervals resulted: $\frac{3}{2}=\frac{9}{6}$ *the fifth mi-si* $\frac{4}{3}=\frac{8}{6}$ *the fourth mi-la* $\frac{2}{1}=\frac{12}{6}$ *the octave mi-mi*

These proportions, identical to the universal proportion, studied on the tuning of a Mi-mi, are identical to the following geometric ratios:

6 – Represents the number of faces of a cube

8 – is the number of the cube peaks

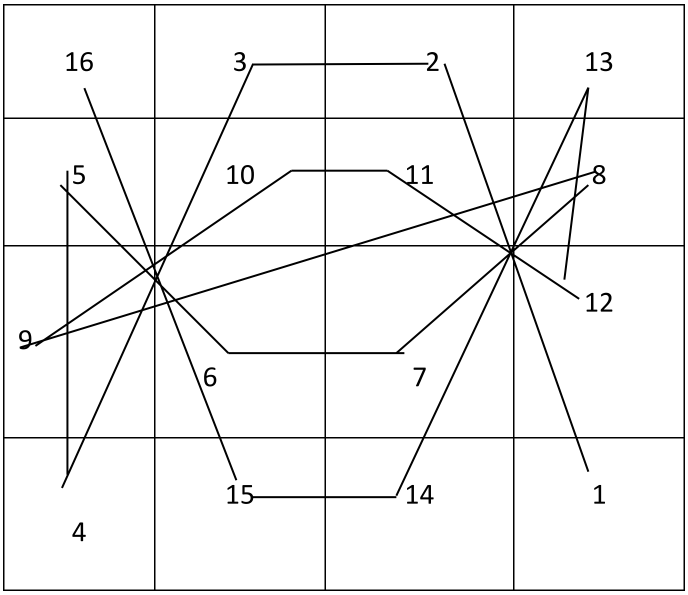
12 – Represents the number of the cube edges

9 – It is the first "masculine number", hence the disturbing conclusion that the interval proportions (length-frequency) are similar to the spatial proportions of geometry.

It is known that in the "New Viennese School" initiated by Arnold Schonberg, Anton Webern was the one who pushed up to an extreme limit the "Microcellular" organization of the 12 Sounds series.

Already in the lyrical suite of Alban Berg – composer who is also part of the "New Viennese School" – the author uses a series in all ranges. At a closer look, we observe a hidden symmetry between the last five sounds, which are nothing but the reversal of the first five around the non-reversible pivot, represented by the reduced fifth. The series is always organized, it is "elastic", by repositioning some sounds and using some cell sections. Sometimes it contains parallel symmetry, with isomorphic figures and non-isomorphic figures.

The whole Viennese School – and especially Webern – was obsessed with the wider possibilities that permutations offer in music, but also in mathematics or even linguistics. The obsession of Durer's magic, a figure of figures whose vertical or diagonal result was always the same, is also felt in many Weberian pages. This line of Durer in *Melancholia* gives rise to the following "magical lines":



The harmony of sounds starts from the harmony of whole numbers. A musical phrase means a string determined by integers, because, as shown above, each sound, each note corresponds to an entire number of Hertz.

And yet you might say that it's not all about respecting the mathematical rules of harmony to create an everlasting music opera. Not every musical phrase arranged by all the rules of composition is also artwork. To really be the work of great value, it is necessary that from the musical composition a strong dynamic and an intrinsic crystalline beauty of the whole and parts, in other words, is a peculiar expressiveness that cannot be translated into words, but it can be understood by the special effect it produces in the souls of the listeners. Mathematics and literary language are not able to explain the message of the problems specific to the music language.

Alongside the composers of Universal Music, many composers approached techniques of musical composition based on mathematical bases. Musicology has also carefully treated all stylistic orientations, of great diversity especially in the twentieth century, aiming to understand and explain the personal stylistic characteristics observed in their creations.

The interference of studies developed by both musicians and mathematicians led to the definition of concepts integrating the two languages with reciprocal effects in the development of approach skills, semantic decoding and interpretation.

The theory of musical agreements today has close ties and the algebra theory of ideals, studied much by the Romanian engineer, George Constantinescu (1881-1965) inventor, who set the basis for the theory of sonicity.

Among Romanian mathematicians concerned about the link between mathematics and music is distinguished Dr. Dan Tudor Vuza (1955) whose passion for music led to the elaboration of new theories of rhythmic structures. The results of his research were published in prestigious International Journals of mathematical Research, and the University of Chicago included in the lessons of musical mathematics a special chapter called *Vuza's rhythmic canons*.

English mathematician James Joseph Sylvester (1814-1897) asks the question: "Could it not be the music characterised as Mathematics of senses and mathematics as music of reason? For the musician senses Mathematics, and the mathematician designs the music. Music is dream, Math life practice!

Starting from the mathematical properties of the music language structure, scientists have built computational algorithms, obtaining computerized programs that transform music into kaleidoscopic images or geometric moving structures.

The use of Fibonacci's string in the techniques of composition of several composers is a well-known fact, and the realization of harmonious proportioned constructions is another example of the application of mathematics in design. (Berger, 1965)

3. Methodical-pedagogical valences of the relationship between music and mathematics

Mathematics is the music of reason, and listening to classical music leads to improved mathematical abilities; In the same sense, mastering elementary notions of mathematics helps in the understanding of musical theory, because the mathematical model that can represent the sounds is given to us by a vector space.

Listening to music involves the human brain in interesting activities, and singing to an instrument simultaneously trains every area of the brain, especially visual, auditory and motor cortex; the activity itself equating with a complete physical training.

The actual chant involves motor skills that are controlled by both hemispheres, combining linguistic and mathematical accuracy, specific to the left hemisphere, with the elements of content and creativity, specific to the right hemisphere.

In the theoretical and creative approach, music can be approached by the intercession of mathematics, giving it a solid foundation of great depth. In support of this idea, computers can be used to mechanize the orchestrations of musical compositions, and in writing programs, the laws of harmony intervene.

In teaching mathematics can be used the concepts of rhythm and measure to highlight the link between multiplication, division and operations with fractions.

Also, ranges and music ranges can be helpful in understanding elementary mathematical notions such as strings, ranges or crowds.

Ioana Steluţa Manea, professor of mathematics at the theoretical High school Emil Racoviţă in Techirghiol, teacher MERITO 2019, testifies that, "at the dorm, in college, classical music was a reliable friend in the study nights and this passion she tries to impregnate now to the pupils. At the more complicated lessons, such as geometry, when she sees that students understand and need only to apply, she pushes play and the calculations begin to sing. She'd like to have multidisciplinary curricula that shows the student the use of mathematics in life — that its important in painting, in music, in architecture, that you find it everywhere." (www.proiectulmerito.ro ioana-steluta-manea)

The number π is a mathematical constant whose value is the ratio between the circumference and the diameter of any circle in a Euclidean space. The fascination of this number has entered the non-math culture. In order for it to be easily memorized, musician David Macdonald managed to transform the mathematical number π into a song by transforming each digit into a note. Thus, the number strings can be easily memorized with the help of music.

Sign systems that underlie music (notations) and mathematics (numbers) can lead to interdisciplinary projects through which mathematics can be used to explain music and vice versa.

European Music Portfolio (EMP) is a Comenius lifelong learning project, a teacher's handbook, which proposes ways to translate sounds into Mathematics, with the aim of using music to improve students' knowledge. Starting from the basics of learning, EMP addresses mathematics and music teachers by detailing the connection between the two study disciplines, demonstrating that music has a mathematical character and mathematics has a musical character. The manual contains activities with contents of the two disciplines. (EMP, 2016)

Contrary to the claims from the beginning of the dissertation that mathematics is a cold discipline and music is an unapproachable language, through an intelligent opening and pedagogical creativity, through the connection of the two languages during the study hours, activities can be carried out that lead to the development of creativity, memory and of intellectual capacity.

References

Andonie, George Ştefan. 1969. *Varia matematica* [Varies mathematics]. Bucureşti: Editura Tineretului.

Berger, Wilhelm Georg. 1965. “Moduri şi proporţii” [Modes and proportions]. *Studii de Muzicologie*, Vol. I, p. 139. București: Editura Muzicală.

Berger, Wilhelm Georg. 1977. “Teoria coralurilor integral cromatice” [The theory of corals completely chromatic]. *Revista MUZICA a Uniunii Compozitorilor din România*, Nr. 12, p. 28. Bucureşti.

Brediceanu, Mihai. 1983. *Le temps modulaire* [Modular time]. *Revista MUZICA a Uniunii Compozitorilor din România*, Nr. 4. Bucureşti.

Ciocan, Dinu. 1973. “*Aplicaţii ale matematicii în cercetarea muzicologică*” [Applications of mathematics in musicology research]. *Studii de Muzicologie*, Vol. IX. București: Editura Muzicală.

Dogaru, Anton. 1978. “Introducere în teoria mulţimilor pentru muzicieni” [Introduction to crowd theory for musicians]. *Revista MUZICA a Uniunii Compozitorilor din România*, Nr. 7. Bucureşti.

Gardner, Howard. 2006. *Inteligențe multiple. Noi orizonturi* [Multiple intelligences. New horizons]. București: Editura Sigma.

Giuleanu, Victor. 1986. *Tratat de teoria muzicii* [Treated by music theory]. Bucureşti: Editura Muzicală.

Mall, Peter, Maria Spychiger, Rose Vogel, Julia Zerlik. 2016. *European Music Portfolio (EMP) – Moduri de transpunere a sunetelor în Matematică* [Modes of sound transposition in Mathematics]. Frankfurt (Main) Universitatea de Muzică și Arte vizuale, Universitatea Goethe, Comenius Lifelong Learning Project.

Misdolea, Tudor. 1979. “Matematica şi muzica” [Mathematics and music]. *Studii de Muzicologie*, Vol. XIV. București: Editura Muzicală.

Urmă, Dem. 1982. *Acustica şi muzica* [Acoustics and music]. Bucureşti: Editura Ştiințifică şi Enciclopedică.

Webography – accessed 10.01.2020
http://www.anulmatematicii.ro
http://aguasonic.com
http://www.scritub.com/
http://likeadrum.wordpress.com
www.proiectulmerito.ro ioana-steluta-manea