

Historical considerations of temperate sound systems

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Abstract: *We instinctively tend to believe that nature has perfectly designed the phenomenon of musical sound, but this is not entirely true. Any sound is not singular, but the result of combining several vibrations, called harmonics. The first new harmonics are the octave and the fifth of the fundamental sound. Starting from this idea, Pythagoras, in one of the oldest sound systems, tried to build a model of thinking and determining the pitch of sounds using fifths. With the second voice (polyphonic singing), another type of sound system had to emerge: the tempered one. By temperament, fifths lose their pureness and become somewhat false. Starting around the 12th century, theorists have tried to find the perfect formula for polyphonic singing. This scientific paper aims to present some tempered sound systems, from the Renaissance and the beginning of the Baroque, which tried to solve the problem of tuning the pitch of the sounds.*

Key-words: *temperament, meantone, tuning, keyboard instruments.*

1. Introduction

Sound temperament is the system that aims to clearly define the distances between sounds. The most commonly used sound system today is the one that divides the octave into 12 equal semitones, which is why it is called “equal-tempered”.

A tempered or untempered sound system is defined by the quality of the fifths, and how they are tempered/adjusted. Tempering a fifth involves altering it compared to its acoustically pure version. The fifth interval in its pure form is, in the case of a string, two-thirds of its length, and can be written using the ratio $3/2$. Most of the time, the sound systems, tempered or untempered, are represented graphically by a succession of fifths arranged in the form of a dodecagon (Figure 1).

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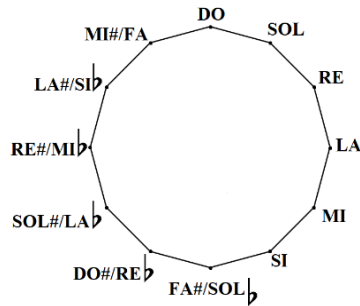


Fig. 1. *The sound systems*

2. Material and methods

In the case of the equal-tempered system, the fifths are tempered and become slightly more diminished than the pure version. If we build a sound system based on pure/untempered fifths, after a succession of 12 fifths, the sound from which the system starts will not be identical to the final one. For example, if the system is built starting from the sound “do”, and we go through 11 pure ascending fifths, the last sound “si#” will not be the sound equivalent of the “do” from the beginning. The “si#” will be higher than the “do” by one Pythagorean comma (figure 2).

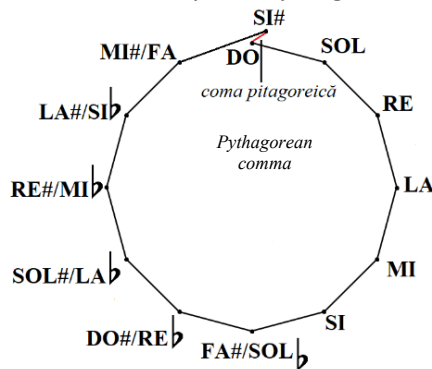


Fig. 2. *The sound systems with si#*

The equal-tempered system solves this “problem” by reducing each fifth with the 12th part of the Pythagorean comma. In this way, each fifth is tempered and becomes impure.

After the fifths, the next category of intervals used to define a sound system is that of major thirds. In the case of the equal-tempered system, they are, like the fifths, impure. Unlike fifths that are tempered by diminution, major thirds are

altered by augmentation. The proportion of the length of a string of a pure third is four-fifths, and is represented by the ratio $5/4$.

The only information regarding the tempered and untempered sound systems from the Middle Ages and the Renaissance comes from the musicological texts of those centuries, but the connection between this theoretical information and the practical part can only be assumed. Until the 15th century, the system most often used for tuning was the Pythagorean one. We use the term “tuning system” because it is not a tempered system. It is untempered, the constituent fifths being pure, and expressed mathematically by the ratio $3/2$. However, major thirds cannot be represented by the ratio $5/4$. In this way, they are impure. The ratio with which major thirds can be expressed in the Pythagorean tuning system is $81/64$. This system consists of a sequence of 12 pure fifths. Unlike the equal-tempered system which can be expressed graphically by a closed dodecagonal shape (see figure 1 above), the twelve Pythagorean fifths are represented graphically by an open shape (figure 3) in which the two extremities do not meet.

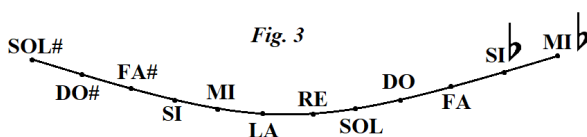


Fig. 4. Closed dodecagonal shape

These two sounds, Sol# and Mib, are not meant to be sung together because in the untempered Pythagorean system they represent an interval impure by augmentation, which was later called in musicological works the “wolf fifth”. The remainder of the fifths are pure, but the major thirds remain impure, being higher than their version mathematically represented by the ratio $5/4$. In some sources, such as *De institutione musica* by Boethius, these impure thirds are called “ditonus”, and are represented by the ratio $81/64$. Most musical works of this period use in their beginning and end pure consonant intervals (octaves and fifths), and the impure consonant intervals are mainly used in cadences to generate the tension that is resolved with the help of pure intervals. Below (figure 4), as an example, is a cadence from the work “Gloria, suscipite Trinitas” by composer Johannes Ciconia.



Fig. 4. Cadence from “Gloria, suscipite Trinitas” by composer Johannes Ciconia.

Starting with the 16th century, during the Renaissance, music and the tuning system underwent changes. The most commonly used sound systems are the meantone ones. The first step is to compromise the fifths in favour of the thirds. In the “extreme” version of the meantone system, called meantone 1/4, major thirds are as purely tuned as possible, but this change involves the diminution of fifths, which become impure. In this tempered system, we find another type of comma, called “syntonic comma” (figure 5).

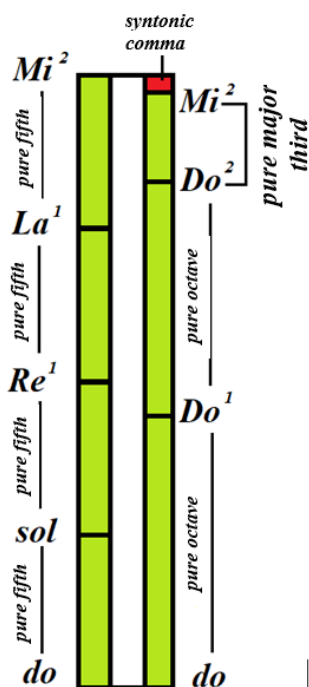


Fig. 5. Syntonic comma

In the image above, four pure fifths are superimposed on the left side on the sound “do”, and, on the opposite side, two pure octaves and a pure major third. The difference between the two superimpositions is highlighted in red, and represents a syntonic comma. In the case of the 1/4 meantone system, each pure fifth is diminished by a quarter of the syntonic comma (figure 6). In this way, the thirds become pure. Among the first sources to describe the 1/4 meantone sound system are those from the end of the 17th century. These are *Lettre touchant le cycle harmonique* published in 1691, and *Oeuvres complètes de Christiaan Huygens, Tome Vingtième, Musique et Mathématique* written by the Dutch mathematician Christiaan Huygens. These works were made possible mainly by the discovery of logarithms by John Napier in 1614 in *Mirifici Logarithmorum Canonis Descriptio*.

There is also the variant of the 1/6 meantone system, in which the thirds are slightly augmented compared to their pure version, and the fifths are tempered only by one sixth of the syntonic comma, but the category of major thirds is favoured (figure7).

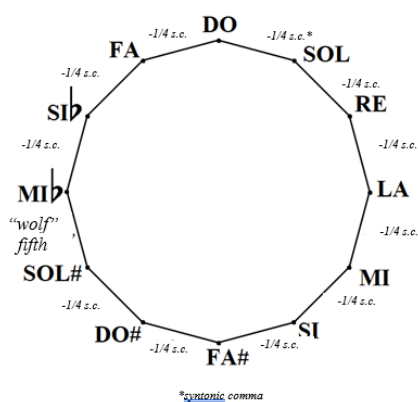


Fig. 6. Quarter-comma meantone temperament

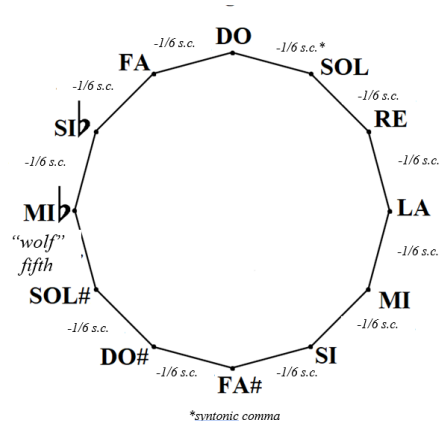


Fig. 7. One-sixth-comma meantone temperament.

Unlike the Middle Ages, when thirds and sixths were considered generators of tension, starting with the Renaissance, these categories of intervals become more important, and are preferred in the beginnings and especially in the cadences of musical works, becoming almost mandatory regardless of the major or minor mode of the piece. This is how the Picardy cadence is born.

The Italian composer and theorist **Francesco Antonio Vallotti** described in his work “Della scienza teorica e practica della moderna musica” a tempered sound system that combines the elements of the Pythagorean system with those of the

1/6 meantone. This system is very often used today for the interpretation of musical works from the Baroque period. In this system, the sounds belonging to the white keys of a keyboard instrument (fa, do, sol, re, la, mi, si) are tuned using the 1/6 meantone pattern, and the sounds positioned on the black keys are tuned using pure Pythagorean fifths (figure 8).

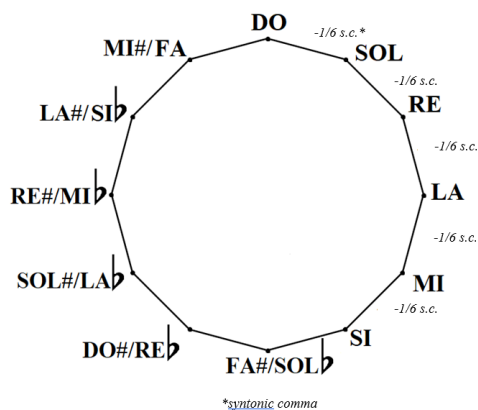


Fig. 8. Vallotti tuning system

Other examples of irregular or uneven sound temperaments:

- The **“Young” system** – is very similar to the Vallotti system, and refers to a pair of circular temperaments (closed shape) described by the scholar Thomas Young in a letter to the Royal Society of London dated 9 July 1799 and read at the meeting of 16 January 1800. The first of Young’s systems, also called Young no. 1 (figure 9), is designed to make the most commonly used tones sound good. The major third of the sound “do”, “mi” is tuned higher by 1/4 of the syntonic comma, and the major third fa# - la# is tuned higher by a full syntonic comma. The fifths “do” – “sol”, “sol” – “re”, “re” – “la”, and “la – mi” are tuned lower by 3/16 of the syntonic comma, and the fifths “fa#” – “do#”, “do#” – “sol#”, “sol#” – “re#” (mib) and “mib” – “sib” are tuned in pure variants. The remaining fifths: “mi” – “si”, “si” – “fa#”, “sib” – “fa”, and “fa” – “do” are tuned lower by 1/12 of the syntonic comma. In the next system, Young no. 2 (figure 10), the fifths “fa#” – “do#”, “do#” – “sol#”, “sol#” – “re#” (mib), “mib” – “sib”, “sib” – “fa, and “fa” – “do” are maintained in the pure version, and the fifths “do” – “sol”, “sol” – “re”, “re” – “la”, “la” – “mi”, “mi” – “si”, and “si” – “fa#” are each tuned lower by 1/6 of a Pythagorean comma. This system, due to its similarities to Vallotti’s, is also called the Vallotti-Young system.

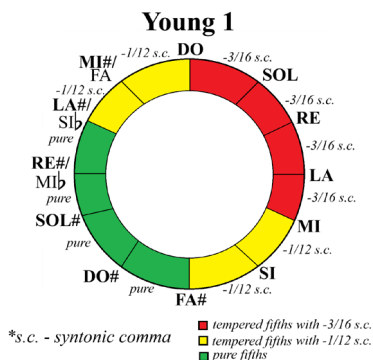


Fig. 9. Young no. 1 system

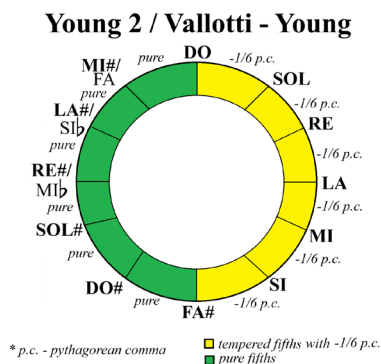


Fig. 10. Young no. 2 system

- The German theorist and composer of the Baroque period, **Andreas Werckmeister**, is one of the first followers of an equal temperament. He designed several tuning systems. The first system (figure 11) mainly uses pure fifths. The only pairs of fifths that are tempered by a 1/4 diminution of the Pythagorean or syntonic comma are: “do” – “sol”, “sol” – “re”, “re” – “la”, and “si” – “fa#”. The wolf fifth does not exist. This system is designed especially for “musica ficta”, which uses chromaticisms. Many of Bach’s works are performed using this type of tuning. In Werckmeister’s second system (figure 12), the fifths “do” – “sol”, “re” – “la”, “mi” – “si”, “fa#” – “do#”, and “sib” – “fa” are tempered by a 1/3 diminution of a comma, and “sol#” – “re#”, and “mib” – “sib” are tempered by a 1/3 augmentation of a comma. The rest of the fifths are pure. This temperament is adapted to the predominantly diatonic musical discourse. The third system (fig. 13) tempers by a 1/4 diminution of a comma the fifths “re” – “la”, “la” – “mi”, “fa#” – “do#”, “do#” – “sol#”, and “fa” – “do”, and the fifth “sol#” – “re#” is augmented by 1/4 of a comma. The remaining pairs of fifths are pure. This tuning system is closest to what is now called “equal-tempered”.

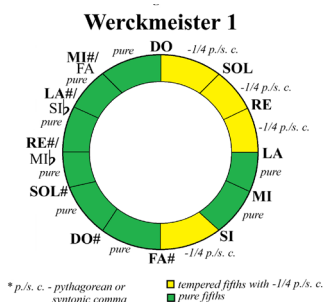


Fig. 11. Werckmeister’s first system

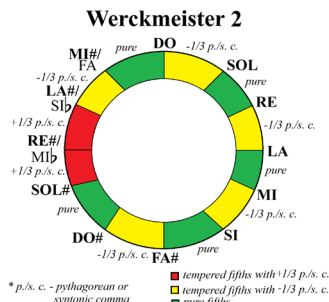


Fig. 12. Werckmeister’s second system

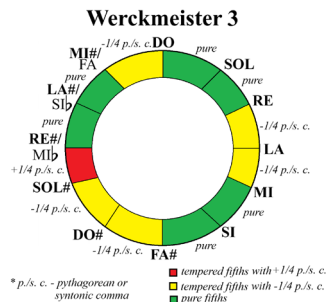


Fig. 13. Werckmeister’s third system

- Along with Werckmeister, **Johann Georg Neidhardt** was another theorist who worked to improve the equal-tempered sound system. Although he was also a composer, he remained in history due to his theoretical-musical research. Neidhardt initially proposed three unequally tempered tuning systems and one equal-tempered tuning system. The latter is based on the division of the Pythagorean comma into 12 equal parts, and their distribution within the circle of fifths (figure 14). Although Neidhardt's equal-tempered system is one of the first attempts to theorise a sound system in which all fifths are equal, an almost perfect system, he continued to calculate and propose more than ten systems. Some of these were designed for certain villages, towns, and for the royal court he recommended the equal-tempered system.

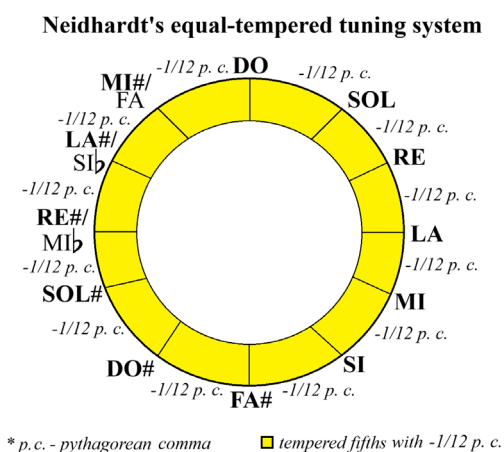


Fig. 14. *Neidhardt's equal-tempered tuning system*

- **Johann Kirnberger**, German composer and theorist, one of Johann Sebastian Bach's students, proposed in his work "Die Kunst des reinen Satzes in der Musik" several types of sound tuning. In Kirnberger I (figure 15), the fifth "re" – "la" is tempered by a 1/4 diminution of the syntonic comma, and can be represented by the ratio 40/27, instead of 3/2. The major thirds "fa" – "la", "do" – "mi", "sol" – "si", and "re" – "fa#" are tuned in their pure version, observing the 5/4 ratio. Kirnberger II (fig. 16) divides the "wolf fifth" into two different pairs of fifths, and then removes 1/2 of a comma from two pairs of pure fifths to make a closed circle of fifths. Kirnberger chooses to temper fifths because, during his time, the major thirds were the priority. In the third type of tuning, Kirnberger III (figure 17), he divides the syntonic comma, and divides it between four pairs of pure fifths. He

retains only one pure major third, “do” – “mi”. This system contains fewer Pythagorean thirds compared to Kirnberger II.

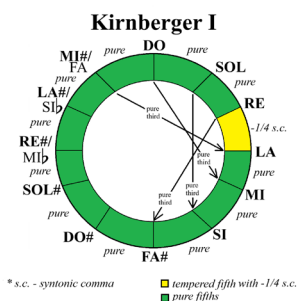


Fig. 15. Kirnberger I

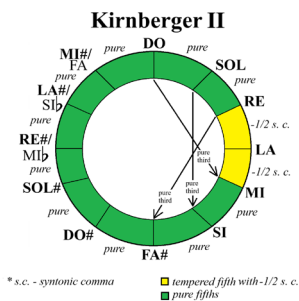


Fig. 16. Kirnberger II

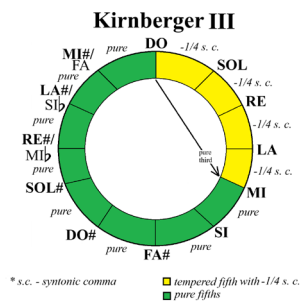


Fig. 17. Kirnberger III

American musicologist **Bradley Lehmann** analysed the spiral drawn on the first page of J.S. Bach’s “Well-Tempered Clavier”, and concluded that drawn loops are indications of how to tune a keyboard instrument. Lehmann’s system is shown in figure 18 below:

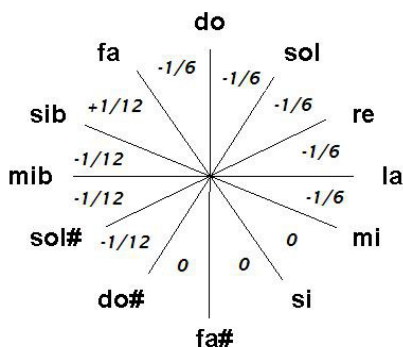


Fig. 18. Lehmann's system

Tuning systems for keyboard instruments

Unlike the aforementioned tempered systems that try to solve the problem of the heights of a sound system by more theoretical or mathematical methods, the following musicological sources propose a pragmatic approach, as follows:

a) The German Renaissance composer and organist Arnolt Schlick proposes in his work entitled “Spiegel der Orgelmacher und Organisten”, published in 1511 in Mainz, in chapter eight, the following steps: make its fifth not high enough nor pure, but somewhat “beating” in the low (nitt hoch genug, oder gantz gerad in,

sonder etwas in die niedere sch weben); - it starts with the sound “fa”, and, using a sequence of ascending fifths, the sounds “do”, “sol”, “re”, “la”, “mi”, “si”, and “fa#” are tuned; - also from the sound “fa”, the fifths of the sounds “sib” and “mib” are tuned in a descending direction; - the diatonic major thirds “fa – la”, “do – mi”, and “sol – si” are more important than those with chromatic elements “sib – re”, “mib – sol”, “la – do#”, and „re – fa#”; - he recommends that the sound “sol#” be tuned a little higher than the major third of the sound “mi”, but not identical to “lab” (fig. 19). The reason for this last action is not very clear from his writings, but he may have wanted to attenuate or even eliminate the “wolf” fifth formed between the sounds sol# and mib. This makes it possible to use all the fifths in the system.

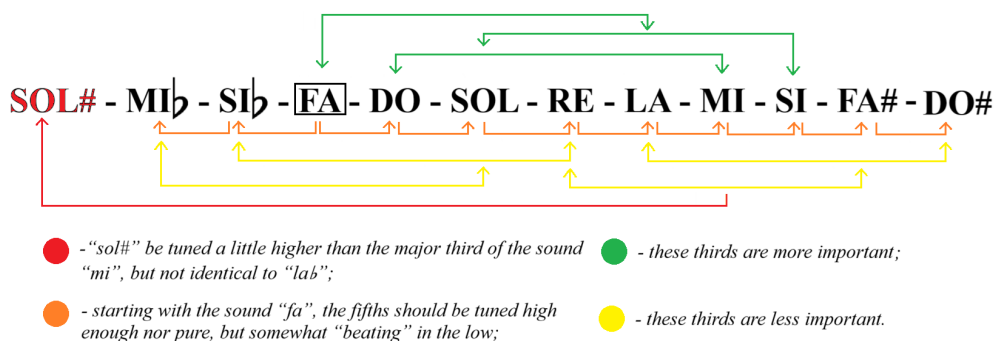


Fig. 19. Arnolt Schlick system

b) The Italian theorist and composer Pietro Aron in his work “Il Toscanello”, 1523, chapter 41, proposes: fifths to be permanently somewhat diminished and lacking perfection (*un poco scarsa, sempre discadono e mancano della sua perfettione*), and the major thirds to be straight sounding, and, as much as possible, united (*sonora e giusta cioe unito al suo possibile*). By “as much as possible”, the author suggests that it is not possible for these intervals to be perfectly tuned; - the tuning begins by building a major chord on the sound “do” (do - mi - sol) after which the fifths of these sounds (do-sol, sol-re, re-la, la-mi) are tuned, and continues this path until to the sound “si” through fifth mi-si is reached; - this is followed by a descending tuning through fifths starting from the sound “do” to “mib” (do-fa, fa-sib, sib-mib); - the sharp sounds (fa#, do#, and sol#) are tuned with the help of major thirds (re-fa#, la-do#, and mi-sol#) (figure 20). In modern musicological literature, this type of tuning is considered to be the first description of a 1/4 meantone sound system, but there is no information to suggest temperament by diminishing the fifths with a quarter of a syntonic comma.

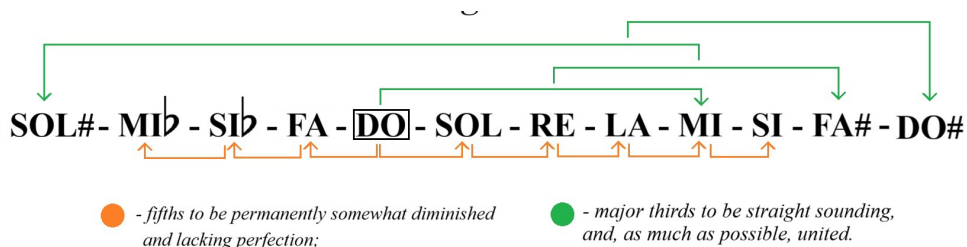


Fig. 20. Pietro Aron system

c) The work *L'arte Organica* by the Italian composer and organ builder Costanzo Antegnati, published in 1608 in Brescia, contains information on tuning similar to that of Pietro Aron: - fifths need not be perfectly tuned, just pleasing to the ear (*non bisogna tirarle tutto quello si potrebbe a perficerle, ne far che l'orecchia sia del tutto contenta*), and the major thirds should be as perfect as possible (*si tira a tutta quella perfettione che si puo*) (figure 21); - what follows is the construction of major chord structures starting with the “fa” sound. He tunes the ascending fifth of “fa”, “do”, then the upper major third of the same sound “fa”, “la”, finally obtaining a major chord on the sound “fa”. He does the same on the sounds “do”, obtaining “do” - “mi” - “sol”, on “sol” - “sol” - “si” - “re”, on “re” - “re” - “fa#” - “la”, on “la” - “la” - “do#” - “mi”, and on “mi” - “mi” - “sol#” - “si”. The sound “mib” is tuned starting from the sound “sol”, and “sib” is tuned as the upper fifth of “mib”.

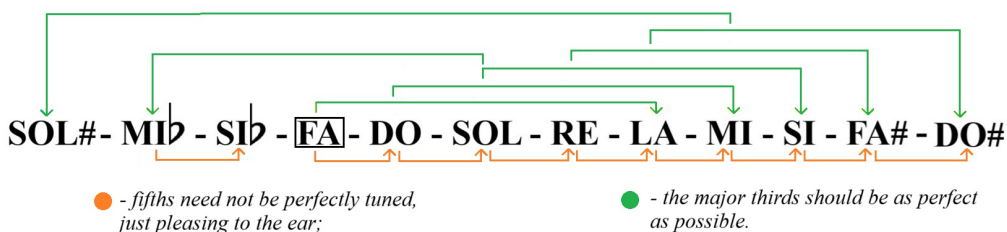


Fig. 21. Costanzo Antegnati system

d) Michael Praetorius, in his work *Syntagma Musicum*, published in 1619 at Wolfenbüttel, offers three detailed methods of tuning:

1. For the first method, major thirds must be clean, pure (*rein*), and the fifths must be slightly more diminished compared to the pure version, so as to produce the effect of “beating” (*nicht gar rein, sondern gegeneinander niedrig schwebend gelassen werden*) (figure 22). Praetorius is the first to refer to the term “beat” (*schwebung*), and explain its usefulness for differentiating between a pure

and an impure interval. "Beat" is not a clear indication of the quality of an interval. Praetorius' method is similar to that of Antegnati, as follows: - it begins with the construction of a major chord using the sound "fa"; - this type of tuning is continued on the sounds "do", "sol", "re", "la", after which a minor chord structure is used starting with the sound "mi". The ascending major third of the sound "mi", "sol#" is tuned, and from this "sol#" its descending fifth is tuned, "do#". This way, the minor chord "do#" - "mi" - "sol#" is obtained. This is a special case, because it is for the first time when the tuning is performed between two sounds positioned at a fifth interval, both with alterations; - a return to the first sound, "fa", occurs, and a major chord structure is built. Starting from "fa", the descending fifth "sib" is tuned. The sound re, from the major chord "sib" - "re" - "fa", was previously tuned; - the sound "mi \flat " as the descendant major third of the sound "sol".

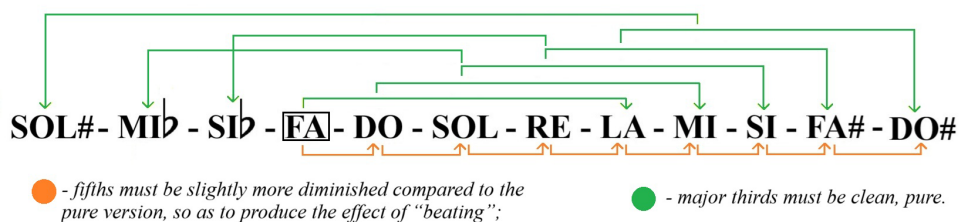


Fig. 22. Michael Praetorius system

2. The second method is similar to Schlick's: - it starts from the sound "fa", and the following six sounds ("do", "sol", "re", "la", "mi", and "si") are tuned by a sequence of fifths; - this is followed by the tuning by descending fifths of the sounds "sib" and "mi \flat ", starting from the sound "fa"; - the sounds "fa#", "do#", and "sol#" are tuned independently by ascending major thirds using the sounds "re", "la", and "mi"; - like Schlick, he recommends tuning the sound sol# slightly higher, but below the pitch of "la \flat " (figure 23).

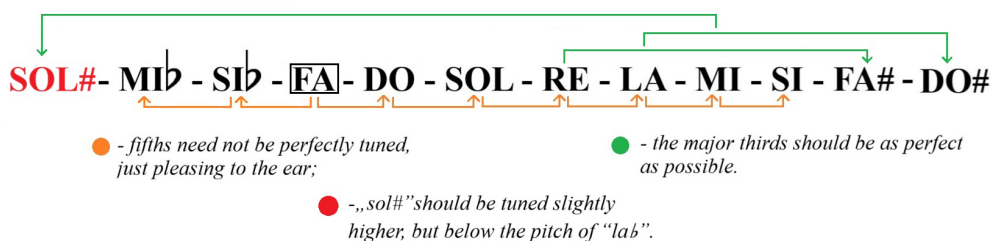


Fig. 23. Werckmeister's second tuning method

3. The last method proposed by Praetorius is also the most “chaotic” in terms of stages, but, at the same time, it is also the most modern up to this point: - all sounds are tuned by fifths starting from the sound “fa”, except for the “wolf” fifth (sol# - mi \flat), and the thirds are tuned both as major thirds and by fifths (figure 24). Through this method, Praetorius aims to build a complete tuning system in which all sounds are interdependent and equal in importance.

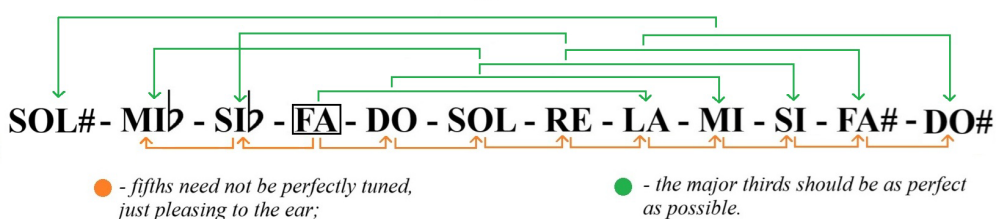


Fig. 24. Praetorius system

In the modern era, one of the most influential musicologists who interpreted the above Renaissance sources as descriptions of the 1/4 meantone system is James Barbour in his book *Tuning and Temperament. A Historical Survey*.

3. Conclusions

The theoretical-musical works presented in this scientific paper are not very clear but some important conclusions can be drawn and formulated from them:

1. Fifths must be tuned lower in comparison with their pure versions, but it is not clear to what extent. None of the musicological works presented above shows the equal tempering of fifths;
2. The altered sharp sounds must be tuned as major thirds of the diatonic sounds, and the resulting fifths are not considered as important for most theorists;
3. The “ideal” major third is not identical with the “pure” one;
4. Praetorius’s tuning system no. 3 is the only source that may be interpreted as what is currently called the 1/4 meantone system;
5. No source designates the sound system. The term “meantone” appeared much later.

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