

EMOTIONS AND THE LEARNING OF SCHOOL MATHEMATICS

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Abstract: *Emotions take central stage between the factors which influence the success of the learning process and, in particular, learning mathematics, along with the cognitive processes involved. In school practice, however, this aspect is not fully recognized and, therefore, not valued to the extent that might serve as important levers to ensure school success. Thus, valuable resources the mathematics teacher might have to streamline the didactic process are being lost, but the teacher must be prepared in this regard. The learning of mathematics must be seen and approached as a complex, dynamic and continuous process, with multiple determiners.*

Key words: *emotion, mathematics anxiety, control strategy, classroom climate*

1. Introduction

Emotions, these “subjective reactions to the environment, accompanied by hormonal and autonomic responses, usually proven to be agreeable or disagreeable and used as adaptative reactions that affect our way of thinking” (Papalia & Olds, as cited in Raynal & Rieunier, 2005, p. 118), are an incontestable reality of the school climate. Student life is full of experiences more or less intense – *joy, pleasure, satisfaction* and so on (positive emotions with *sthenic* effect, helping to increase exercise capacity), and *hatred, sadness, fear, anger, anxiety, disgust, fear, suffering* and so on (negative emotions, with *asthenic* consequences on the individual, contributing to the decrease of its exercise capacity). Some authors would add to these emotions, guilt and shame, phenomena with a high incidence in school. Therefore, “emotions are complex motivational systems, the affective echo of the relationship between subject and objects, beings, phenomena. They have spontaneous manifestation and create an affective tension [...]” (Golu, 2014).

Emotions are part of the affective structure of the individual’s psychic system, located at the interface between cognition and motivation. Cognition mediates and provides signals in order to activate and trigger emotion. Between emotion and cognition there is a bilateral relationship of mutual conditioning, but the referent of emotional experience is always granted by the cognitive experience related to surrounding situations and objects. The sign and the intensity of the feelings change according to the perception of the situation or to its anticipated mental representation. The temporal rapport between the cognitive processes (perception, representation or judgement) and the generation of emotion is not determined. In general, cognition foregoes and conditions the generation

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of emotion, but in situations of informational deficiency, affective reflection *outruns/foregoes* cognition. On the other hand, cognition may take place on the background of an active emotional state (fear, depression or joy, euphoria), being influenced by it, both in terms of dynamics, and of the content (Golu, 2014).

Research on the role of emotions in intellectual functioning (Bandura, 1986; Boekaerts, 2006; Kuhl, 1985; Pekrun, 1992; Schultz & Lanehart, 2002, as cited in Crahay & Dutrévis, 2010, p.78) showed that „emotions are intimately involved in learning, in the way of preparing for learning, in order to react to difficulties and obstacles and to appreciate the value of achievements”. According to the research mentioned, fear of failure is linked to a deficit in self-regulation practiced during the task (use of metacognitive strategies). Artino showed that successful students reported less negative emotions (Artino & Stephens as cited in Bouffard et Vezeau, 1998) and were more satisfied with their learning experiences.

Pekrun and colab. (as cited in Crahay & Dutrévis, 2010) proposed a sociocognitive theory of „academic” emotions, building a dynamic model that stresses on the links between characteristics of the learning context, student’s motivational beliefs (perception of competence, task value), its emotions (positive or negative), knowledge and performance achieved. Thus, in learning, student’s motivational beliefs help generate positive emotions (such as excitement and hope), and negative emotions (such as boredom and frustration); in turn, emotions felt in a given situation may influence motivational beliefs in a further situation. According to the authors, emotions’ effects on learning and performance would be publicized through several cognitive and motivational mechanisms such as use of strategies, effort and perseverance. This theory may be linked to that of the sense of personal efficacy (Bandura, as cited in Crahay & Dutrévis, 2010), according to which the assessment of personal capacity to face a situation generates negative emotions (such as anxiety or discouragement, when the student is deemed incapable to fulfill the task successfully) and positive (such as excitement or pleasure, when the student believes it can succeed). The nature of these emotions determines the intensity of motivation. For Bandura, this process is essentially bidirectional, the sense of subsequent personal efficacy being determined by the nature of cognitive assessment of the information transmitted through emotions. Govaerts (as cited in Crahay & Dutrévis, 2010) showed even that the influence of the way of perception of task’s value on performance in mathematics would be fully publicized by the nature of emotions regarding the subject.

As already seen, the affective process is triggered by the action of an external stimulus, recorded and evaluated on a cortical level, where cortical signals on the significance with impact on affectivity of the stimulus are sent to the limbic system, implicitly to hypothalamus. Sousa (2017, p. 94) explains the physiological processes that occur in the brain that influence student’s behaviour in relation to the act of learning. At the central nervous system level, information of emotional nature are processed with priority: the older limbic system takes a major role, and the complex cerebral processes are suspended. The author make reference to the experience „known by all of us when anger, fear of the unknown, or joy quickly overcame our rational thoughts”. Sousa states forwards that „strong emotions can shut down conscious processing during the event while enhancing our memory of it. Emotion is a powerful and misunderstood force in learning and memory.” Sousa (2017, p. 94). Math teacher should acknowledge the dynamic of the relationship between affective processes and cognitive ones which run in

the act of learning, because, traditionally, the teacher focuses on mathematical contents, ignoring learning's aspects of contextual nature, the frame within emotions manifest.

"Teachers cannot deny anymore the role of affectivity in learning mathematics, be it anxiety or pleasure. We should learn to manage these emotions and to show this to students" (Lafortune & St-Pierre, 1994). However, many math teachers are convinced that "intervention on more psychological aspects is not part of their role", arguing that feeling of incompetence in using emotional skills because of lack of training in this regard. In addition, math class is traditionally meant for purely mathematical contents and excludes the approach of subjects related to students' feelings on what happens during these classes.

The range of emotions linked to mathematics learning specific situations with impact on students' cognitive engagement and its performance in learning is relatively wide, some of these having a higher incidence and a higher intensity compared to others. Experience has shown that students' feelings during math class varies from delight (pleasure, satisfaction), felt when solving a complicated math problem, to anxiety (anger, disgust), caused by the inability to do math under pressure or not at all. Students often try restlessness or anxiety before an exam, being able to develop into fear or panic.

The great majority of the studies made on emotions' school („academic”) functioning showed that anxiety is the „emotion associated most with difficulties encountered in learning mathematics" (Lafortune & St-Pierre, 1994; Bouffard & Vezeau, 1998, Crahay & Dutrévis, 2010; s.a.). Tall refers to *mathematical anxiety* as „feelings of tension and anxiety that interfere with the manipulation of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suin, as cited in Tall, 2015, p. 122). Lafortune defines *anxiety* as an “emotional state characterized by feelings of restlessness, insecurity and diffuse physical disorders regarding an undefined danger which makes the individual feel helpless" (Lafortune, as cited in Lafortune & St-Pierre, 1994, p. 48). According to these authors, anxiety towards mathematics knows various forms of manifestation (restlessness, malaise and fear, panic, paralysis, mental disorganization) and different levels of intensity. For example, the intensity of emotions increases when the individual has to solve a math problem (Hendel, as cited in Lafortune & St-Pierre, 1994). Lam (as cited in Lafortune & St-Pierre, 1994) signals three forms of anxiety: *anxiety on the class' organization*, *anxiety on the fear of seeming stupid*, *anxiety towards exam/assessment*. Each one of these has a higher or lower incidence in math classes. Anxiety can be a serious obstacle in learning mathematics (for some students, the mere evocation of mathematics can trigger panic, a natural consequence of this being avoiding contact with this subject). In situations still quite frequent, in which assessment is a goal of learning, this panic has a greater frequency and a higher intensity.

Research on mathematics anxiety sources reveals a variety of factors: „personal attributes of confidence or insecurity; attitudes of parents, teachers, parents and peers; poor teaching or inadequate preparation for the exam" (Tall, 2015, p. 118), to which are added „anxiety at being asked to do mathematical problems in front of the class; fear to failure; poor self-image; poor memory and so on" (Furner & Berman, as cited in Tall, 2015, p. 123). In addition, emotional problems can be caused by factors related to the specific of the learning of mathematics, respectively *instrumental understanding* (*knowing how* to perform mathematical operations) and *relational understanding* (*knowing why*) (Skem, as cited in Tall, 2015, p. 119): instrumental learning may cause anxiety or fear of failure when the calculations become too complicated; relational

understanding may involve confusion or frustration when the mathematics fail to make sense.

The successive experiences lived by the student regarding mathematics are one aspect that needs to be given more attention by all factors involved in the learning process and responsible for its success. These experiences may have either supportive aspects, which encourage student's progress, or concerning aspects, which prevent this progress, depending on the personal way in which previous experiences were perceived. In this regard, „the emotional reactions have long-term effects on learners' personal development, encouraging some to build on their confidence to enjoy the challenge of more sophisticated ideas while others are impeded in their progress and may resort to rote-learning procedures without meaning, or become disaffected and develop mathematical anxiety” (Tall, 2015, p. 119). The same author states that mathematics anxiety is a phenomenon more or less widely encountered in all school systems (even reaching the threshold of „hatred and a deep fear of math”) and at all levels of schooling, from elementary school through high school and it is manifested by physiological factors (increased heart rate, sweaty palms) as well as „a horrendous inability to recall facts under pressure and a genuine antipathy to mathematics” (Tall, 2015, p. 122).

Of particular interest for the present study is the dynamics of the relationship between mathematics anxiety and difficulties in learning mathematics: it has a cyclic evolution, in that difficulties cause anxiety and anxiety causes difficulties. „An individual who begin to fear failure may develop a level of anxiety that makes it far more difficult to cope in stressful situations.” (Tall, 2015, p. 122).

Another aspect regarding the manifestation of emotions in the learning process is their *adjustment* (ability defined by Goleman through *emotional intelligence*). According to studies, this could be a predictor of school success. „To be successful learner and productive citizens, we need to know how to use our emotions intelligently” (Sousa, 2017, p. 51). Adjusting emotions refers to individual's efforts made in order to influence the emotions he/she feels, their nature, the moment when they occur, as well as the way they are lived and expressed (Gross, as cited in Crahay & Dutrévis, 2010, p. 80). According to these authors, most common strategies for adjusting emotions in school environment are cognitive reassessment and suppression of expression of emotion, with specific cognitive consequences, but also with influence on the effect a negative feedback has on the student (Raftery & Bizer, as cited in Crahay & Dutrévis, 2010, p. 81). Thus: users of reassessment strategies achieve better performances at further assessments after receiving negative feedback than those who received a relatively neutral feedback. The authors conclude that the strategy of cognitive reassessment of emotions may be a positive resource in situations of negative feedback, leading to a maintained or even improved performances due to a negative feedback. This strategy can be taught to students and they should be encouraged to use it.

If negative emotions hinder the student to learn mathematics, the positive ones are valuable resources in the process. We mention, only as an example, *self-reliance*, which plays a leading role in mathematics. It is a feeling by which the individual proves courage and confidence regarding the success of an experience (Sillamy, as cited in Lafortune & St-Pierre, 1994, p. 45). In order for positive emotions to become reliable „allies” in students' fight with mathematics, as many mathematics learning experiences must be caused as to generate such emotions. Any learning situation, whatever the nature of its contents, is characterized by a climate of whose planning, organization and development

the teacher is primarily responsible. Classroom climate is defining for students' emotional feelings. Sousa (2017, p. 50) says that „students must feel physically safe and emotionally secure before they can focus on the curriculum”. The teacher can ensure a positive climate, generating sthenic emotions which are necessary for reaching the targetted learning objectives and, in this regard, Sousa (2017) proposes two action levers, accesible to the mathematics teacher:

- a) Ensure a positive classroom climate by: promoting positive relationships among the students so they are kind to each other, listen to each other, and respect different viewpoints; cultivating a positive relationships with all of your students so they feel you not only care about their academic success but also care about them as individuals; developing and reinforcing classroom norms and rules that are simple, clear, and provide a physically and emotionally seif learning environment.
- b) Occasionally interview students individually and ask questions such as whether they feel seif in the class, if students are kind to each other, and do they feel welcomed and included in the class. Use the students'feedback to make sure any necessary adjustments to improve the climate.

These proposals define the paradigm within which was designed and developed the current research.

2. Objectives

This study is part of a much wider research that was carried on to identify and to analyze obstacles that middle school students face in learning mathematics (Căprioară, 2011). The reason on which this research was based was the need of explaining the relatively week results recorded at the mathematics task from National Assessment Exam, with the purpose of coming up with practical solutions for supporting the ones involved in the didactic act (students, teachers, parents, decision factors in educational politics and so on).

The main objective to which this endeavour is subordinated aims at *studying the motivational level of middle school students for learning mathematics and identifying factors with affective-motivational impact on their results/performance*.

The hypothesis underlying the research is that *middle school students face in learning mathematics obstacles of psychological nature with a negative impact on school performance*.

3. Material and Methods

Research data was recorded based on an *opinion survey omnibus type* applied on a number of 350 students from the 8th Grade (ending middle school studies), the equivalent of 22 classes of students from 19 areas (12 rural and 7 urban) and included 4 Counties from south-eastern Romania. The reason for choosing 8th graders is twofold: the experience accumulated in relation to school mathematics (at least 8 years of study), respectively the possibility of formulating and expressing opinions on mathematical training, due to the maturity level reached and the perspective on the education system. Information drawn from respondents' answers was completed by the conclusions drawn

from two focus-groups in which took part teachers with extensive teaching experience. Information interpretation was nuanced by personal experience as a mathematics teacher.

4. Results

A first set of items targeted the affective dimensions of the learning context:

During math classes it happens that the teacher:

I 1: *only works with students good in maths;*

I 2: *helps me when I am in difficulty;*

I 3: *mocks me/embarrasses me in front of classmates because of a wrong answer;*

I 4: *punishes me with a bad mark for a wrong problem solving.*

The item I1 is targeting a common practice in the traditional Romanian mathematical learning system, which focuses on preparing students with special mathematical skills and their training in order to achieve high performance in math competitions. This is proved by the excellent results achieved in national and international mathematics competitions by Romanian competitors (otherwise a laudable fact), but amid worrying general results in national mathematics assessments. This means that traditional mathematical learning system is more oriented towards performance, towards the identification of “peaks”, risking the loss of important resources by neglecting other students. Redressing this situation means involving in the didactic practice strategies of differentiation and individualization of teaching on a higher scale, so that each student has the chance to capitalize on the cognitive and affective-motivational potential it owns. Ignoring this potential more or less leads to students’ alienation from everything that means mathematics and the generation/confirmation of some misconceptions on what training in mathematics means.

In figure 1 there are presented the frequency distributions for item I1.

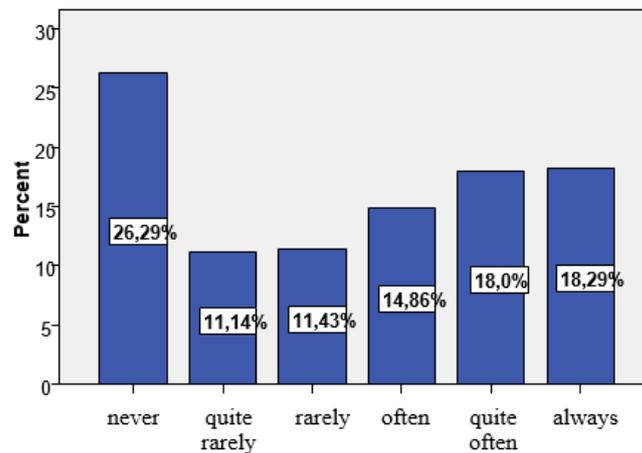


Fig. 1. *The distribution of responses for item I 1*

It is noticed that over 50% of students said that it is worked more with the good students in math classes, which reflects negatively on the motivation state of students, especially for those with lower results in mathematics.

The next item, I2, targets teacher's availability to answer students' questions, when these occur. Registered results (see fig.2) may be thought of as pleasant because more than 80% answered that, when facing a difficulty, the teacher helps them. The issue, however, is to which extent students acknowledge the need of support from the teacher and how often do they ask for it. Obtained results reflect the situation on a sample of middle school students, at the age where, in general, students do not acknowledge the level of knowledge they need to reach in mathematics, the mark being the level of request from the teacher. Unfortunately, they are not rare the situations in which students who do not achieve great performances in mathematics and who are not requested in class by the teacher have the tendency to minimize the training effort and thus ask for teacher's attention only to a small extent.

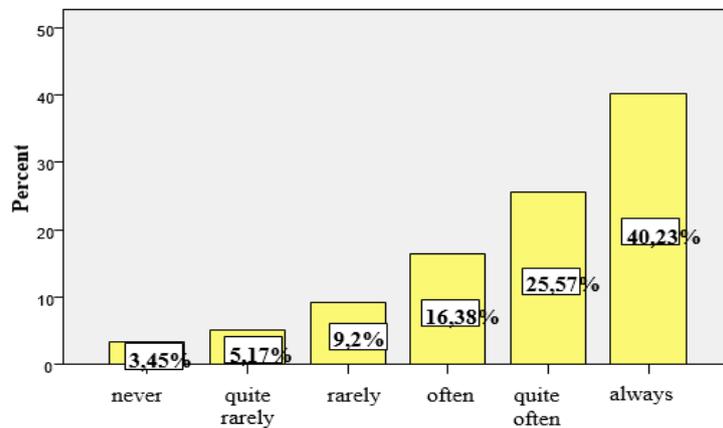


Fig. 2. *The distribution of responses for item I 2*

The item I 3 refers to another aspect of the teacher-student relationship with impact on students' attitude regarding mathematics: teacher's reactions to wrong/erroneous answers given by the student. Students' answers to this item are detailed in figure 3.

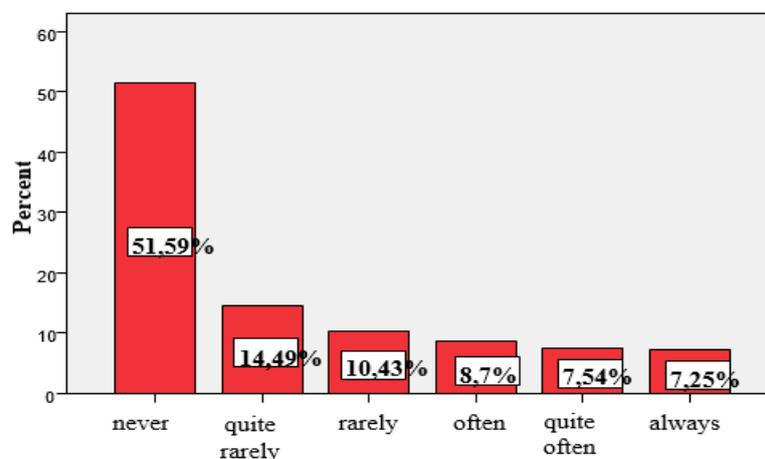


Fig. 3. *The distribution of responses for item I 3*

In general, it can be noticed that students who give wrong answers are not mocked by the teachers. However, almost a quarter of the subjects questioned (23,49%) lived, more or less, such an experience. We believe that this issue is among the factors that generate the highest level of anxiety in students in math class. Students' disapproval, expressed differently (irony, reprimand, threat, punishment and so on) in front of other students generate a climate of emotional insecurity for them. Therefore, students' defense reactions (resistance behaviors expressed in different forms: negativism, rebellion, arrogance, aggressiveness and so on) are absolutely natural, but have an effect on building students' mathematical competences (knowledge, skills, attitudes).

The last item, I4, of this first set refers to the use of the mark as a punishment instrument for unsatisfactory results achieved by students in solving work tasks. As shown in fig. 4, although over 30% of the students questioned were never penalized with low mark for mistakes made, but it is recorded, however, a significant number of cases (35%) when the mistake is punished, to an extent more or less, by mark. The mark, respectively the classbook, still represents for most teachers the number one "allie" in the fight for persuading students to learn mathematics. We believe the effects of this strategy are far from the expected ones. The message sent by the teacher through this practice is, among others, that the mark is the most important and the finality of the training process is the achievement of marks as high as possible, generating this way errors in students' motivational area, switching student's goals from learning to achieving performance, with all effects resulting from this.

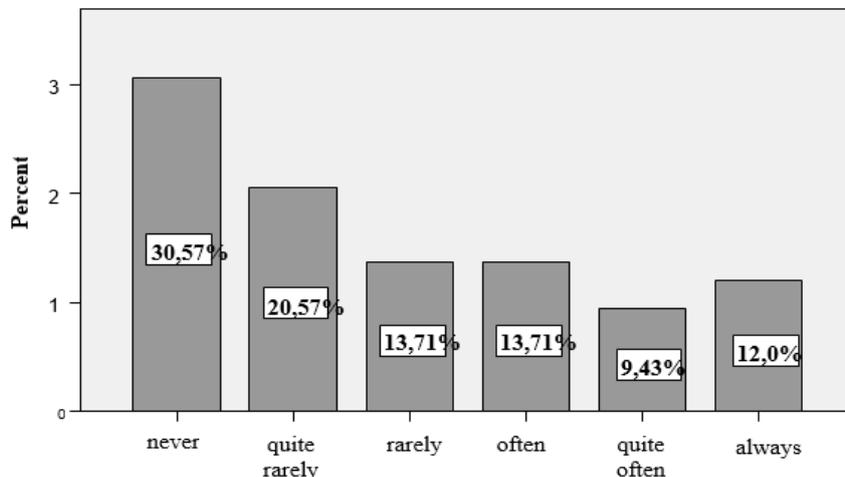


Fig. 4. The distribution of responses for item I 4

The above data explains largely the students' answers to the next set of items:

In math classes, I feel:

I 5: scared by the mark;

I 6: scared by the teacher;

I 7: scared that I will be asked a question and I will not know the correct answer.

Based on *Wilcoxon statistic index*, we can achieve a first ranking of the factors that constitute sources of affective-motivational blocking for students, related to school

mathematics. The ranking scale is decreasing, depending on the intensity of the effect (the differences are significant at a threshold $p < .01$):

- *fear of the mark*
- *fear that I will be asked a question and I will not know the correct answer*
- *fear of the teacher.*

We think that these results are consistent with data previously presented. The fear of mark proves the importance the teacher (and parents) attach to the mark achieved by the student in maths. What happens, though, when the mark is not a real measure of the level achieved by the student in the formation of mathematical skills, but is used by the teacher to punish the mistakes/errors made by students in learning mathematics? Mistakes/errors are natural events in a learning process, generated by various obstacles which students face in this endeavor and should be the expression of a need for help from the teacher, intervention for support instead of punishment. What emotional states generate such situations? We believe we need to reflect deeply on the phenomenon and to have a better math teacher training in this regard.

Finally, in order to identify anxiogenic factors for students during math class, the items below were introduced in the survey.

During math classes, when I am required to answer, I am scared of:

- I 8: *the teacher;*
- I 9: *the classmates (because they will laugh at me);*
- I 10: *giving a wrong answer;*
- I 11: *the task's difficulty;*
- I 12: *the mark I will get.*

Comparing the statistic averages and taking into account other indices of central tendency and of result spread, it is revealed that the biggest fear students feel during math classes is related to *the mark they will obtain*, at the other pole being found, with minimum values, *the fear of social factors* (teacher, as well as peers). Through an analysis based on TNW (*Wilcoxon statistic index*) it is shown that anxiogenic factors contained in this set of items can be grouped into three categories, depending on the intensity with which they act upon students during math classes. Thus, at the highest level would be *fear of the mark*, then at a medium level, *fear of making mistakes* and *fear of task's difficulty*, and at the minimal level, *fear of teacher* and *fear of classmates' reaction*. Differences between these categories are only significant at a threshold $p < .01$.

Therefore, it is not the teacher who causes anxiety (which is very well), but the practices used by it. A change in the attitude of the math teacher regarding the students and a review of the strategies used, help the cognitive and affective needs expressed by students, providing chances of a successful teaching process, as it results from the answers given by students at the following item: *I wish my math teacher would pay more attention to me/give more support in my preparation*. Thus, over half of those surveyed (50,57%) expressed the felt need of receiving more support from the teacher.

5. Conclusions and Discussion

As Sousa stated (2017, p. 135), „emotions play too much of an important part in learning and memory to be ignored in the teaching-learning process.” In learning mathematics, teacher can and must control a series of factors that affect the emotional lives of their students. The results of this study show that middle school students are

sensitive to this issues and need a positive learning climate. The nature of the climate created by the teacher (student-teacher relationship, relationships between students, respect for students' opinions, giving support when required by student, without ironies or humiliation and so on) generate emotions which determine, through their content, meaning and intensity, students' feelings on the learning situation, orienting them towards or against the teacher or future learning experiences. „How a person *feel* about a teaching or a learning situation determines the amount of attention devoted to it. Emotions interact with reason to support or inhibit learning. [...] Thus, we need to explore what and how we teach students about their emotions. We could teach about controlling impulses, delaying gratification, expressing feelings, managing relationships, and reducing stress. Students should recognize that they can manage their emotions for greater productivity and can develop emotional skills for greater success in life” (Sousa, 2017, p. 51).

For this, teachers' initial and continuous training should include training and developing skills that would allow them to act upon the emotional dimensions of students related to learning and to school experience in general, complementarily with programmes of developing students' emotional competence.

In conclusion, learning mathematics should be seen and treated not only as a transmission of mathematical knowledge, but also as a complex, dynamic and continuous process, with multiple determinations (cognitive, metacognitive, emotional and motivational), which should be taken into account by all factors involved in its development and responsible for the results.

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