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An Application of Freight systems

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Abstract: *There are important applications of the maximal dynamic flow problem and its variations in the areas of freight systems, material handling systems and building evacuation. The early freight system application of maximal dynamic network flow models arose in the railroad industry in the scheduling of freight cars. In this paper we present an application of his model.*

Key-words: *dynamic network flows, maximum flow, freight systems*

1. Introduction

The theory of flow is one of the most important parts of Combinatorial Optimization. The static network flow models arises in a number of combinatorial applications that on the surface might not appear to be optimal flow problems at all. The problem also arises directly in problems as far reaching as machine scheduling, the assignment of computer modules to computer processor, tanker scheduling etc. (Ahuja, Magnanti and Orlin, 1993). However, in some applications, the time is an essential ingredient (Aronson 1989), (Cai, Sha and Wong, 2007), (Ford and Fulkerson, 1962), (Hamacher and Tjandra, 2001), (Tjandra, 2003). In this case we need to use the dynamic network flow model (Wilkinson, 1971).

In Section 2 of this paper we present some notions and results for maximum flow problem in general dynamic networks. Section 3 deals with a freight system application for the problem presented in Section 2.

2. Maximum flows in dynamic networks

Dynamic network models arise in many problem settings, including production distribution systems, economic planning, energy systems, traffic systems, and building evacuation systems.

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Let $G = (N, A, u)$ be a static network with the set of nodes $N = \{1, \ldots, n\}$, the set of arcs $A = \{a_1, \ldots, a_m\}$, the upper bound (capacity) function u, 1 the source node and n the sink node. Let $\mathbb{N}$ be the natural number set and let $H = \{0, 1, \ldots, T\}$ be the set of periods, where T is a finite time horizon, $T \in \mathbb{N}$. Let use state the transit time function $h: A \times H \to \mathbb{N}$ and the time upper bound function q: $A \times H \to \mathbb{N}$. The parameter $h(i,j;t)$ is the transit time needed to traverse an arc (i,j). The parameter $q(i,j;t)$ represents the maximum amount of flow that can travel over arc (i,j) when the flow departs from node i at time t and arrives at node j at time $\theta = t + h(i,j;t)$.

The maximal dynamic flow problem for T time periods is to determine a flow function $g: A \times H \to \mathbb{N}$, which should satisfy the following conditions in dynamic network $D = (N, A, h, q)$:

$$\sum_{t=0}^{T} \left(g(1, N; t) - \sum_{\tau} g(N, 1; \tau) \right) = w \tag{1a}$$

$$g(i, N; t) - \sum_{\tau} g(N, i; \tau) = 0, \qquad i \neq 1, n, t \in H \tag{1b}$$

$$\sum_{t=0}^{T} \left(g(n, N; t) - \sum_{\tau} g(N, n; \tau) \right) = -w \tag{1c}$$

$$0 \leq g(i,j;t) \leq q(i,j;t), \qquad (i,j) \in A, \qquad t \in H \tag{2}$$

$$\max \ w \tag{3}$$

where $\tau = t - h(k, i; \tau)$, $w = \sum_{t=0}^{T} v(t)$, $v(t)$ is the flow value at time t and $g(i,j;t) = 0$ for all $t \in \{T - h(i,j;t) + 1, \ldots, T\}$.

Obviously, the problem of finding a maximum flow in the dynamic network $D = (N, A, h, q)$ is more complex than the problem of finding a maximum flow in the static network $G = (N, A, u)$. Happily, this complication can be resolved by rephrasing the problem in the dynamic network D into a problem in the static network $R_1 = (V_1, E_1, u_1)$ called the reduced expanded network.

The static expanded network of dynamic network $D = (N, A, h, q)$ is the network $R = (V, E, u)$ with $V = \{i_t | i \in N, t \in H\}$, $E = \{(i_t, j_\theta) | (i,j) \in A, t \in \{0, 1, \ldots, T - h(i,j;t)\}, \ \theta = t + h(i,j;t), \ \theta \in H\}$, $u(i_t, j_\theta) = q(i,j;t), (i_t, j_\theta) \in E$. The number of nodes in the static expanded network R is $n(T+1)$ and the number of arcs is limited by $m(T+1) - \sum_A h'(i,j)$, where $h'(i,j) = min\{h(i,j;0), \ldots, h(i,j;T)\}$. It is easy to see that any flow in dynamic network D from the source node 1 to the sink node n is equivalent to a flow in static expanded network R from the source nodes $1_0, 1_1, \dots, 1_T$ to the sink nodes $n_0, n_1, \dots, n_T$ and vice versa. We can further reduce the multiple source, multiple sink problem in the static expanded network R to a single source, single sink problem by introducing a supersource node 0 and a supersink node $n+1$ constructing static super expanded network $R_2 = (V_2, E_2, u_2)$, where $V_2 = V \cup \{0, n+1\}$, $E_2 = E \cup \{(0, 1_t) | t \in H\} \cup \{(n_t, n+1) | t \in H\}$, $u_2(i_t, j_\theta) = u(i_t, j_\theta)$, $(i_t, j_\theta) \in E$, $u_2(0, 1_t) = u_2(n_t, n+1) = \infty, t \in H$.

We construct the static reduced expanded network $R_1 = (V_1, E_1, u_1)$ as follows. We denote the function $h_2 : E_2 \to \mathbb{N}$, with $h_2(0, 1_t) = h_2(n_t, n+1) = 0, t \in H$, $h_2(i_t, j_\theta) = h(i, j; t)$, $(i_t, j_\theta) \in E$. Let $d_2(0, i_t)$ be the length of the shortest path from the source node 0 to the node i_t, and $d_2(i_t, n+1)$ the length of the shortest path from node i_t to the sink node $n+1$, with respect to h_2 in network R_2. The computation of $d_2(0, i_t)$ and $d_2(i_t, n+1)$ for all $i_t \in V$ are performing by means of the usual shortest path algorithms. The network $R_1 = (V_1, E_1, u_1)$ have

$$V_1 = \{0, n+1\} \cup \{i_t | i_t \in V, d_2(0, i_t) + d_2(i_t, n+1) \leq T\},$$
$$E_1 = \{(0, 1_t) | d_2(1_t, n+1) \leq T, t \in H\} \cup \{(i_t, j_\theta) \mid (i_t, j_\theta) \in E, d_2(0, i_t) + h_2(i_t, j_\theta) + d_2(j_\theta, n+1) \leq T\} \cup \{(n_t, n+1) \mid d_2(0, n_t) \leq T, \; t \in H\}$$
and u_1 are restrictions of u_2 at E_1.

Now, we construct the static reduced expanded network $R_1 = (V_1, E_1, u_1)$ using the notion of dynamic shortest path. The dynamic shortest path problem is presented in (Cai, Sha and, Wong 2007). Let $d(1, i; t)$ be the length of the dynamic shortest path at time t from the source node 1 to the node i and $d(i, n; t)$ the length of the dynamic shortest path at time t from the node i to the sink node n, with respect to h in dynamic network D. Let as consider $H_i = \{t | t \in H, d(1, i; t) \leq t \leq T - d(i, n; t)\}, i \in N$, and $H_{i,j} = \{t | t \in H, d(1, i; t) \leq t \leq T - h(i, j; t) - d(j, n; \theta)\}, (i, j) \in A$. The multiple source, multiple sinks static reduced expanded network $R_0 = (V_0, E_0, l_0, u_0)$ have $V_0 = \{i_t | i \in N, t \in H_i\}$, $E_0 = \{(i_t, j_\theta) | (i, j) \in A, t \in H_{i,j}\}$, $u_0(i_t, j_\theta) = u_1(i, j; t)$, $(i_t, j_\theta) \in E_0$. The static reduced expanded network $R_1 = (V_1, E_1, l_1, u_1)$ is constructed from network R_0 as follows: $V_1 = V_0 \cup \{0, n+1\}$, $E_1 = E_0 \cup \{(0, 1_t) | 1_t \in V_0\} \cup \{(n_t, n+1) | n_t \in V_0\}$, $u_1(0, 1_t) = u_1(n_t, n+1) = \infty, 1_t, n_t \in V_0$
and $u_1(i_t, j_\theta) = u_0(i_t, j_\theta), (i_t, j_\theta) \in E_0$.

We remark the fact that the static reduced expanded network R_1 is always a partial subnetwork of the static super expanded network R_2. In references (Aronson, 1989) (Ciurea 2002) it is shown that a dynamic flow for T periods in the dynamic network D is equivalent with a static flow in a static reduced expanded network R_1. Since an item released from a node at a specific time does not return to the location at the same or an earlier time, the static networks R, R_2, R_1 cannot contain any circuit, and are therefore acyclic always (Schiopu, 2014).

In the most general dynamic model, the parameter $h(i) = 1$ is waiting time at node i, and the parameter $q(i;t)$ is upper bound for flow $g(i;t)$ that can wait at node i from time t to $t+1$.

The maximum flow problem for T time periods in the dynamic network D formulated in conditions (1), (2), (3) is equivalent with the maximum flow problem in static reduced expanded network R_1 as follows:

$$f_1(i_t, V_1) - f_1(V_1, i_t) = \begin{cases} v_1, & if\ i_t = 0 \quad (4a) \\ 0, & if\ i_t \neq 0, n+1 \quad (4b) \\ -v_1, & if\ i_t = n+1 \quad (4c) \end{cases}$$

$$0 \leq f_1(i_t, j_\theta) \leq u_1(i_t, j_\theta), \quad (i_t, j_\theta) \in E_1 \quad (5)$$

$$\max v_1 \quad (6)$$

where by convention $i_t = 0$ for $t = -1$ and $i_t = n+1$ for $t = T+1$.

If T is very large, then the static reduced expanded network R_1 becomes very large and the number of calculations required to find a maximum flow in network R_1 becomes prohibitively large. Happily, Ford and Fulkerson (Ford and Fulkerson 1962) have devised an algorithm that generates a maximum flow in dynamic network D. This algorithm works only when h and q are constant over time. If h and q are constant over time, then a dynamic network D is said to by stationary.

The algorithm for maximum dynamic flow in stationary dynamic network $D = (N, A, h, q)$ is presented in Figure 1.

1: MDFSDN;
2: BEGIN
3: AMVMCSF $(G, \hat{f}^*)$;
4: ADSFEF $(\hat{f}^*, r(P_1), \dots, r(P_k))$;
5: ARPF $(\hat{f}^*, r(P_1), \dots, r(P_k))$;
6: END.

Fig. 1. *Algorithm for maximum dynamic flow in stationary dynamic network.*

The procedure AMVMCSF performs the algorithm for maximum value and minimum cost flow $\hat{f}^*$ in static network $G = (N, A, c, u)$, where $c(i,j) = h(i,j), u(i,j) = q(i,j), (i,j) \in A$. The procedure AMVMCSF has complexity $O(n, m, \bar{h}, \bar{q})$ with $\bar{h} = \max\{h(i,j) | (i,j) \in A\}, \bar{q} = max\{q(i,j) | (i,j) \in A\}$.

The procedure ADSFEF performs the algorithm for decomposition of the static flow $\hat{f}^*$ in elementary flows (path flows) with $r(P_s)$ the flow along of path $P_s, s = 1, \dots k,$ from source node 1 to sink node n. This algorithm has the complexity $O(m^2)$. We remark the fact that is necessary that $c(P_s) \leq T, s = 1, \dots k$. The procedure ARPF performs the algorithm for to repeat each path flow, starting out from source node 1 at time periods 0 and repeat it after each time period as long as there is enough time left in the horizon for the flow along the path to arrive at the sink node n. This algorithm has complexity $O(nT)$. Hence, the algorithm MDFSDN has complexity $O(\max\{O(n, m, \bar{h}, \bar{u}), nT\})$. The dynamic flow obtained with the algorithm MDFSDN is called temporally repeated flow and has the value:

$$\hat{w} = (T+1)\hat{v}^* - \sum_A h(i,j)\hat{f}^*(i,j) \qquad (12)$$

where $\hat{v}^*$ is value of maximum flow and minimum cost static flow $\hat{f}^*$.

In stationary case the dynamic distances $d(1, i; t), d(i, n; t)$ become the static distances $d(1, i), d(i, n)$.

3. An application of freight systems

The early freight system application of maximal dynamic network flow models arose in the railroad industry in the scheduling of freight cars. Many authors solve a variation of Maximal Dynamic Network Flow Problem that includes time-varying arc capacities, and commodity hold-over prevented in nodes during certain time intervals. The application of his model include to: distribute the maximum number of heavy equipment units from several sinks within a fixed number of periods, maximize the utilization of freight trains, maximize the utilization of a conveyor system in a production plant, maximize the quantity of a perishable product distributed, and maximize the throughput of a communication system over a fixed number of periods. In all cases, the commodity may have to leave a source or arrive at a sink at a fixed interval, perhaps due to loading and unloading new crew availabilities, and certain nodes may not be available for hold-over storage due to parking restrictions (Hamacher and Tjandra, 2001).

From the different cities (towns) $c_1, c_2, \dots, c_{n-1}$, buses leave to go a single destination c_n. For any direct road joining town c_i to c_j, we are given the number $q(i,j)$ of buses that can leave city c_i for c_j in a unit of time; we also know the time $h(i,j)$ required to make this journey. Given the number $b(i)$ of buses available at town c_i at he start of our planning horizon and the number $q(i)$ of buses that can be parked at city c_i, we went to organize traffic routes so that in a given interval of time T, the number of buses arriving at c_n will be as large as possible .

This problem can be formulated as a maximal dynamic network flow problem. We mention that this transportation problem is a special case of classical transportation problem because nodes corresponding cities $c_1, c_2, \ldots, c_{n-1}$ are both source and transfer nodes. The corresponding node for city c_n is the stock node.

This problem can be generalized if considered upper bound (capacities) $q(i,j), q(i)$ and the travel time $h(i,j)$ as functions of time variable t, i.e. we consider
$q(i,j;t), q(i;t)$ and $h(i,j;t)$.

The problem can be solved either through a dynamic approach using dynamic network $D(N,A,h,q)$ either through a static approach using static reduced expanded network $R_1(V_1,E_1,u_1)$.

Example. For the problem presented above we associate a node i for each city c_i. We consider $n = 5$ and $H = \{0,1,2,3,4,5\}$, so $T = 5$. For any direct road joining town i and town j, we are given the arc (i,j).

The support digraph of planar dynamic network is presented in Figure 2. The transit times $h(i,j)$ for all arcs are indicated in Table 1. In Figure 3 is presented the static super expanded network $R_2 = (V_2, E_2, u_2)$ and in figure 4 is presented the static reduced expanded network $R_1 = (V_1, E_1, u_1)$ constructed using the notion of dynamic shortest path.

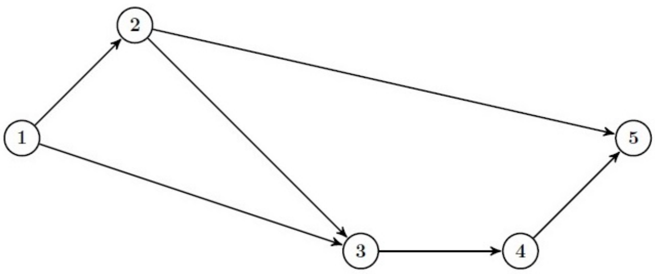

Fig. 2. *The support digraph of network* $D(N,A,h,q)$

(i, j)	(1, 2)	(1,3)	(2, 3)	(2,5)	(3,4)	(4,5)
h (i, j)	1	2	1	3	1	1

Table 1. *The transit times for dynamic network*

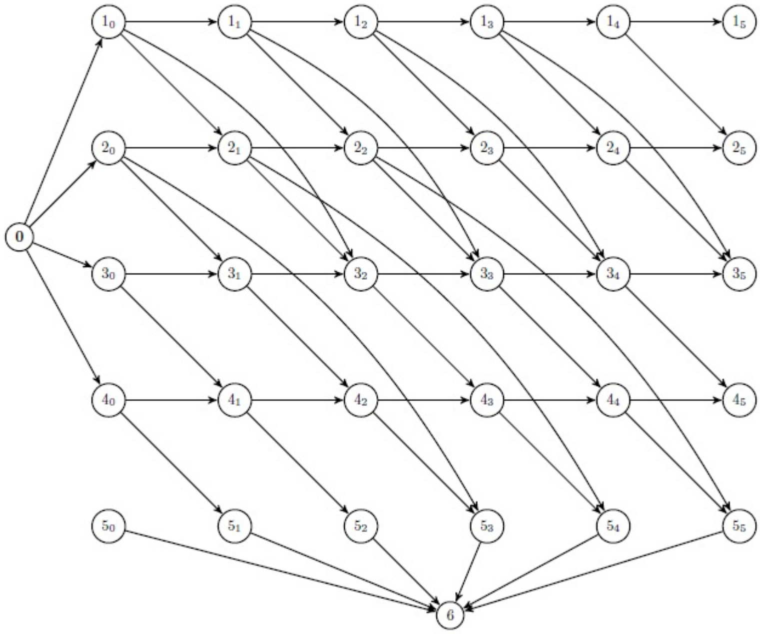

Fig. 3. *Static super expanded network* $R_2 = (V_2, E_2, u_2)$

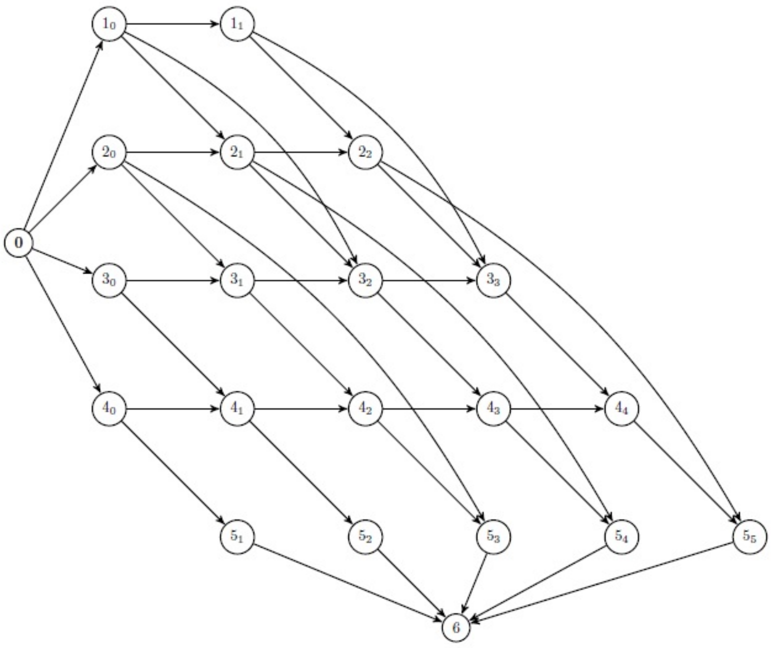

Fig. 4. *Static reduced expanded network* $R_1 = (V_1, E_1, u_1)$.

$u_1(0, i_0) = b(i), \ \ i = 1,2,3,4$

$u_1(i_t, j_\theta) = q(i,j), \ \ (i,j) \in A, \qquad t = 0,1,2,3,4,5$

$u_1(i_t, i_{t+1}) = q(i), \ \ \mathrm{i} = 1,2,3,4.$

The maximum flow problem for T time periods in dynamic network D formulated in conditions (1), (2), (3) is equivalent with the maximum flow problem in static reduced expanded network R_1.

As a conclusion maximum dynamic flows can be used to resolve many practical problems.

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