An assignment model based on similarity measures of intuitionistic fuzzy sets

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Abstract: *First, this paper presents some distance and similarity measures of intuitionistic fuzzy sets. These similarity measures can be applied in models of multi-attribute decision. We propose an assignment model based on similarity measures of intuitionistic fuzzy sets, where the elements of sets are weighted. A numerical example is also given.*

Key-words: *Intuitionistic fuzzy sets, similarity measures, Distance measures, Assignment model*

1. Introduction

The notion of intuitionistic fuzzy sets (IFS), introduced by Atanasov (1986), generalized the concept of fuzzy sets (FS) introduced by Zadeh (1965). The measures of distance and similarity are used to estimate the degree of closeness between two sets. In the models of multi-attribute decision, the distance and the similarity between two IFS is very important (Lixăndroiu and Lixăndroiu, 2013).

Szmidt and Kacprzyk (2000), Hung and Yang (2004, 2008) show several measures for the distance between two IFS and the way the associated similarity measure is constructed. Li Qin and Olson (2007) make a comparative analysis of different defined measures of similarity between two IFS. Xu (2007) develop some similarity measures of IFS and define the notions of positive ideal IFS and negative ideal IFS. These similarity measures are applied to multiple attribute decision making based on intuitionistic fuzzy information.

This article presents some measures for the distance between two IFS and the possibility of obtaining similarity measures. It is known that the two concepts of distance and similarity are dual concepts.

In Ejegwa, Akubo, Joshua (2014) a model of allocation is built based on the distance between two IFS, in which the elements have the same importance.

In this article, based on the measure of similarity between two IFS, which also considers the weights of the elements, we build a weighted allocation model.

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2. Basic concepts

A fuzzy set *(FS)* is defined as follows (Zadeh, 1965): let $X = \{x_1, x_2, ..., x_n\}$ be a universe of discourse, a fuzzy set A is characterised by a membership function $\mu_A : X \to [0,1]$, which associates the degree of membership $\mu_A(x_j)$ to each element $x_j \in X$,

$$A = \{(x_j, \mu_A(x_j)), x_j \in X\} \tag{1}$$

In the particular case, when μ_A only takes the values *0* or *1,* the fuzzy set A is a classical subset of *X.*

Definition 2.1. An intuitionistic fuzzy set *(IFS) A* in *X* is (Atanasov, 1999):

$$A = \{(x_j, \mu_A(x_j), v_A(x_j)), x_j \in X\} \tag{2}$$

which is characterized by a *membership function* μ_A and a *non-membership function* v_A, where:

$$\mu_A : X \to [0,1], \quad x_j \in X \to \mu_A(x_j) \in [0,1] \tag{3}$$
$$v_A : X \to [0,1], \quad x_j \in X \to v_A(x_j) \in [0,1] \tag{4}$$

on condition that

$$\mu_A(x_j) + v_A(x_j) \le 1 \text{ for all } x_j \in X$$

For each *IFS A* in *X,* if

$$\pi_A(x_j) = 1 - \mu_A(x_j) - v_A(x_j) \tag{5}$$

then $\pi_A(x_j)$ is called the *degree of indeterminacy* (or a *hesitation margin*) of x_j to A.
If $\pi_A(x_j) = 1 - \mu_A(x_j) - v_A(x_j) = 0$, for each $x_j \in X$ the *IFS A* is reduced to a classical fuzzy set.

Definition 2.2. (Hung and Yang, 2008)
   If *A* and *B* are two IFS in X**,** then

(i) $A \subseteq B$ if and only if $\forall x \in X$, $\mu_A(x) \le \mu_B(x)$ and $v_A(x) \ge v_B(x)$;
(ii) $A = B$ if and only if $\forall x \in X$, $\mu_A(x) = \mu_B(x)$ and $v_A(x) = v_B(x)$;

(iii) $A^C = \{(x, v_A(x), \mu_A(x))/ x \in X\}$, where A^C denotes the complement of A;

(iv) $A \cap B = \{(x, \min(\mu_A(x), \mu_B(x)), \max(v_A(x), v_B(x)))| x \in X\}$;

(v) $A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)), \min(v_A(x), v_B(x)))| x \in X\}$.

Let $\Phi(X)$ be the set of all IFSs of *X*. We introduce the concepts of *distance measure* and *similarity measure* between two IFSs:

Definition 2.3. (Hung and Yang, 2008)

The real function $d : \Phi(X) \times \Phi(X) \to R^+$ is called a *distance measure* if it satisfies the following properties:

(D1) $d(A, B) = d(B, A), \quad \forall A, B \in \Phi(X)$;

(D2) $d(A, A) = 0, \quad \forall A \in \Phi(X)$;

(D3) $d(D, D^C) = \max\limits_{A, B \in \Phi(X)} d(A, B)$, if *D* is a crisp set;

(D4) If $A \subset B \subset C$, then $d(A, B) \leq d(A, C)$ and $d(B, C) \leq d(A, C)$,

$\forall A, B, C \in \Phi(X)$.

Definition 2.4. (Hung and Yang, 2008)

The real function $S : \Phi(X) \times \Phi(X) \to R^+$ is called a *similarity measure* if it satisfies the following properties:

(S1) $S(A, B) = S(B, A), \quad \forall A, B \in \Phi(X)$;

(S2) $S(D, D^C) = 0, \quad$ if *D* is a crisp set;

(S3) $S(E, E) = \max\limits_{A, B \in \Phi(X)} S(A, B), \quad \forall E \in \Phi(X)$;

(S4) If $A \subset B \subset C$, then $S(A, B) \geq S(A, C)$ and $S(B, C) \leq S(A, C)$,

$\forall A, B, C \in \Phi(X)$.

Remark. Generally, normalized expressions are used for the measures considered:
- for the *distance measure*: $d : \Phi(X) \times \Phi(X) \to [0,1]$

    and $d(A, B) \in [0,1], \quad \forall A, B \in \Phi(X)$;
- for the *similarity measure*: $S : \Phi(X) \times \Phi(X) \to [0,1]$

    and $S(A, B) \in [0,1], \quad \forall A, B \in \Phi(X)$.

In Xu (2007), the property (S3) is replaced by:
$S(A,B) = 1$ if and only if $A = B$.

We may use the *distance measure* to define a *similarity measure.*
   Let f be a monotone decreasing function.
   From $0 \leq d(A,B) \leq 1$ we have $f(0) \geq f(d(A,B)) \geq f(1)$. This implies:

$$0 \leq \frac{f(d(A,B)) - f(1)}{f(0) - f(1)} \leq 1.$$

The *similarity measure* between $A, B \in \Phi(X)$ as follows:

$$S(A,B) = \frac{f(d(A,B)) - f(1)}{f(0) - f(1)} \tag{6}$$

Hung and Yang (2004) give several possibilities for the selection of f.

3. Distance measures and similarity measures of IFSs

Szmidt and Kacprzyk (2000) proposed several distances for IFSs based on the geometric distance model. Xu (2007) generalizes these distances, as follows:

$$d(A,B) = \left[\frac{1}{2} \sum_{j=1}^{n} \left(\left| \mu_A(x_j) - \mu_B(x_j) \right|^\alpha + \left| \nu_A(x_j) - \nu_B(x_j) \right|^\alpha + \left| \pi_A(x_j) - \pi_B(x_j) \right|^\alpha \right) \right]^{\frac{1}{\alpha}} \tag{7}$$

and

$$d(A,B) = \left[\frac{1}{2n} \sum_{j=1}^{n} \left(\left| \mu_A(x_j) - \mu_B(x_j) \right|^\alpha + \left| \nu_A(x_j) - \nu_B(x_j) \right|^\alpha + \left| \pi_A(x_j) - \pi_B(x_j) \right|^\alpha \right) \right]^{\frac{1}{\alpha}} \tag{8}$$

where $\alpha > 0$.

Remark. If $\alpha = 1$, then (7) and (8) are the *Hamming distance* and the *normalized Hamming distance* respectively. If $\alpha = 2$, then (7) and (8) are the *Euclidian distance* and the *normalized Euclidian distance* respectively.

If the weight of the element $x_j \in X$ is considered as $w_j \in (0,1)$, a weighted distance can be defined:

$$d(A,B)=\left[\frac{1}{2}\sum_{j=1}^{n}w_j\left(\left|\mu_A(x_j)-\mu_B(x_j)\right|^\alpha+\left|v_A(x_j)-v_B(x_j)\right|^\alpha+\left|\pi_A(x_j)-\pi_B(x_j)\right|^\alpha\right)\right]^{\frac{1}{\alpha}} \qquad (9)$$

where $w=(w_1,w_2,...,w_n)^T$ is the weight vector of x_j, $j=1,2,...,n$, with the property $\sum_{j=1}^{n}w_j=1$ and $\alpha>0$. The vector w of the weights reflects the relative importance given to each x_j, $j=1,2,...,n$.

Remark. If $\alpha=1$, then (9) is reduced to the *weighted Hamming distance.*

If $w=\left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)^T$, then (9) is reduced to (8). If $\alpha=2$ and $w=\left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)^T$, then (9) is reduced to the *normalized Euclidian distance*.

According to Szmidt and Kacprzyk (2000), these distance measures satisfy the conditions specified in the Definition 2.3.

From (8), a *similarity measure* of A and $B\in\Phi(X)$, $S(A,B)$ can be defined as:

$$S(A,B)=1-\left[\frac{1}{2n}\sum_{j=1}^{n}\left(\left|\mu_A(x_j)-\mu_B(x_j)\right|^\alpha+\left|v_A(x_j)-v_B(x_j)\right|^\alpha+\right.\right.$$
$$\left.\left.+\left|\pi_A(x_j)-\pi_B(x_j)\right|^\alpha\right)\right]^{\frac{1}{\alpha}}. \qquad (10)$$

If we take the weight of each x_j, $j=1,2,...,n$, $x_j\in X$ into account, then

$$S(A,B)=1-\left[\frac{1}{2}\sum_{j=1}^{n}w_j\cdot\left(\left|\mu_A(x_j)-\mu_B(x_j)\right|^\alpha+\left|v_A(x_j)-v_B(x_j)\right|^\alpha+\right.\right.$$
$$\left.\left.+\left|\pi_A(x_j)-\pi_B(x_j)\right|^\alpha\right)\right]^{\frac{1}{\alpha}}. \qquad (11)$$

Remark. If each element has the same importance, i.e. $w = \left(\dfrac{1}{n}, \dfrac{1}{n}, ..., \dfrac{1}{n} \right)^T$, then (11) is reduced to (10).

The similarity measures defined by (10) and (11) satisfy the conditions specified in the Definition 2.4.

4. The assignment model based on similarity measures of IFSs

The classical assignment problem is a special type of linear programming problem where assignees are being assigned to perform tasks (Hillier and Lieberman, 2005).

We reformulate the problem in a way that satisfies the following hypotheses:

1. The number of assignees *(A)* is *m*.
2. The number of tasks *(T)* is *n*.
3. Each task T_j, $j = 1, 2, ..., n$ is characterized by *s* attributes (characteristics) noted by $C = \{C_1, C_2, ..., C_s\}$. The performance level desired for each attribute (characteristic) and for each task is represented by the IFSs, shown as follows:

$$T_j = \left\{ \left(C_k, \mu_{T_j}(C_k), \nu_{T_j}(C_k) \right) \mid C_k \in C \right\} \quad j = 1, 2, ..., n . \tag{12}$$

4. For each assignee, we have the individual performance level for each attribute (characteristic) of the task. This is represented by the IFSs, shown as follows:

$$A_i = \left\{ \left(C_k, \mu_{A_i}(C_k), \nu_{A_i}(C_k) \right) \mid C_k \in C \right\}, \quad i = 1, 2, ..., m . \tag{13}$$

5. The weight vector of the attributes:

$$w = (w_1, w_2, ..., w_s)^T, \text{ where } w_k \geq 0, k = 1, 2, ..., s \text{ and } \sum_{k=1}^{s} w_k = 1 .$$

To solve the problem, we calculate the degree of similarity of assignee A_i and the task T_j, for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$, with the relationship (11) (supposing that $\alpha = 1$):

$$S(A_i, T_j) = 1 - \left[\frac{1}{2} \sum_{k=1}^{s} w_k \cdot \left(\left| \mu_{A_i}(C_k) - \mu_{T_j}(C_k) \right| + \left| \nu_{A_i}(C_k) - \nu_{T_j}(C_k) \right| + \right. \right. \\ \left. \left. + \left| \pi_{A_i}(C_k) - \pi_{T_j}(C_k) \right| \right) \right] \tag{14}$$

The assignee A_i is assigned to task T_j that achieves the maximum the degree of similarity, i.e. $S(A_i, T_j) = \max_i S(A_i, T_j)$.

5. Numerical example

Let $A = \{A_1, A_2, A_3, A_4\}$ be the set of IT graduates, $T = \{T_1, T_2, T_3, T_4\}$ be the IT jobs (e.g. IT programmer, IT tester, Database consultant, Web designer) and $C = \{C_1, C_2, C_3\}$ be the set of the skills needed to the jobs.

The jobs and the performance level desired for each skill are represented by the IFSs and shown as follows:

$$T_1 = \{(C_1, 0.8, 0.1), (C_2, 0.7, 0.1), (C_3, 0.4, 0.2)\}$$
$$T_2 = \{(C_1, 0.7, 0.1), (C_2, 0.9, 0.1), (C_3, 0.8, 0.1)\}$$
$$T_3 = \{(C_1, 0.5, 0.2), (C_2, 0.8, 0.1), (C_3, 0.9, 0.1)\}$$
$$T_4 = \{(C_1, 0.9, 0.1), (C_2, 0.6, 0.2), (C_3, 0.8, 0.1)\}$$

After the various examinations, the graduates obtained the following performance level represented by the IFSs:

$$A_1 = \{(C_1, 0.7, 0.2), (C_2, 0.7, 0.2), (C_3, 0.6, 0.2)\}$$
$$A_2 = \{(C_1, 0.8, 0.1), (C_2, 0.4, 0.1), (C_3, 0.7, 0.1)\}$$
$$A_3 = \{(C_1, 0.6, 0.2), (C_2, 0.8, 0.1), (C_3, 0.7, 0.2)\}$$
$$A_4 = \{(C_1, 0.7, 0.1), (C_2, 0.8, 0.2), (C_3, 0.8, 0.1)\}$$

The weight of the skills (attributes) are: $w = (0.5, 0.2, 0.3)^T$.

We utilize (14) to calculate the degree of similarity between jobs and graduates. It follows that:

Degree of similarity	A_1	A_2	A_3	A_4
T_1	**0.87**	0.85	0.79	0.79
T_2	0.85	0.82	0.90	**0.98**
T_3	0.79	0.71	**0.89**	0.85
T_4	0.82	**0.86**	0.78	**0.86**

The optimal solution is to assign the IT graduate A_1 to the IT job T_1, graduate A_2 to the job T_4, graduate A_3 to the job T_3 and graduate A_4 to the job T_2.

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