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COINTEGRATED-BASED FORECAST OF LONG-RUN RELATIONSHIPS

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Abstract: *The Vector Error Correction Model (VECM) and the Autoregressive Distributed Lag Model (ARDL) are used to estimate the cointegration in the case of long-run relationship of quarterly GDP and Final Consumption in Romania during the period 1995 – 2019. The actual data of 2020 Q1 and Q2 were used to check the best model's validity. The static and dynamic approaches of the ARDL model were used to forecast the Final Consumption for Q3 and Q4 of the year 2020. Applying the cointegration model shows the long term relationship of GDP and Final Consumption, but also the effects of other factors, seen in the differences of Final Consumption from its Long-Run evolution, and comprised in the cointegrating terms.*

Key words: *cointegrating equation, Vector Error Correction Model (VECM), Autoregressive Distributed Lag Model (ARDL), forecasting.*

1. Introduction

The non-stationary time series might generate spurious regressions with consequences over the estimation and inference of the linear model.

The linear regression model of time series requires all variables to be I(0) in order to hold the statistical results. Otherwise, if some or all of the time variables in the regression are I(1) then the usual results could be affected. It is the case of the spurious regression in which the statistical results do not hold when all the explanatory variables are non-stationary, i.e. I(1), and not cointegrated. The regression model with I(1) time series makes sense only when they are cointegrated.

The Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests (1981) for the null hypothesis of non-stationarity of time-series have great success in empirical practice. Some authors agreed that these tests for a unit root against alternative hypotheses of a deterministic trend have low power. Zivot and Andrews (1992) showed that when deterministic trends with structural breaks are considered, the low power problem of these tests is evident. The actual studies of Maddala and Kim (1998), Zivot (2003), and Favero (2014) moved from the debate on deterministic versus stochastic trends within

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the univariate models to multivariate modelling of non-stationary time series. The non-stationarity may be solved with the dynamic multivariate time-series models. Cointegration is important in forecasting when dynamic models are estimated.

2. Data and Methodology

2.1. Objectives and data

The non-stationary time series are cointegrated if a linear combination of them exists, and this should be stationary, i.e. I(0). This linear combination is the long-run equilibrium relationship. The explanation is that I(1) time series having a long-run equilibrium relationship are influenced by the economic forces which correct their evolutionary behaviour, bringing them close to the equilibrium relationship. The correction time of deviations from equilibrium depends on the analysed span size and on the frequency of data. The models of cointegrated time series use long periods of low frequencies data: monthly, quarterly or annual data.

This study is about the cointegration between consumption and income at macroeconomic level, using annual data and quarterly time series of GDP and Final Consumption, in Romania, during the period 1995 Q1 – 2019 Q4.

The main objective is to use the cointegrating relationship when forecasting the quarterly data of Final Consumption depending on GDP.

2.2. Estimating and forecasting with the cointegrating relationship

A regression-based estimation of error correction model is also VECM. A VECM is a VAR model for non-stationary series, known as being cointegrated. The cointegration relations of VECM change the long-run behaviour of the endogenous variables for converging to their cointegrating relationships simultaneous with allowing the short-run adjustments.

The connection between VAR models and cointegration is proved by Granger and Johansen. Granger theorem links cointegration to Error Correction Models (ECM). The Johansen's approach of cointegrating modelling consists in putting together the cointegrating and Error Correction Models into the framework of a VAR model (Zivot, 2003).

Forecasting with VECM supposes firstly to transform the VECM to a VAR model, and then using its algorithms to obtain either the changes in the variables, ΔY_t, or the levels of the variables Y_t (Zivot, 2003).

The ARDL models can be used to identify the long-run relationship between cointegrated variables (Pesaran and Shin, 1999).

3. Results and Discussions

For annual indicators of GDP and FCONS, the number of observations is too small, and when analysing their long-run relationship we will look further to use the quarterly indicators for the same period 1995 -2019 and the names of variables are GDPQ and FCONSQ.

3.1. Non-stationarity of GDP and final consumption in Romania during 1995 - 2019

Seeing the chart from Figure 1, we may conclude that the two series have a similar evolution over time and they are non-stationary.

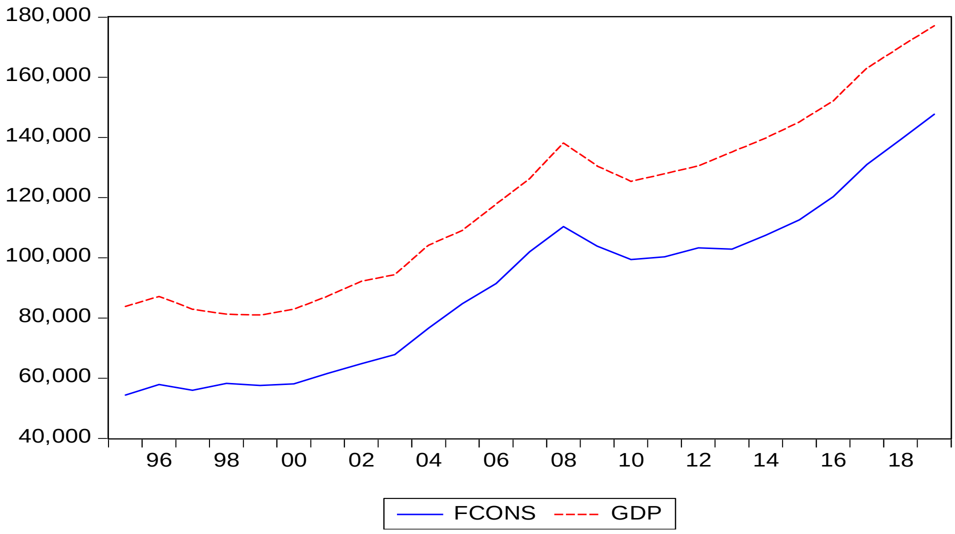


Fig. 1. *Evolution of annual GDP and FCONS during 1995-2019*

The economic crisis which started in 2008 was felt in Romania in 2009 with a deeper decline from previous year than in 1997. The decrease of FCONS variable in 2009 from the year 2008 (-5.9%) was higher than that of GDP (-5.5%).

Fig. 2. *Annual dynamic growth rates (%) of GDP and FCONS during 1995-2019*

The Dickey-Fuller test for both annually series of GDP and Final Consumption (FCONS) conduct us to accept that both have unit roots. The identified *lag length* is 1 for both series. The same results are obtained when applying the Augmented Dickey-Fuller for both series with intercept and 1 lag length identified. For the series of FCONS and GDP we accept H_0 of non-stationarity.

Considering the annual growth rates of GDP and of Final Consumption, in Figure 2, we can see that both series are stationary. When testing the stationarity using the Dickey-Fuller test, we reject H_0, meaning that the series of RFCONS and RGDP do not have unit roots; they are stationary or I(0).

The majority of the economic series are non-stationary in levels, but their growth rates are covariance stationary (Diebold, 2017).

We check the non-stationarity also for the quarterly data of GDP (GDPQ) and the Final Consumption (FCONSQ) between 1995 Q1- 2019 Q4, and the results sustain the non-stationarity of the time series.

3.2. Using VECM to find the cointegrating relationship of FCONSQ and GDPQ

As we have already established, we use the quarterly data of the two series GDP and the Final Consumption to identify the cointegrating relationship. The Granger test shows the causality at 1 lag and we decide to use VECM with 1 lag, in Table 1.

VECM for FCONSQ and GDPQ with 1 lag Table 1

Vector Error Correction Estimates
Sample (adjusted): 1995Q3 2019Q4
Included observations: 98 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
FCONSQ(-1)	1.000000	
GDPQ(-1)	-1.047717	
	(0.02697)	
	[-38.8538]	
C	8384.858	
Error Correction:	D(FCONSQ)	D(GDPQ)
CointEq1	-0.283008	0.015709
	(0.10203)	(0.07289)
	[-2.77378]	[0.21553]
D(FCONSQ(-1))	-0.120756	0.266376
	(0.10478)	(0.07485)
	[-1.15243]	[3.55867]
D(GDPQ(-1))	0.156834	0.045352
	(0.14963)	(0.10689)
	[1.04817]	[0.42430]
C	237.2652	162.8253
	(75.7370)	(54.1030)
	[3.13275]	[3.00954]

VAR Model - Substituted Coefficients:
===============================
D(FCONSQ) = **- 0.2830*(FCONSQ(-1) - 1.0477*GDPQ(-1) + 8384.8580)** - 0.1208*D(FCONSQ(-1)) +
+ 0.1568*D(GDPQ(-1)) + 237.2652

D(GDPQ) = 0.0157***(FCONSQ(-1) - 1.0477*GDPQ(-1) + 8384.8580)** + 0.2664*D(FCONSQ(-1)) +
+ 0.0453*D(GDPQ(-1)) + 162.8253

Adding more lags will eliminate the effects of shocks and the model will continue to decrease the values of information criteria: Akaike and Schwarz, indicating a better model. The system of VECM equations are described in eqn. (1).

$$\begin{aligned} \Delta y_t &= \alpha_{11}\left(\beta_{11} y_{t-1} + \beta_{12} x_{t-1} + \beta_{13}\right) + \gamma_{11}\Delta y_{t-1} + \gamma_{12}\Delta x_{t-1} + \gamma_{14} \\ \Delta x_t &= \alpha_{21}\left(\beta_{11} y_{t-1} + \beta_{12} x_{t-1} + \beta_{13}\right) + \gamma_{21}\Delta y_{t-1} + \gamma_{22}\Delta x_{t-1} + \gamma_{24} \end{aligned} \tag{1}$$

The identified cointegrating term is $FCONSQ_{t-1}$ - 1.0477*$GDPQ_{t-1}$ + 8384.8580 with the cointegrating coefficient $\beta_{12} = 1.0477$ (eq. 1) and the adjustment speed to the equilibrium is the coefficient $\alpha_{11} = -0.2830$ (eq. 1). At any period *t*, 28.3% of the error at *t-1* is subtracted, in order to keep the long-run equilibrium path.

3.3. Modelling the Cointegration with Autoregressive Distributed Lag Model (ARDL)

We used ARDL - Autoregressive Distributed Lag model, which is OLS regression considering the lags of the dependent variable as explanatory variables together with other independent variables and their lags (Greene, 2008). In Table 2, we used ARDL with the automatic selection of the best model.

Table 2

The ARDL for analysing the cointegration between FCONSQ and GDPQ

Dependent Variable: FCONSQ
Method: ARDL
Sample (adjusted): 1995Q3 2019Q4
Included observations: 98 after adjustments
Maximum dependent lags: 4 (Automatic selection)
Model selection method: Akaike info criterion (AIC)
Dynamic regressors (4 lags, automatic): GDPQ
Fixed regressors: C
Number of models evaluated: 20
Selected Model: ARDL(2, 1)
Note: the final equation sample is larger than the selection sample

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
FCONSQ(-1)	0.378386	0.097869	3.866244	0.0002
FCONSQ(-2)	0.300439	0.087559	3.431283	0.0009
GDPQ	0.806689	0.120794	6.678244	0.0000
GDPQ(-1)	-0.469290	0.146417	-3.205166	0.0019
C	-2594.069	684.3363	-3.790635	0.0003

R-squared	0.994603	Mean dependent var	22730.34
Adjusted R-squared	0.994371	S.D. dependent var	7395.437
S.E. of regression	554.8492	Akaike info criterion	15.52494
Sum squared resid	28630764	Schwarz criterion	15.65683
Log likelihood	-755.7222	Hannan-Quinn criter.	15.57829
F-statistic	4284.886	Durbin-Watson stat	1.469349
Prob(F-statistic)	0.000000		

*Note: p-values and any subsequent tests do not account for model selection.

FCONSQ = C(1)*FCONSQ(-1) + C(2)*FCONSQ(-2) + C(3)*GDPQ + C(4)*GDPQ(-1) + C(5)
FCONSQ = 0.3784*FCONSQ(-1) + 0.3004*FCONSQ(-2) + 0.8067*GDPQ - 0.4693*GDPQ(-1) - 2594.0695
Cointegrating Equation:
D(FCONSQ) = -0.3004*D(FCONSQ(-1)) + 0.8067*D(GDPQ) **- 0.3212*(FCONSQ - (1.0505*GDPQ(-1) - 8076.8163))**

In Table 2, Eviews selected the best model, i. e. ARDL (2,1), with 2 lags for FCONSQ and 1 lag for GDPQ, described by eqn. (2a) and the cointegrating relationship, in eqn. (2b).

$$y_t = \alpha_0 + \alpha_1 y_{t-} 1 + \alpha_2 y_{t-2} + \alpha_3 x_t + \alpha_4 x_{t-1} + v_t \quad (2a)$$

$$\Delta y_t = \beta_1 \Delta y_{t-1} + \beta_2 \Delta x_t + \beta_3 \left(y_{t-1} - \left(\tau x_{t-1} + \beta_0\right)\right) + u_t \quad (2b)$$

The cointegrating term or the error correction term, is: $FCONSQ_{t-1}$ - (1.0505*$GDPQ_{t-1}$ - 8076.8163), which ensures the partial short adjustments to the long-run equilibrium. The *cointegrating coefficient* τ which describes the long-run relationship between the two variables is 1.050516. The speed of adjustment to the equilibrium is the coefficient β_3= -0.321175 which shows the proportion of the error of period *t-1* which is considered to correct the path at present period *t*. So, at period *t*, -32.12% of the error at *t-1* is added, in order to be on the long-run equilibrium path.

Equation (2c) started from eqn. (2b) to make the correspondence between the α's coefficients of eqn. (2a) and the β's of eqn. (2b).

$$y_t = -\beta_0 \beta_3 + \left(1 + \beta_1 + \beta_3\right) y_{t-1} - \beta_1 y_{t-2} + \beta_2 x_t - \left(\beta_2 + \tau \beta_3\right) x_{t-1} + v_t \quad (2c)$$

The coefficient of adjustment to the equilibrium from eqn. (2b) β_3 can be obtained based on the coefficients from eqn. (2a) and eqn. (2c):

$$\alpha_1 = 1 + \beta_1 + \beta_3; \ \alpha$$
$$\beta_3 = \alpha_1 + \alpha_{2-1} \text{ i.e. } \beta_3 = 0.3784 + 0.3004 - 1 = -0.3212$$

$$\alpha_1 = 1 + \beta_1 + \beta_3;$$

$\beta_3 = \alpha_1 + \alpha_2 - 1$ i.e. $\beta_3 = 0.3784 + 0.3004 - 1 = 0.3212$.

We see that: $\alpha_3 = \beta_2$ and $\alpha_4 = -\beta_2 - \tau\,\beta_3$.

The coefficient of long-run equilibrium, τ is:

$$\tau = \frac{\alpha_4 + \beta_2}{-\beta_3} = \frac{\alpha_3 + \alpha_4}{-\beta_3} = \frac{\alpha_3 + \alpha_4}{-(\alpha_1 + \alpha_2 - 1)} = \frac{\alpha_3 + \alpha_4}{(1 - \alpha_1 - \alpha_2)} \text{ i.e.}$$

$$\tau = \frac{0.8067 - 0.4693}{(1 - 0.3784 - 0.3004)} = 1.0505\,.$$

The intercept of the cointegrating term is:

$$\beta_0 = \frac{-\alpha_0}{\beta_3} = \frac{-(2594.069)}{-0.3212} = -807.8163\,.$$

The coefficients β's and τ can be found in Table 3, where the information about the *Cointegrating* and *Long-Run Form* is presented.

Table 3

ARDL coefficients - Cointegrating and Long-Run Form

ARDL Cointegrating And Long-Run Form
Dependent Variable: FCONSQ
Selected Model: ARDL(2, 1)
Sample: 1995Q1 2019Q4
Included observations: 98

Cointegrating Form

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(FCONSQ(-1))	-0.300439	0.087559	-3.431283	0.0009
D(GDPQ)	0.806689	0.120794	6.678244	0.0000
CointEq(-1)	-0.321175	0.092624	-3.467511	0.0008

Cointeq = FCONSQ - (1.0505*GDPQ -8076.8163)

Long-Run Coefficients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
GDPQ	1.050516	0.029705	35.364846	0.0000
C	-8076.8163	811.9002	-9.9480	0.0000

If we use ARDL to identify the best number of lags to use, the information criteria are lower in case of ARDL(2,1) compared to VECM with 0 and then with 1 lag.

3.4. Choosing the best model of cointegrating the quarterly GDP and final consumption in Romania

We may conclude that the best model shows that the quarterly consumption is influenced by the consumption of the previous quarter and the two quarters before and by the quarterly GDP of the previous quarter. The two series are cointegrated and they have a long-run relationship.

The informational criteria of the two models used to identify the cointegrating equation which best fits the data are presented in Table 4.

AIC and SC to identify the best model (*) Table 4

Informational Criterion	VECM with 1 lag	ARDL(2,1)*
Akaike AIC	15.8939	15.5249
Schwarz SC	15.9995	15.6568

The cointegrating equation obtained with ARDL is better than that with VECM, because the fit indicators sustain this conclusion. The problem with VECM is that it doesn't allow the selection of different lags for the endogenous and exogenous variables.

The cointegrating equation with ARDL is:

$$\Delta FCONSQ_t=-0.3004\Delta FCONSQ_{t-1}+0.8067\Delta GDPQ_t-0.3212(FCONSQ_{t-1}–1.0505GDPQ_{t-1}+8076.8163)$$

The cointegrating equation with VECM with 1 lag is:

$$\Delta FCONSQ_t=-0.1208\Delta FCONSQ_{t-1}+0.1568\Delta GDPQ_{t-1}-0.2830(FCONSQ_{t-1}- 1.0477GDPQ_{t-1}+8384.858) +237.2652$$

The error correction terms are differently obtained, but approximately close to each other. The error correction terms with VECM improved with 1 lag is $-0.2830(FCONSQ_{t-1}–(1.0477*GDPQ_{t-1}-8384.8580))$ in Table 1, and with ARDL(2,1) it is $-0.3212(FCONSQ_{t-1}-(1.0505*GDPQ_{t-1}–8076.8163))$, in Table 3. The chart of the two cointegrating relationships is presented in Figure 3.

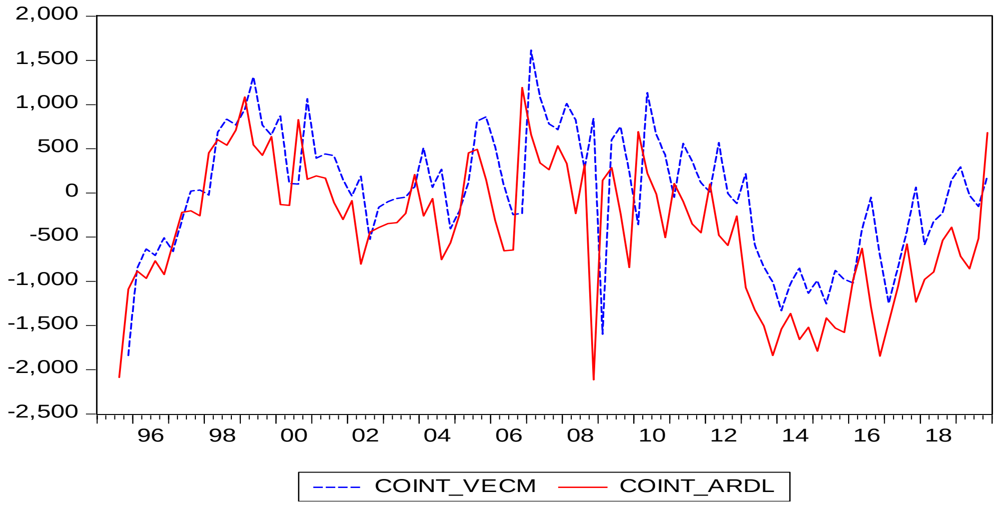


Fig. 3. *The cointegrating terms with ARDL(2,1) and with VECM(1,1)*

The differences between the two approaches consist in one more lag of Y variable considered in the ARDL(2,1) compared to VECM(1,1).

We will further consider the ARDL(2,1) based ECM in this case, to verify the validity of the model and to forecast the future values of the FCONSQ when we know the GDPQ scenarios.

3.5. Using the cointegrating model to forecast final consumption on short term

We accept the model ARDL (2,1) offering the cointegrating equation (eqn. 2b) of the best model:

$$\Delta FCONSQ_t = -0.3004 \times \Delta FCONSQ_{t-1} + 0.8067 \times \Delta GDPQ_t - 0.3212 \times$$
$$\times \left(FCONSQ_{t-1} - 1.0505 \times GDPQ_{t-1} + 8076.8163 \right)$$

So the cointegrating term from Table 3 is: $FCONSQ_{t-1}$-(1.0505*$GDPQ_{t-1}$-8076.8163).

We see in Table 3, that the Long-Run coefficients are significantly different from 0. The equation (3) defines the long-run relationship between FCONSQ and GDPQ:

$$Longr\_FCONSQ_t = 1.0505 \times GDPQ_t - 8076.8163 \tag{3}$$

Figure 4 shows the evolution of FCONSQ, the Long-Run relationship between FCONSQ and GDPQ, described by eqn. (3) and the calculated theoretical values of the quarterly Final Consumption (THEO_FCONSQ) using eqn. (2a) of ARDL(2,1).

The forecasts for the next period can either be calculated by the levels of the variables Y_t using eqn. (2a) or obtaining the changes in the variable, ΔY_t, using eqn. (2b) and then rebuilding the levels (Zivot, 2003).

Having the updated GDPQ and FCONSQ values for 2020 Q1 and 2020 Q2, we check the model validity for these two quarters; the effect of the COVID-19 pandemic can be identified by our model as shown in Table 5; we choose to present the dynamic rates (%) from the previous quarter and use year-on-year basis (y-o-y). We obtain the theoretical quarterly levels of $FCONSQ_t$ using the model ARDL(2,1) from eqn. (2a); we also may use the cointegrating equation from eqn. (2b) when we first obtain the changes $\Delta FCONSQ_{t/t-1}$ and then we rebuild the levels $FCONSQ_t$. The theoretical values THEO_FCONSQ are the same with both approaches.

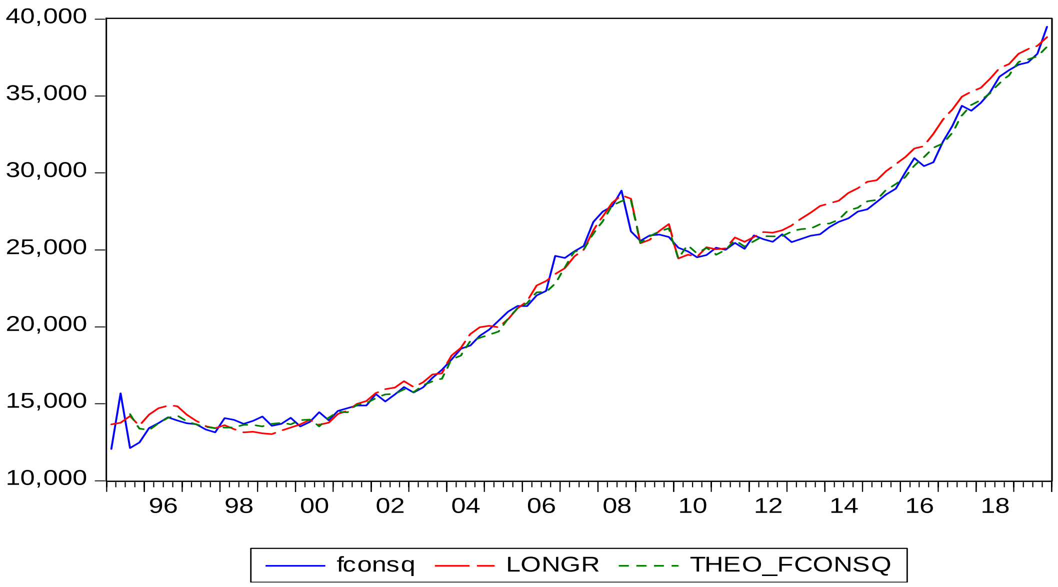


Fig. 4. *FCONSQ, the Long-Run and the Theoretical FCONSQ, during 1995Q1-2019Q4*

To forecast FCONSQ for the next two quarters Q3 and Q4 in 2020 we used two approaches: a static approach when we consider the Long-Run being unchanged and a dynamic one, when the Long-Run is changing after each quarter by considering the previous quarter in a new estimate.

The economic growth of GDP in 2020 was set by different prognoses of specialists. The National Romanian Bank established the economic growth decline at -4.7%, the IMF at -5%, the EU expects the Romanian economy to decline by 6%, and the World Bank has reduced the economic growth forecast from 3.8% to 0.3% for 2020.

We take the scenario of an annual GDP rate of -5%, as we can see in Table 5. Looking at the GDP evolution in the first two quarters, we expect a recovery in the next two quarters of the year 2020, as a result of the economic measures with positive effects undertaken by the government. Further for the next quarters Q3 and Q4, we can make some optimistic scenarios depending on the expectations of GDPQ evolution. We assume a GDPQ increase by 5% in Q3 compared to Q2 and by 2% in Q4 compared to Q3, in order to obtain an annual GDP growth rate of -5%.

In Table 5, the rows corresponding to 2020 Q3 and Q4 are forecast. The GDPQ increase in Q3 by 5% from Q2 means a decline of 6.5% from 2019 Q3. The GDPQ increase by 2% in Q4 compared to Q3 means a decline of 5.8% from 2019 Q4.

Considering the dynamic rates compared to the previous period, when in 2020 Q1 GDPQ slowly increased by 0.3% from 2019 Q4, the Long-Run value based on eqn. (3) showed a FCONSQ change of 1.5% for 2020 Q1, the model indicated an increase of 1.8% and the effective change of FCONSQ was a decline by 2.6%. So the COVID-19 pandemic has already acted in Q1 and the proactive behaviour of the Final Consumption preceded the decline of GDPQ by 12.3% in the next quarter, Q2.

Table 5

Forecasts of dynamic growth rates of Final Consumption for year 2020 [%]

Variables	FCONSQ		GDPQ		Model ARDL for FCONSQ		Long-Run	
Year & quarter	r t/t-1	r t/t-4	r t/t-1	r t/t-4	r t/t-1	r t/t-4	r t/t-1	r t/t-4
Static approach in forecasting for Q3 and Q4								
2020 Q1	-2.6	3.9	**0.3**	**2.7**	1.8	4.5	1.5	4.7
2020 Q2	-14.6	-11.7	**-12.3**	**-10.5**	-11.2	-7.7	0.4	3.2
2020 Q3	*10.3*	*-4.0*	*5.0*	*-6.5*	*5.0*	*-3.6*	*-14.8*	*-12.7*
2020 Q4	*-1.8*	*-10.0*	*2.0*	*-5.8*	*-1.8*	*-6.8*	*6.2*	*-7.9*
Annual growth rate		***-5.4***		***-5.0***		***-3.4***		***-3.2***
Dynamic approach in forecasting for Q3 and Q4								
2020 Q1	-2.6	3.9	**0.3**	**2.7**	1.8	4.5	1.5	4.7
2020 Q2	-14.6	-11.7	**-12.3**	**-10.5**	-11.4	-7.9	0.1	2.9
2020 Q3	*11.3*	*-3.2*	*5.0*	*-6.5*	*5.9*	*-2.7*	*-15.7*	*-13.6*
2020 Q4	*-1.8*	*-9.9*	*2.0*	*-5.8*	*-1.8*	*-6.8*	*5.1*	*-8.8*
Annual growth rate		***-5.2***		***-5.0***		***-3.2***		***-3.7***

The Long-Run in 2020 Q2 indicated a FCONSQ change of 0.4%, the ARDL model offered a deeper decline of 11.2% by considering the cointegrating term, and the effective decline was 14.6%.

We notice that the declines of FCONSQ are higher than those of GDPQ.

When using the quarterly dynamic rates compared to the same quarter of the previous year, i.e. year-on-year basis, we see that the increase of 2.7% of GDPQ in 2020 Q1 compared to GDPQ in 2019 Q1. The GDPQ decreased in 2020 Q2 by 10.5% (y-o-y). The dynamic rates y-o-y of the FCONSQ Long-Run were 4.7% in Q1 and 3.2% in Q2.

Meantime, ARDL model showed FCONSQ changes in Q1 of 4.5% y-o-y in Q1, and -7.7% y-o-y in Q2. The effective FCONSQ decreased in 2020 Q2 by 11.7% y-o-y.

3.5.1. Static approach in forecasting final consumption for 2020 Q3 and Q4

The static approach supposes that the model is that obtained for the sampling period is 1995 Q1 – 2019 Q4. We may obtain the forecast only for a quarter ahead. The forecast can be only for 2020 Q1. When considering the value of FCONSQ for Q1, we can obtain the forecast for 2020 Q2, based on the model estimated for the initial period. Continuing by completing the variable of FCONSQ with its actual value for Q2, we can obtain the theoretical value for Q3. And then, in order to forecast FCONSQ for Q4, we need to add the theoretical value of Q3 at FCONSQ. Having the actual values of FCONSQ only for Q1 and Q2 and the forecasts for Q3 and Q4, we can compare the results offered by the model with the actual dynamic rates, as in Table 5.

The Long-Run of FCONSQ shows a change of -14.8% in Q3 compared to Q2, meaning -12.7% compared to 2019 Q2. The ARDL model gives a FCONSQ increase of 5% in Q3 compared to Q2, but the FCONSQ effective change is expected to increase by 10.3%, i.e. a change of -4.0% y-o-y. These expectations about FCONSQ are considering the theoretical value obtained with the ARDL model for FCONSQ in 2020 Q3.

We can only make FCONSQ forecast for 2020 Q3. If we want to continue forecasting FCONSQ for Q4, then we may suppose the theoretical FCONSQ of Q3 becomes the effective value of FCONSQ in Q3 in order to continue applying the ARDL model to obtain the theoretical FCONSQ for Q4.

Based on the GDPQ growth forecast of 2% in 2020 Q4 from Q3, the Long-Run records a FCONSQ increase of 6.2% compared to Q3, i.e. a change of -7.9% y-o-y. The model shows a FCONSQ change of -1.8% from Q3, and the effective change could be that of the model, -1.8%, i.e. a change of -10% y-o-y.

The annual growth rate of the Long-Run for 2020 is -3.2%; for the theoretical values of FCONSQ obtained with the ARDL model is -3.4%. The economic decline of GDP expected to be -5% in 2020 conducts to an expected average dynamic rate of -5.4% for FCONS.

3.5.2. Dynamic approach in forecasting the final consumption for 2020 Q3 and Q4

In the dynamic approach, for each new quarterly forecast, the model is updating by considering the previous actual quarterly value in the new sample of observations, based on which we estimate the new cointegrating equation and the new value of the Long-Run. The Long-Run is changing after each new quarterly forecast, when changing the sample, this being one more quarter larger. In accordance with the always new cointegrating coefficient, the ARDL model gives new theoretical values.

The dynamic rates of quarterly changes of Long-Run are calculated in Table 5, considering the above idea; also those from the ARDL model.

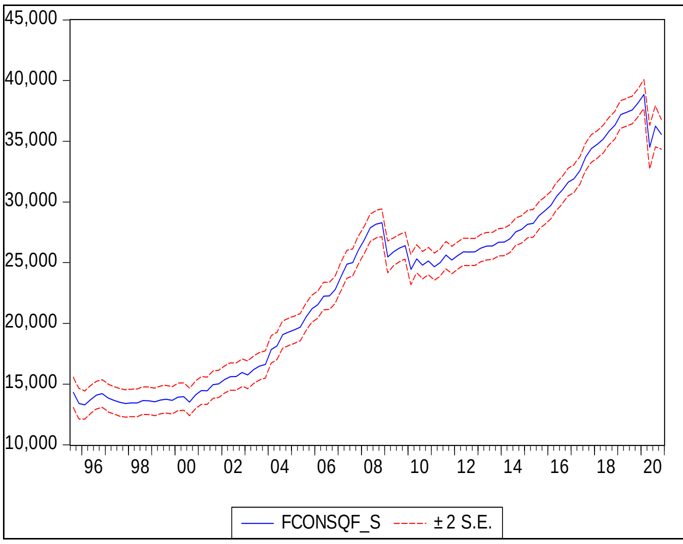
It can be seen in Table 5 that there are not too many differences between the two approaches: static and dynamic, or at quarterly level, for Q3 and Q4, for annual average change rates of Final Consumption.

The annual growth rate of the Long-Run is lesser for the dynamic approach being -3.7% in 2020, but the ARDL model gives an annual change of FCONS of about -3.2% and the expected average dynamic rate of FCONS could be -5.2% in 2020, compared to -5.4% of the static approach.

3.6. The Forecasts quality

The quality of forecasting can be interpreted by looking at the indicators RMSE, MAE, MAPE, Theil Inequality Coefficient and the decomposition of Mean Square Error (MSE) into: bias proportion, variance proportion and covariance proportion (Papell and Prodan, 2018).

Figure 5 presents the quality indicators for the static approach, when using the theoretic value of Q3 as actual for Q3, in order to get the forecast for Q4.



Forecast: FCONSQF_S	
Actual: FCONSQ	
Forecast sample: 1995Q1 2020Q4	
Adjusted sample: 1995Q3 2020Q4	
Included observations: 101	
Root Mean Squared Error	558.8393
Mean Absolute Error	385.9205
Mean Abs. Percent Error	1.806281
Theil Inequality Coefficient	0.011477
Bias Proportion	0.001324
Variance Proportion	0.000472
Covariance Proportion	0.998205

Fig. 5. *The theoretical values of the quarterly Final Consumption – static approach*

Figure 6 contains the quality indicators for the dynamic approach, when using the theoretic the sample 1995Q1 – 2019Q4, in order to get the forecasts for Q3 and Q4.

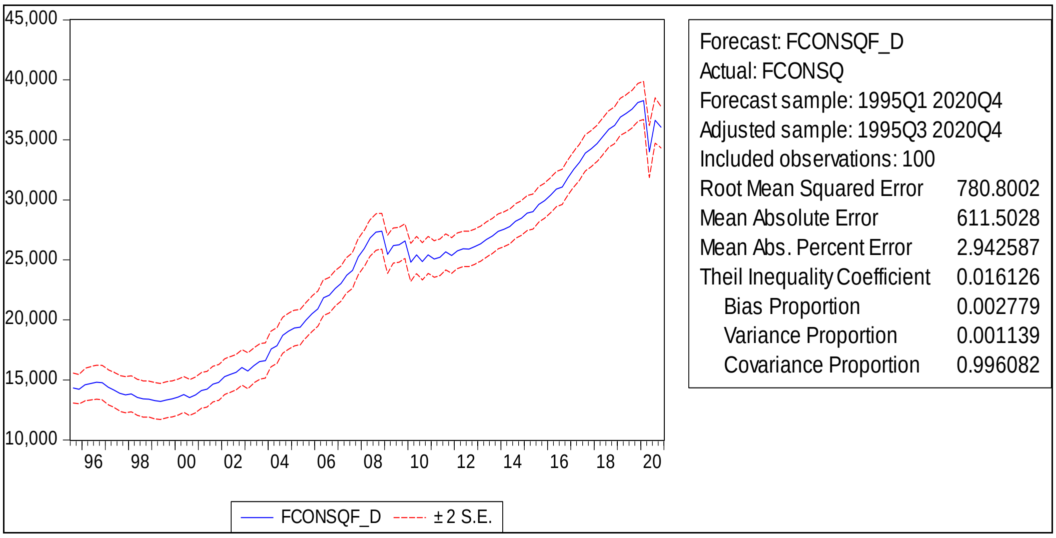


Fig. 6. *The theoretical values of the quarterly Final Consumption - dynamic approach*

The Bias Proportion shows how large the difference is of the mean of the forecast FCONSQF_S, FCONSQF_D respectively, from the mean of the series FCONSQ. The Variance Proportion shows the same thing but for the variation of the forecast FCONSQF_S, FCONSQF_D respectively, from the variation of FCONSQ. These two proportions should be small for a good model. Here we see that both bias and variance proportions are very small, indicating that the models are providing good forecasts. The Covariance Proportion shows the remaining unsystematic forecasting error; this component should represent the greatest part of the mean square error. In the tables of Figures 5 and 6, it can be seen that the covariance proportions are over 99%; we conclude that ECM based on ARDL (2,1) is a very good model. We may decide that the static approach has better quality indicators because the RMSE, MAE, MAPE and Theil Inequality Coefficient are all lower than for the dynamic approach.

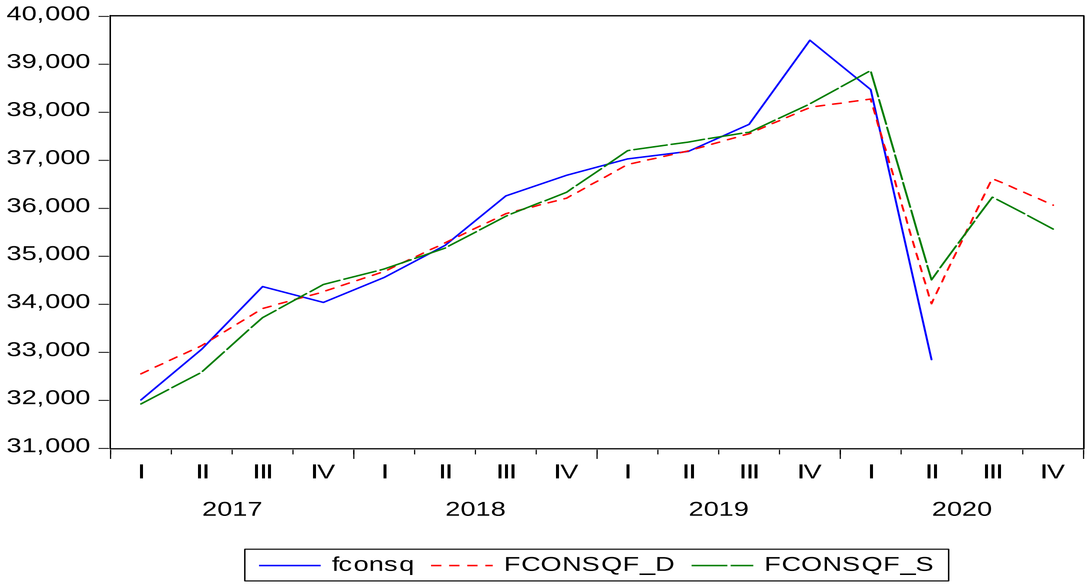


Fig. 7. *The forecasts of the quarterly Final Consumption – static and dynamic ARDL*

Figure 7 presents the forecasts of the quarterly Final Consumption for Q3 and Q4, obtained with the two forecast approaches; they are quite close to each other.

4. Conclusions

We conclude that the two variables GDP and Final Consumption are non-stationary both for the annual and the quarterly data in levels. We cannot say if they are also cointegrated and they keep the characteristics of their Long-Run relationship no matter the data frequency because of the small number of annual data. In this paper, it was proved that the variables are non-stationary and that they are cointegrated for their quarterly data.

The theoretical presentation was applied for the case of the Romanian quarterly GDP and Final Consumption, in levels, for analysing the Long-Run relationship of the non-stationary time series. The undertaken ways of determining the best model conducted us to a dynamic model ARDL. We used the findings to forecast the future value of the Final Consumption, considering the continuously adaptation of the Long-Run to the recent history. The forecasts are important for economists and politicians in establishing the governmental economic policies. The economic decision staff should be interested in measuring the Long-Run relationship of economic variables, in order to take measures to diminish the effects of the random factors.

The theoretical aspects presented in this study are important for the specialists in economic forecasting and modelling the economic relationships. Our study can be useful for everyone wishing to understand and use the cointegrating concept in their scientific research.

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