

WILL THE USE OF *GEOGEBRA* IN GEOMETRY BE FACILITATED BY PROGRAMMING?

Ernest SCHEIBER¹

Abstract

In *GeoGebra*, the development method based on programming for geometry is exemplified by solving some Apollonius problems. For each problem we review the method used to build a *GeoGebra* tool and we reproduce the code. Finally, we indicate findings to facilitate the use of this approach.

2012 ACM Subject Classification: G.4, D.1.m

Key words: *GeoGebra*, programming for geometry, Apollonius problem.

1 Introduction

In the last 30 years geometry software was created with the aim of supporting learning geometry and of visualizing geometric realities. At https://en.wikipedia.org/wiki/List_of_interactive_geometry_software there is a list of such software. An user has the possibility to obtain geometric drawings / constructions, to verify some statements and to visualize some geometric locus. The paper page is replaced by the monitor screen and the ruler and compass are replaced by commands to trace straight lines and circles.

We shall refer to the software *GeoGebra*, [5], [3], currently in continuous development. *GeoGebra* is a free product and is available in the cloud to be used. Plus there is a version for some Android devices. Interactively, picking an action from a menu and the required arguments *GeoGebra* executes the command. If it is the case, *GeoGebra* draws the generated object.

The purpose of this note is to underline another way of usage, closer to the traditional programming. For *GeoGebra* three facilities allow this approach:

1. For any action that can be picked from a menu there is a command as a call of a subroutine. The family of these commands allow users to solve a large set of problems.

¹Faculty of Mathematics and Computer Science, *Transilvania* University of Braşov, Romania, e-mail: scheiber@unitbv.ro

2. There is an *Input* line where it is possible to edit / copy a command.
3. Finally, the geometric construction may be transformed into a tool, a kind of subroutine.

The technique isn't new. It is the development mode for *Geometry Constructions L^AT_EX Converter* (GCLC), [1].

In the next section we exemplify this method giving codes to solve some cases of Apollonius problem. In the last section we indicate findings to facilitate the use of this approach.

2 The Apollonius problem

The Apollonius problem is to find / draw circles tangent to three *things*. The *things* may be points (*P*), (straight) lines (*L*) and circles (*C*). The required circles must be tangent to the given circles and / or lines and pass through given points. It yields 10 problems:

No.	The problem	Abbr.
1	Circle passing through three points	PPP
2	Circle tangent to three lines	LLL
3	Circle passing through two points and tangent to a line	PPL
4	Circle passing through a point and tangent to two lines	PLL
5	Circle tangent to two lines and a circle	LLC
6	Circle passing through a point and tangent to a line and a circle	PLC
7	Circle tangent to a line and two circles	LCC
8	Circle passing through two points and tangent to a circle	PPC
9	Circle passing through a point and tangent to two circles	PCC
10	Circle tangent to three circles	CCC

Here we present the solutions for the last three problems. These problems are correlated, as it is indicated in Fig. 1.

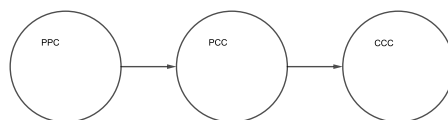


Fig. 1: Dependency graph

The same approach is found in [2], [4], but without providing codes for geometric software. For each problem, we review the method used to build a *GeoGebra* tool and finally we reproduce the result of using that tool.

We used *GeoGebra* 5 (*GeoGebra* classic). With given data the lines of a script must be copied in turn to the *Input* line and executed.

2.1 PPC

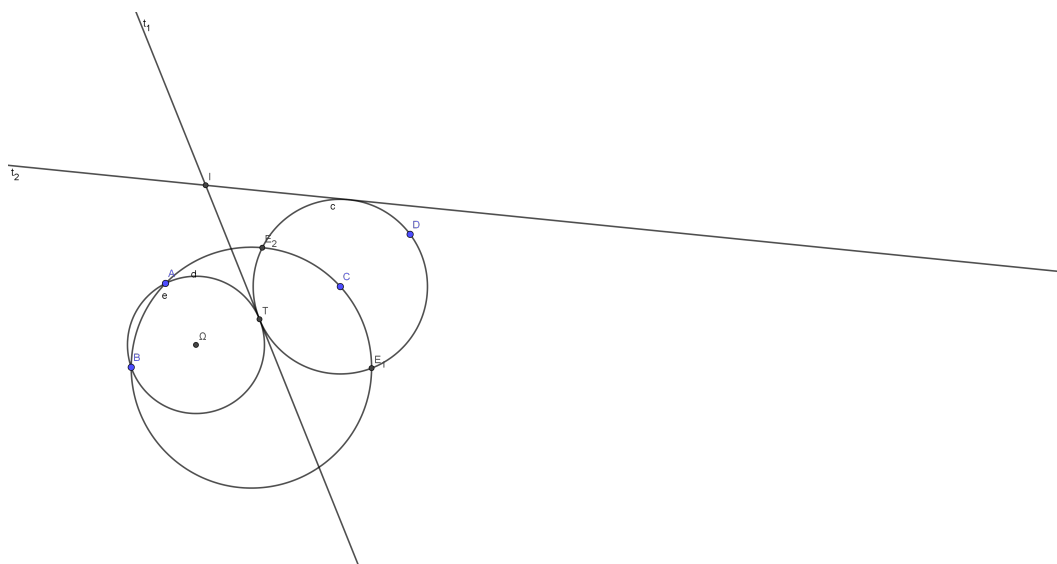


Fig. 2: Reasoning image for solving the PPC problem

Let A and B be two points and the circle $c = \mathcal{C}(C)$, Fig. 2. The circle passing through the points A, B, C intersects c in the points E_1, E_2 . It is supposed that the lines AB and E_1E_2 are not parallel, case that occurs when the point C is on the perpendicular bisector of segment AB .

If $\mathcal{C}(\Omega)$ is a solution of the PPC problem, then the lines AB, E_1E_2 and the exterior tangents to the circle c intersect in I , the radical center of $c, \mathcal{C}(\Omega)$ and $\mathcal{C}(A, B, C)$.

If T is the intersection between an exterior tangent and c then $\mathcal{C}(\Omega)$ is the circle passing through A, B and T . The following script results

1	$d = \text{circle}(A, B, C)$
2	$E = \text{intersect}(c, d)$
3	$I = \text{intersect}(\text{line}(E_1, E_2), \text{line}(A, B))$
4	$t = \text{tangent}(I, c)$
5	$e = \text{circle}(\text{intersect}(t_1, c), A, B)$
6	$g = \text{circle}(\text{intersect}(t_2, c), A, B)$

The results of the PPC tool are given in Fig. 3.

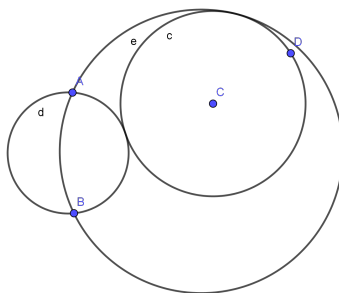


Fig. 3: Results of calling $PPC(A, B, C, D)$

2.2 PCC

We need an additional tool for the similarities centers of two circles. Two circles $\mathcal{C}(O_1, r_1), \mathcal{C}(O_2, r_2)$, not included one in other, have an exterior similarity center. If the circles do not intersect, then there exists an interior similarity center, too. In Fig. 4, the point S_e is the exterior similarity center, while S_i is the interior similarity center.

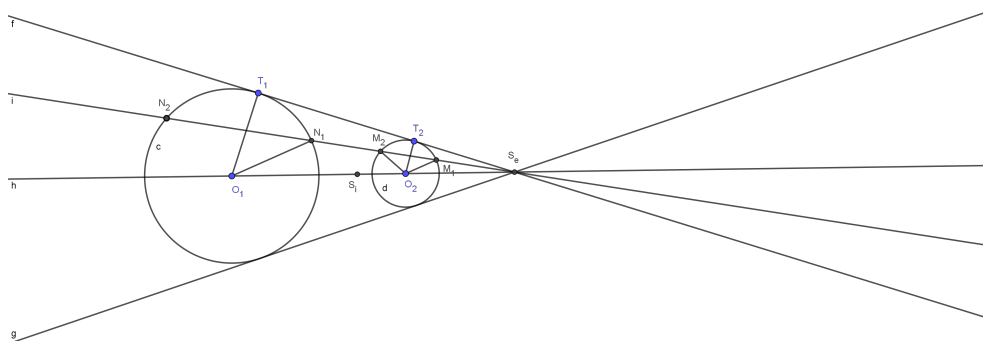


Fig. 4: The similarity centers

If $r_1 > r_2$, then the following relations hold

$$|S_e O_2| = \frac{r_2}{r_1 - r_2} |O_1 O_2|, \quad |S_i O_2| = \frac{r_2}{r_1 + r_2} |O_1 O_2|.$$

Based on the above relations, the *GeoGebra* script to fix the similarity centers is

1	$f = \text{line}(A, C)$
2	$r1 = \text{radius}(c)$
3	$r2 = \text{radius}(d)$
4	$re = \text{min}(r1, r2) / \text{abs}(r1 - r2)$
5	$E = \text{intersect}(f, \text{circle}(if(r1 > r2, C, A), re * \text{segment}(A, C)))$
6	$Se = if(r1 > r2, E2, E1)$
7	$ri = r2 / (r1 + r2)$
8	$F = \text{intersect}(f, \text{circle}(C, ri * \text{segment}(A, C)))$
9	$Si = F1$

We remind some properties related to the similarity centers:

- If the line i intersects the circles c and d in the points N_1, N_2 and respectively M_1, M_2 , Fig. 4, then the triangles $\triangle S_e O_1 N_1$ and $\triangle S_e O_2 M_2$ are similar. As a consequence it results that $O_1 N_1 \parallel O_2 M_1$. A similar result occurs in the case of interior similarity center.
- If a circle $\mathcal{C}(\Omega, \rho)$ is tangent to the circles $\mathcal{C}(O_i, r_i)$, $i \in \{1, 2\}$ with the tangent points T_1, T_2 , Fig. 5, then the points T_1, T_2, S_e are colinear. This may be proved using the Menelaus's Theorem in $\triangle \Omega O_1 O_2$.
- Computing the power of S_e w.r. to the circle $\mathcal{C}(\Omega)$, Fig. 5, we obtain

$$\begin{aligned} \wp(S_e) &= |S_e T_1| \cdot |S_e T_2| = |S_e T_2| \cdot |S_e T'_2| \cdot \frac{|S_e T_1|}{|S_e T'_2|} = \\ &= \wp_{\mathcal{C}(O_2)}(S_e) \cdot \frac{|S_e T_1|}{|S_e T'_2|} = (|S_e O_2|^2 - r_2^2) \cdot \frac{r_1}{r_2} = |S_e O_1| \cdot |S_e O_2| - r_1 r_2. \quad (1) \end{aligned}$$

There is another meaning of the above equality. If we consider the inversion with center S_e mapping the circle $\mathcal{C}(O_2, r_2)$ onto $\mathcal{C}(O_1, r_1)$ then the power of inversion is

$$\rho^2 = |S_e T_1| \cdot |S_e T_2| = |S_e O_1| \cdot |S_e O_2| - r_1 r_2.$$

We turn to the PCC problem.

Point A and circles $\mathcal{C}(O_1, r_1)$ and $\mathcal{C}(O_2, r_2)$ are given. Fig. 5. Let be S_e the exterior similarity center and suppose that the circle $\mathcal{C}(\Omega)$ is a solution to the PCC problem. We denote by

- T_1 and T_2 the tangent points between $\mathcal{C}(\Omega)$ and the given circles.
- B the second intersection between the line AS_e and $\mathcal{C}(\Omega)$.

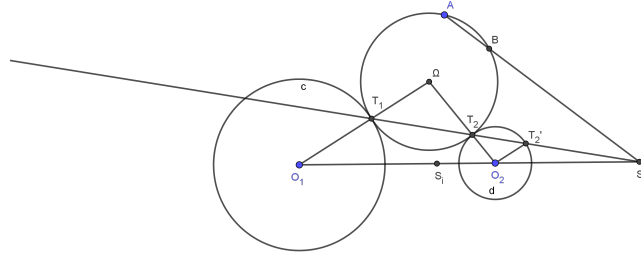


Fig. 5: Reasoning image for solving the PCC problem

The points T_1, T_2, S_e are collinear, (see 2.2).

Computing the power of S_e w.r. to the circle $\mathcal{C}(\Omega)$ we get from (1)

$$\wp(S_e) = (|S_e O_2|^2 - r_2^2) \cdot \frac{r_1}{r_2}$$

and also

$$\wp(S_e) = |S_e A| \cdot |S_e B|.$$

From the above two equalities it results the position of the point B

$$|S_e B| = (|S_e O_2|^2 - r_2^2) \cdot \frac{r_1}{r_2 \cdot |S_e A|}$$

Of course two possible solutions for the point B result. In this way the PCC problem is reduced to the PPC problem: construct the circles passing through the points A, B and tangent to circle $\mathcal{C}(O_1, r_1)$.

Considering the interior similarity center other solutions of the PCC problem result.

The *GeoGebra* script is

1	$S = \text{simCenters}(B, C, D, E)$
2	$r1 = \text{radius}(c)$
3	$r2 = \text{radius}(d)$
4	$s1 = (\text{segment}(S_1, D)^2 - r2^2) * r1 / (r2 * \text{segment}(S_1, A))$
5	$F_1 = \text{intersect}(\text{ray}(S_1, A), \text{circle}(S_1, s1))$
6	$o1 = \text{PPC}(A, F_1, B, C)$
7	$s2 = (\text{segment}(S_2, D)^2 - r2^2) * r1 / (r2 * \text{segment}(S_2, A))$
8	$A' = \text{reflect}(A, S_2)$
9	$F_2 = \text{intersect}(\text{ray}(S_2, A'), \text{circle}(S_2, s2))$
10	$o2 = \text{PPC}(A, F_2, B, C)$

Calling the corresponding *GeoGebra* tool the results are given in Fig. 6.

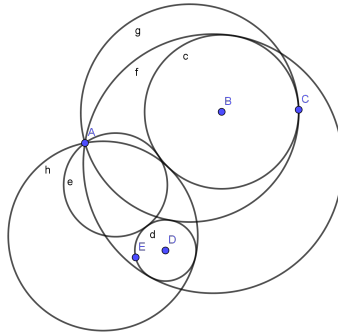


Fig. 6: Results of calling $PCC(A, B, C, D, E)$

2.3 CCC

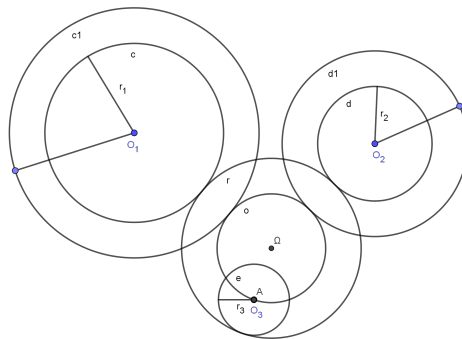


Fig. 7: Reasoning image for solving the CCC problem

Three circles $\mathcal{C}(O_i, r_i)$, $i \in \{1, 2, 3\}$ with $r_1 > r_2 > r_3$ are given. See Fig. 7. If $\mathcal{C}(\Omega, \rho)$ is a solution, then circle $\mathcal{C}(\Omega, \rho - r_3)$ passes through the point O_3 and are tangent to the circles $\mathcal{C}(O_i, r_i + r_3)$, $i \in \{1, 2\}$. This is called the Viète method. Thus the problem CCC is reduced to the problem PCC.

The solutions will be obtained solving four PCC problems, generate the circles passing through O_3 and tangent to circles $\mathcal{C}(O_1, r_1 \pm r_3)$ and $\mathcal{C}(O_2, r_2 \pm r_3)$, considering all combinations of + and - operations.

The corresponding *GeoGebra* script is

1	$c1 = \text{circle}(A, \text{radius}(c) + \text{radius}(e))$
2	$d1 = \text{circle}(C, \text{radius}(d) + \text{radius}(e))$
3	$B1 = \text{point}(c1)$
4	$D1 = \text{point}(d1)$
5	$o1 = \text{PCC}(E, A, B1, C, D1)$
6	$q1 = \text{circle}(\text{center}(o1_2), \text{radius}(o1_2) - \text{radius}(e))$
7	$c2 = \text{circle}(A, \text{radius}(c) - \text{radius}(e))$
8	$d2 = \text{circle}(C, \text{radius}(d) - \text{radius}(e))$
9	$B2 = \text{point}(c2)$
10	$D2 = \text{point}(d2)$
10	$o2 = \text{PCC}(E, A, B2, C, D2)$
11	$q2 = \text{circle}(\text{center}(o2_1), \text{radius}(o2_1) - \text{radius}(e))$
12	$r2 = \text{circle}(\text{center}(o2_2), \text{radius}(o2_2) + \text{radius}(e))$
13	$o3 = \text{PCC}(E, A, B1, C, D2)$
14	$q3 = \text{circle}(\text{center}(o3_3), \text{radius}(o3_3) + \text{radius}(e))$
15	$r3 = \text{circle}(\text{center}(o3_4), \text{radius}(o3_4) - \text{radius}(e))$
16	$o4 = \text{PCC}(E, A, B2, C, D1)$
17	$q4 = \text{circle}(\text{center}(o4_3), \text{radius}(o4_3) + \text{radius}(e))$
18	$r4 = \text{circle}(\text{center}(o4_4), \text{radius}(o4_4) - \text{radius}(e))$

The results of the CCC tool are given in Fig. 8.

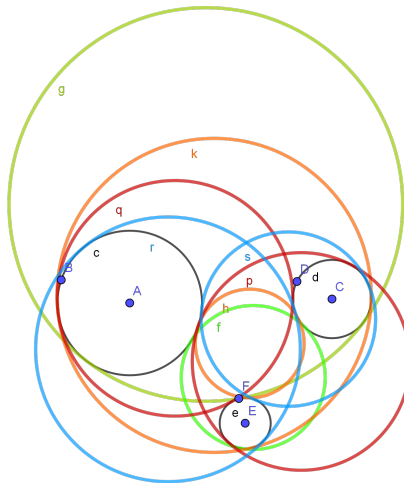


Fig. 8: The results of $\text{CCC}(A, B, C, D, E, F)$

3 Conclusions

Instead of using the mouse to pick command from menus we prefer to edit / copy commands in the *Input* line. That is a more programmable style of using *GeoGebra*, at least formally. It would be useful for *GeoGebra* to offer the following possibilities:

- To upload and process a command file.
- The *Manage Tool* does not allow the possibility to upload tools. If a construction needs two custom tools then we shall start with a file with that tools, created successively, that's an annoying procedure.
- To introduce functions to set properties of objects, i.e. hide an object or a label, the parameters of lines, colors, e.t.c.
- To allow polymorphism tools.
- To offer a public Java API to be used outside *GeoGebra* but accessing *GeoGebra*.

References

- [1] Janičić P., <http://poincare.matf.bg.ac.rs/~janicic//gcllc/>.
- [2] Pamfilos P., *Apollonius problem*. <http://users.math.uoc.gr/~pamfilos/eGallery/problems/ApolloniusProblem.pdf>
- [3] Venema A. G., *Exploring Advanced Euclidean Geometry with GeoGebra*. Mathematical Association of America, 2013.
- [4] * * *, *Construction de cercles*. http://www.debart.fr/pdf/construc_cercle.pdf, 2009.
- [5] * * *, www.geogebra.org.

