Bulletin of the *Transilvania* University of Braşov
Series III: Mathematics and Computer Science, Vol. 4(66), No. 2 - 2024, 1-18
https://doi.org/10.31926/but.mif.2024.4.66.2.1

INFLUENCE OF GRAVITY AND MECHANICAL STRIP LOAD ON MICROPOLAR THERMOELASTIC MEDIUM IN THE CONTEXT OF MULTI-TEMPERATURES THEORY

Elasyed M. ABD-ELAZIZ[1], **Marin MARIN**[2,3] **and Mohamed I. A. OTHMAN**[*,4]

Abstract

A new model of multi-temperatures for a generalized micropolar thermoelastic medium has been established in this paper. A medium is affected by a gravitational field and two types of mechanical strip load (continuous load and impact load). The technique called Laplace Fourier transform has been utilized to obtain the analytical expressions of variables under deliberation. The numerical and graphical illustration of the results has been carried out to indicate the differences among one temperature model, the classical dual-temperature model, and the hyperbolic dual-temperature model upon the Lord and Shulman theory. Also, in the case of Coupled Theory (CT) and Lord and Shulman theory (L-S), we discussed the effect of the gravitational field and mechanical strip load. The most significant points are highlighted. The current investigation has led us to deduce some particular cases of special interest. When it comes to heat conduction's new general model then this study will be extremely beneficial in developing a better understanding of the ingrained features.

2020 Mathematics Subject Classification: 74BXX, 35QXX, 65NXX, 65TXX.

Key words : gravitational field; mechanical strip load; micropolar thermoelasticity; multi-temperatures model; Laplace-Fourier transforms technique.

[1]Ministry of Higher Education, Higher Institute of Eng. & Tech., Zagazig, Egypt, e-mail: sayed_nr@yahoo.com

[2]Department of Mathematics and Computer Science, Transilvania University of Brașov, 500036 Brașov, Romania

[3]Academy of Romanian Scientists, Ilfov Street, No. 3, 050045, Bucharest, Romania

[4*] *Corresponding author*, Department of Mathematics, Faculty of Science, Zagazig University, P.O. Box 44519, Zagazig, Egypt, e-mail: m_i_a_othman@yahoo.com

1 Introduction

The Coupling Theory of thermoelasticity (CT Theory) is the result of mutual coupling between strain and thermal fields. As soon as the coupling fades, no dependence is left between these two fields and thus rises the static problem. The coupling effects were considered in the works of Lesson [12] and Weiner [34]. Taking the usual framework of linear coupled thermoelasticity into account, governing equations of thermoelasticity comprises two types of equations namely "parabolic equation (diffusion-type) of heat conduction" and "hyperbolic equation (wave-type) of motion." The close observation of classical theory reveals that in the case of the energy equation, one particular part of its solution tends to infinity. Thus, it can be inferred that when mechanical or thermal disturbances are present in an "Isotropic homogeneous elastic medium" then even at infinite distances from the disturbance source, there is an instant effect of these displacement and temperature fields. It points out towards physically impossible phenomena of the propagation's velocity being infinite for a part of the solution. To overcome this problem, Lord and Shulman [13] came up with heat conduction's hyperbolic differential equation as a result of modifying Fourier's heat conduction law. It includes the time required for heat flow's acceleration as well as necessary adjustments that have been made for the coupling of strain and temperature fields. A discussion over this paradox in "Coupled Theory of thermoelasticity" has also been carried out by Boley [3]. This ever-new theory is termed as "Generalized theory of thermo-elasticity." The contradiction regarding the propagation at infinite velocity has been eliminated and instead, it now revolves around a comparatively general linear functional relationship between temperature gradients and flow of heat. The term generalized thermoelasticity stands for hyperbolic thermoelasticity is now termed as generalized thermoelasticity according to which whenever a body undergoes a thermo-mechanical load then its transmission proceeds in the form of a wave throughout the body. A "Generalized dynamical theory of thermoelasticity (LS theory)" has been proposed by Lord and Shulman by utilizing Maxwell-Cattaneo law which introduces a single relaxation time to fulfill the acceleration purposes of heat flow by generalizing the "Fourier's equation of heat conduction." The model in a more general parameter, called two thermal relaxation times was introduced by Green and Lindsay [10].

One can assume the micropolar elastic solids to be made up of dumbbell-shaped molecules and these molecules may undergo a linear displacement and rotation around their mass when collected together in the element. Subsequently, the forces stress, as well as couple stress properties, might be present in the micropolar solids at the same time. As per the theory, a couple of stress vectors and a force vector cause the transmission of load across a surface element and to characterize this motion, six degrees of freedom are used in which three for micro-rotation and the rest three for translation. A monography displays the historical development of the theory of micropolar elasticity (see Eringen [8]). The micropolar theory was extended to include thermal effects by Nowacki [16–18] (1966a,b,c), Eringen [7]. The basic equations of the linear theory of micropolar thermoelasticity were derived by Tauchert et al. [33]. Several authors have directed their efforts toward the investigation of problems in the micropolar isotropic thermoelastic media by [2, 14, 15, 19–21, 25, 26, 31, 32]. In the case of deformable bodies, the heat conduction theory relies on dual temperatures namely thermodynamical temperature T, and the conductive temperature θ, was established by Chen and Williams [4]. The conductive temperature is because of thermal processes and thermodynamical temperature is owing to mechanical processes which are inherited between layers of elastic materials and their particles. Chen et al. [5] specified that under certain conditions, equality may exist between these temperatures. The relation between these two temperatures is $T = \theta - a\nabla^2\theta$, generally. In the case of time-dependent scenarios, there is a considerable difference between these temperatures. It is nothing but the temperature discrepancy (a), parameter that separates the classical theory of thermoelasticity from the theory of two-temperature. The theory of two-temperature generalized thermoelasticity together with its uniqueness theorem was constructed by Youssef [35]. The immanent relations and the governing equation for the "Theory of two-temperature generalized micropolar thermoelasticity" were derived by Ezzat and Awad [9]. In multiple research works in [1, 6, 11, 22–24, 27–30] the effect of different fields on generalized thermoelastic problems.

The present problem discussed the 2D problem of a "micropolar thermoelastic medium" with the multi-temperatures model in regards to Lord and Shulman's theory. A medium is affected by a gravitational field and two types of mechanical stip load (continuous load and impact load). The analytical solutions for the field variables of interest are obtained by using Fourier and Laplace transforms. Numerically simulated results are obtained and presented graphically to depict the effect of gravity parameter, continuous load and impact load, and the difference between the two-temperature models on wave propagation in a micropolar thermoelastic.

2 Basic equations

Following Lord and Shulman [13], Tauchert et al. [3] and Youssef [10], the field equations and the constitutive relations in micropolar generalized thermoelastic solid without body forces, body couples and heat sources can be written as

2.1 The constitutive relations

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + 2\mu e_{ij} + k\left(u_{j,i} - \varepsilon_{ijk}\phi_r\right) - \nu T\delta_{ij}, \tag{1}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}. \tag{2}$$

2.2 The strain-displacement relation

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{3}$$

2.3 The equations of motion

$$\sigma_{ji,j} + F_i = \rho u_{i,tt}. \tag{4}$$

2.4 The couple stress equation of motion

$$(\alpha+\beta+\gamma)\nabla(\nabla\cdot\varphi)_i-\gamma\nabla\times(\nabla\times\varphi)_i+k(\nabla\times u)-2k\varphi=j\rho\varphi_{i,tt}. \tag{5}$$

2.5 The heat conduction equation with two-temperature

$$K\theta_{,ii}=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E T+T_0\nu e_{kk}\right), \tag{6}$$

2.6 The equation of multi-temperatures

$$b_0(\theta-T)+c_1(\theta_{,tt}-T_{,tt})=(a+c^2)\theta_{,ii}, \tag{7}$$

where λ and μ indicate to the Lame's constants, the elastic medium density is ρ, K refers to the coefficient of thermal conductivity, the specific heat at constant strain is C_E, the reference temperature is T_0, the thermodynamic temperature is θ, the conductive temperature T, the stress components are σ_{ij}, δ_{ij} is Kroneker delta, k, α,β,γ are the material constants, φ is the micro rotation vector, m_{ij} is the couple stress tensor, j is the microinertia, u_i is the displacement vector, ε_{ijr} is the alternate tensor. The volume thermal expansion is $\nu=(3\lambda+2\mu+k)\alpha_t$, α_t the coefficient of linear thermal expansion, τ_0 is the relaxation time, and b_0, c_1, a, c are constants.

3 Formulation of the problem

Consider a homogeneous, isotropic, rotated thermoelastic half-space under the purview of multi-temperatures theory. Rectangular Cartesian coordinates are introduced having the surface of the half-space as the plane $z=0$, with z-axis pointing vertically downwards into the medium. The present formulation is restricted to $xz-$plane and thus all the field variables are independent of the space variable y. So the displacement vector $u=(u,0,w)$, the microrotation vector $\varphi=(0,\phi_2,0)$, and the dilation is $e=u_{,x}+w_{,z}$. It is also assumed that the plane is under the effect of the gravitational field with gravity force $F_i=(\rho g w_{,x},0,-\rho g u_{,x})$.

Taking into account the above assumptions, the governed and constitutive equations can be written in the form

$$\rho\frac{\partial^2 u}{\partial t^2}=(\lambda+\mu)\frac{\partial e}{\partial x}+(\mu+k)\nabla^2 u-k\frac{\partial\phi_2}{\partial z}-\nu\frac{\partial T}{\partial x}+\rho g\frac{\partial w}{\partial x}, \tag{8}$$

$$\rho\frac{\partial^2 w}{\partial t^2}=(\lambda+\mu)\frac{\partial e}{\partial z}+(\mu+k)\nabla^2 w+k\frac{\partial\phi_2}{\partial x}-\nu\frac{\partial T}{\partial z}-\rho g\frac{\partial u}{\partial x}, \tag{9}$$

$$j\rho\phi_{2,tt}=k\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)+\gamma\nabla^2\phi_2-2k\phi_2, \tag{10}$$

$$K\theta_{,ii}=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\rho C_E T+T_0\nu e_{kk}\right), \tag{11}$$

$$\left(b_0+c_1\frac{\partial^2}{\partial t^2}\right)T=\left[\left(b_0+c_1\frac{\partial^2}{\partial t^2}\right)-\left(a+c^2\right)\nabla^2\right]\theta, \tag{12}$$

$$\sigma_{xx} = \lambda e + (2\mu + k)\frac{\partial u}{\partial x} - \nu T, \tag{13}$$

$$\sigma_{yy} = \lambda e - \nu T, \tag{14}$$

$$\sigma_{zz} = \lambda e + (2\mu + k)\frac{\partial w}{\partial z} - \nu T, \tag{15}$$

$$\sigma_{xz} = \mu\frac{\partial u}{\partial z} + (k + \mu)\frac{\partial w}{\partial x} + k\phi_2, \tag{16}$$

$$\sigma_{zx} = \mu\frac{\partial w}{\partial x} + (k + \mu)\frac{\partial u}{\partial z} - k\phi_2, \tag{17}$$

$$m_{xy} = \gamma\frac{\partial \phi_2}{\partial x}, \tag{18}$$

$$m_{zy} = \gamma\frac{\partial \phi_2}{\partial z}. \tag{19}$$

4 Solution of the problem

For simplifications we shall use the following non-dimensional variables:

$$\begin{gathered} x_i' = \frac{\eta_0}{C_0}x_i,\ \left(u',\, w'\right) = \frac{\rho\eta_0 C_0}{\gamma_1 T_0}\left(u,\, w\right), \\ \left(t',\, \tau_0'\right) = \eta_0\left(t,\, \tau_0\right), \sigma_{ij}' = \frac{\sigma_{ij}}{\nu T_0}, \phi_2' = \frac{\rho C_0^2}{\nu T_0}\phi_2, \\ \left(\theta', T'\right) = \frac{1}{T_0}\left(\theta, T\right), m_{ij}' = \frac{\eta_0}{C_0 \nu T_0}m_{ij}, \eta_0 = \frac{\rho C_E C_0^2}{K}, C_0^2 = \frac{\lambda + 2\mu + k}{\rho}. \end{gathered} \tag{20}$$

Eqs. (18) - (21) take the following form (dropping the dashed for convenience)

$$a_1\nabla^2 u + a_2\frac{\partial e}{\partial x} - a_3\frac{\partial \phi_2}{\partial z} - \frac{\partial \theta}{\partial x} + g\frac{\partial w}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{21}$$

$$a_1\nabla^2 w + a_2\frac{\partial e}{\partial z} + a_3\frac{\partial \phi_2}{\partial x} - \frac{\partial \theta}{\partial z} - g\frac{\partial u}{\partial x} = \frac{\partial^2 w}{\partial t^2}, \tag{22}$$

$$\nabla^2\phi_2 - a_4\phi_2 + a_5\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = a_6\frac{\partial^2 \phi_2}{\partial t^2}, \tag{23}$$

$$\nabla^2\theta = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left(T + a_7 e\right), \tag{24}$$

$$\left(b_0 + c_1\eta_0^2\frac{\partial^2}{\partial t^2}\right)T = \left[\left(b_0 + c_1\eta_0^2\frac{\partial^2}{\partial t^2}\right) - \left(a + c^2\right)\eta_0^2\nabla^2\right]\theta, \tag{25}$$

where $a_1 = \frac{k+\mu}{\rho C_0^2}$, $a_2 = \frac{\lambda+\mu}{\rho C_0^2}$, $a_3 = \frac{k}{\rho C_0^2}$, $a_4 = \frac{2kC_0^2}{\gamma\eta_0^2}$, $a_5 = \frac{kC_0^2}{\gamma\eta_0^2}$, $a_6 = \frac{j\rho C_0^2}{\gamma}$, $a_7 = \frac{\nu^2 T_0}{\rho\eta_0 K}$.

Introducing the displacement potentials $q(x,\, z,\, t)$ and $\psi(x,\, z,\, t)$ which are related to displacement components, we obtain

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{26}$$

The problem is solved using the Laplace and Fourier transform defined by

$$\overline{f}\left(x,z,b\right)=\int_{0}^{\infty}f\left(x,z,t\right)e^{-bt}dt, \tag{27}$$

$$f^{*}\left(s,z,b\right)=\int_{-\infty}^{\infty}\overline{f}\left(x,z,b\right)e^{isx}dx. \tag{28}$$

Using Eqs. (26)-(28) in Eqs. (21)-(25), we get

$$\left[\left(a_1+a_2\right)\left(D^2-s^2\right)-b^2\right]q^{*}-isg\psi^{*}-T=0, \tag{29}$$

$$\left[a_1\left(D^2-s^2\right)-b^2\right]\psi^{*}-a_3\phi_2^{*}+isgq^{*}=0, \tag{30}$$

$$\left[D^2-s^2-a_6b^2-a_4\right]\phi_2^{*}+a_5\left(D^2-s^2\right)\psi^{*}=0, \tag{31}$$

$$\left(D^2-s^2\right)\theta^{*}+\left(b-\tau_0b^2\right)\left[T^{*}+a_7\left(D^2-s^2\right)q^{*}\right]=0, \tag{32}$$

$$T^{*}=\left[1-a_0\left(D^2-s^2\right)\right]\theta^{*}, \tag{33}$$

where $a_0=\frac{(a^2+c^2)\eta_0^2}{b_0+c_1\eta_0^2b^2}$.

Substituting from Eq. (33) into Eqs. (29)-(32), we obtain

$$\left(N_1D^2-N_2\right)q^{*}+N_3\psi^{*}+\left(a_0D^2-N_4\right)\theta^{*}=0, \tag{34}$$

$$\left(a_1D^2-N_5\right)\psi^{*}-a_3\phi_2^{*}+N_3q^{*}=0, \tag{35}$$

$$\left(D^2-N_6\right)\phi_2^{*}+a_5\left(D^2-s^2\right)\psi^{*}=0, \tag{36}$$

$$(N_7D^2-N_8)\theta^{*}+N_9(D^2-s^2)q^{*}=0, \tag{37}$$

where $D=\frac{d}{dz}$, $N_1=a_1+a_2$, $N_2=s^2(a_1+a_2)+b^2$, $N_3=isg$, $N_4=a_0s^2+1$, $N_5=a_1s^2+b^2$, $N_6=s^2+a_4+a_6b^2$, $N_7=1+a_0(\tau_0b^2-b)$, $N_8=N_7s^2-(\tau_0b^2-b)$, $N_9=a_7(b-\tau_0b^2)$.

Eliminating ϕ_2^{*}, ψ^{*} and θ^{*} between Eqs. (34) - (37), we obtain

$$(D^8-A_1D^6+A_2D^4-A_3D^2+A_4)q^{*}(z)=0. \tag{38}$$

In similar manner we arrive at

$$(D^8-A_1D^6+A_2D^4-A_3D^2+A_4)\{\phi_2^{*}(z),\theta^{*}(z),\psi^{*}(z)\}=0, \tag{39}$$

where

$$
\begin{aligned}
A_0 &= \frac{1}{N_9 a_1 a_0 - N_1 a_1 N_7},\\
G_3 &= N_9 N_5 a_0 N_6 - N_2 N_5 N_7 - N_2 a_1 N_6 N_7 + N_2 a_5 a_3 N_7,\\
G_1 &= N_9 a_1 a_0 N_6 + N_9 N_5 a_0 - N_9 a_5 a_3 a_0 - N_2 a_1 N_7 + N_9 a_1 N_4 + N_1 a_5 a_3 N_7,\\
G_2 &= N_9 s^2 a_1 a_0 - N_1 N_5 N_7 - N_1 a_1 N_6 N_7 - N_1 a_1 N_8,\\
A_2 &= A_0(G_3 + G_4 + G_5 + G_6),\ A_1 = A_0(G_1 + G_2),\\
G_4 &= N_9 s^2 a_1 a_0 N_6 + N_9 s^2 a_1 N_4 - N_2 a_1 N_8 + N_1 a_5 a_3 s^2 N_7,\\
A_3 &= A_0(G_7 + G_8 + G_9),\ A_4 = A_0(G_{10} + G_{11}),\\
G_5 &= -N_1 N_5 N_6 N_7 + N_9 a_1 N_4 N_6 - N_1 a_1 N_6 N_8 + N_9 N_5 N_4 + N_9 s^2 N_5 a_0,\\
G_6 &= -N_9 a_5 a_3 N_4 - 2N_9 a_5 a_3 s^2 a_0 + N_1 a_5 a_3 N_8 - N_1 N_5 N_8 - N_3^2 N_7,\\
G_7 &= N_9 s^2 N_5 N_4 - N_3^2 N_8 - N_9 s^4 a_5 a_3 a_0 + N_1 a_5 a_3 s^2 N_8 - N_2 N_5 N_8,\\
G_8 &= N_9 s^2 N_5 a_0 N_6 - N_3^2 N_6 N_7 + N_2 a_5 a_3 N_8 + N_2 a_5 a_3 s^2 N_7 - N_1 N_5 N_6 N_8,\\
G_9 &= N_9 N_5 N_4 N_6 - N_2 N_5 N_6 N_7 - N_2 a_1 N_6 N_8 - 2N_9 a_5 a_3 s^2 N_4 + N_9 s^2 a_1 N_4 N_6,\\
G_{10} &= -N_3^2 N_6 N_8 + N_2 a_5 a_3 s^2 N_8 + N_9 s^2 N_5 N_4 N_6,\\
G_{11} &= -N_9 s^4 a_5 a_3 N_4 - N_2 N_5 N_6 N_8.
\end{aligned}
$$

Eqs. (38) and (39) can be factorized as:

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)\{\psi^*(z),\ q^*(z),\ \phi_2^*(z), \theta^*(z)\} = 0, \quad (40)$$

where k_n^2 $(n = 1, 2, 3, 4)$ are roots of the characteristic Equations of (38) and (39).

The solutions of Eqs. (38), (39) which bounded as $(z \to \infty)$ are given by

$$(q^*, \theta^*, \psi^*, \phi_2^*)\,(z) = \sum_{n=1}^{4} (1, H_{1n}, H_{2n}, H_{3n})\, M_n \exp(-k_n z)\,. \quad (41)$$

From Eq. (41) into Eq. (33) we get the component of conductive temperature

$$T^*(z) = \sum_{n=1}^{4} H_{4n} M_n \exp(-k_n z)\,. \quad (42)$$

Substituting from Eq. (41) into Eq. (26), with the aid of Eqs. (27) and (28), we obtain the components of displacements

$$(u^*, w^*)\,(z) = \sum_{n=1}^{4} (H_{5n}, H_{6n})\, M_n \exp(-k_n z)\,, \quad (43)$$

$$e^*(z) = \sum_{n=1}^{4} M_n H_{7n} \exp(-k_n z)\,. \quad (44)$$

Substituting from Eq. (20) into Eqs. (13)-(19) and with the help of Eqs. (26)-(27) and (41)-(44), we obtain the components of stresses and tangential couple stress

$$(\sigma_{xx}^*, \sigma_{zz}^*, \sigma_{xz}^*, \sigma_{zx}^*)\,(z) = \sum_{n=1}^{4} (H_{8n}, H_{9n}, H_{10n}, H_{11n})\, M_n \exp(-k_n z)\,, \quad (45)$$

$$\left(m_{xy}^*, m_{zy}^*\right)(z) = \sum_{n=1}^{4} (H_{12n}, H_{13n})\, M_n \exp(-k_n z)\,, \quad (46)$$

where $r_1 = \frac{\lambda}{\rho C_0^2}$, $r_2 = \frac{k+2\mu}{\rho C_0^2}$, $r_3 = \frac{\mu}{\rho C_0^2}$, $r_4 = \frac{k+\mu}{\rho C_0^2}$, $r_5 = \frac{k}{\rho C_0^2}$, $H_{13n} = \frac{-\gamma\eta_0^2}{\rho C_0^4} k_n H_{3n}$, $H_{1n} = \frac{N_7 k_n^2 - N_8}{s^2 - N_9 k_n^2}$, $H_{2n} = \frac{N_1 k_n^2 - N_2 + (a_0 k_n^2 - N_4) H_{1n}}{N_3}$, $H_{3n} = \frac{a_5 (k_n^2 - s^2) H_{2n}}{k_n^2 - N_6}$, $H_{4n} = [1 - a_2(k_n^2 - s^2)] H_{1n}$, $H_{5n} = is - k_n H_{2n}$, $H_{6n} = -k_n - isH_{2n}$, $H_{7n} = k_n^2 - s^2$, $H_{8n} = r_1 H_{7n} + isr_2 H_{5n} - H_{4n}$, $H_{9n} = r_1 H_{7n} - k_n r_2 H_{6n} - H_{4n}$, $H_{12n} = \frac{\gamma\eta_0^2}{\rho C_0^4} isH_{3n}$, $H_{10n} = -r_3 k_n H_{5n} + isr_4 H_{6n} + r_5 H_{3n}$, $H_{11n} = isr_3 H_{6n} - r_4 k_n H_{5n} - r_5 H_{3n}$, $n = 1, 2, 3, 4$.

5 Boundary conditions

When boundary surface of half-space is subjected to mechanical strip load, the boundary conditions on the surface at $z = 0$ are as follows:

$$\sigma_{xz} = m_{xy} = 0,\ \sigma_{xx} = -p_1 \delta\left(x\right) F\left(t\right),\ \frac{\partial \theta}{\partial z} = 0, \tag{47}$$

where p_1 is a constant, $\delta(x)$ is the Dirac-delta function.

We consider two types of loads on the plane boundary, for which F is defined below

$$F\left(t\right) = \begin{cases} H(t) & \text{for continuous load} \\ \delta^*(t) & \text{for impact load} \end{cases}, \tag{48}$$

where $H(t) = e^{-t}$ and $\delta^*(t)$ is the heavside function.

5.1 Continuous load

Using the expressions of the variables considered into the above boundary conditions Eqs. (47), we can obtain the following equations satisfied by the parameters M_n $n = 1, 2, 3, 4$:

$$\begin{gathered} \sum\nolimits_{n=1}^{4} H_{10n} M_n = 0, \\ \sum\nolimits_{n=1}^{4} H_{12n} M_n = 0, \\ \sum\nolimits_{n=1}^{4} H_{8n} M_n = -\frac{p_1}{b}, \\ \sum\nolimits_{n=1}^{4} - k_n H_{1n} M_n = 0. \end{gathered} \tag{49}$$

Invoking Eqs. (49), we obtain a system of four equations. After applying the inverse of the matrix method, we have the values of the four constants M_n $(n = 1, 2, 3,\ 4)$.

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{101} & H_{102} & H_{103} & H_{104} \\ H_{121} & H_{122} & H_{123} & H_{124} \\ H_{81} & H_{82} & H_{83} & H_{84} \\ k_1 H_{11} & k_2 H_{12} & k_3 H_{13} & k_4 H_{14} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ -\frac{p_1}{b} \\ 0 \end{pmatrix}. \tag{50}$$

5.2 Impact load

We can obtain the following equations satisfied satisfied by the parameters M_n $n = 1, 2, 3, 4$:

$$\begin{aligned}&\sum\nolimits_{n=1}^{4} H_{10n} M_n = 0,\\ &\sum\nolimits_{n=1}^{4} H_{12n} M_n = 0,\\ &\sum\nolimits_{n=1}^{4} H_{8n} M_n = -p_1,\\ &\sum\nolimits_{n=1}^{4} - k_n H_{1n} M_n = 0.\end{aligned} \tag{51}$$

Thus, we have the values of the four constants M_n $(n = 1, 2, 3,\ 4)$.

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{101} & H_{102} & H_{103} & H_{104} \\ H_{121} & H_{122} & H_{123} & H_{124} \\ H_{81} & H_{82} & H_{83} & H_{84} \\ k_1 H_{11} & k_2 H_{12} & k_3 H_{13} & k_4 H_{14} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ -p_1 \\ 0 \end{pmatrix}. \tag{52}$$

6 Special and particular cases

6.1 If $a = c = c_1 = 0,\ b_0 = 1$, we get the model of one-temperature theory.

6.2 If $c = c_1 = 0,\ b_0 = 1,\ a > 0$, we get the model of classical two-temperature theory.

6.3 If $a = b_0 = 0,\ c_1 = 1,\ c > 0$, we get the model of hyperbolic two-temperature theory.

6.4 Neglecting gravitational field effect i.e $g = 0$ in Eqs. (34)-(37), we get

$$\left(N_1 D^2 - N_2\right) q^* + \left(a_0 D^2 - N_4\right) \theta^* = 0, \tag{53}$$

$$\left(a_1 D^2 - N_5\right) \psi^* - a_3 \phi_2^* = 0, \tag{54}$$

$$\left(D^2 - N_6\right) \phi_2^* + a_5 \left(D^2 - s^2\right) \psi^* = 0, \tag{55}$$

$$\left(N_7 D^2 - N_8\right) \theta^* + N_9 \left(D^2 - s^2\right) q^* = 0. \tag{56}$$

Eliminating q^* and θ^* between Eqs. (53) and (56), we obtain

$$(D^4 - B_1 D^2 + B_2)\{q^*(z), \theta^*(z)\} = 0, \tag{57}$$

The solutions of Eq. (57), which bounded as $(z \to \infty)$ are given by

$$\left(q^*, \theta^*\right)(z) = \sum\nolimits_{n=1}^{2} \left(1, H_{1n}^*\right) R_n \exp\left(-h_n z\right), \tag{58}$$

where h_1^2 and h_2^2 are roots of the characteristic equation of (57).

Also, eliminating ψ^* and ϕ_2^* between Eqs. (54) and (55), we obtain

$$(D^4 - E_1 D^2 + E_2)\{\psi^*(z), \phi_2^*(z)\} = 0. \tag{59}$$

The solutions of Eq. (59), which bounded as $(z \to \infty)$ are given by

$$(q^*, \theta^*)(z) = \sum\nolimits_{m=3}^{4} (1, H_{2m}^*) R_m \exp(-h_n z), \tag{60}$$

where h_3^2 and h_4^2 are roots of the characteristic equation of (60).

The components of displacement, strain, stresses and tangential couple stress are

$$u^*(z) = \sum\nolimits_{n=1}^{2} isR_n \exp(-h_n z) - \sum\nolimits_{m=3}^{4} h_m R_m \exp(-h_m z), \tag{61}$$

$$w^*(z) = \sum\nolimits_{n=1}^{2} - h_n R_n \exp(-h_n z) - \sum\nolimits_{m=3}^{4} isR_m \exp(-h_m z), \tag{62}$$

$$e^*(z) = \sum\nolimits_{n=1}^{2} H_{3n}^* R_n \exp(-h_n z), \tag{63}$$

$$\sigma_{xx}^*(z) = \sum\nolimits_{n=1}^{2} H_{4n}^* R_n \exp(-h_n z) + \sum\nolimits_{m=3}^{4} H_{3m}^* R_m \exp(-h_m z), \tag{64}$$

$$\sigma_{zz}^*(z) = \sum\nolimits_{n=1}^{2} H_{5n}^* R_n \exp(-h_n z) + \sum\nolimits_{m=3}^{4} H_{4m}^* R_m \exp(-h_m z), \tag{65}$$

$$\sigma_{xz}^*(z) = \sum\nolimits_{n=1}^{2} H_{6n}^* R_n \exp(-h_n z) + \sum\nolimits_{m=3}^{4} H_{5m}^* R_m \exp(-h_m z), \tag{66}$$

$$\sigma_{zx}^*(z) = \sum\nolimits_{n=1}^{2} H_{7n}^* R_n \exp(-h_n z) + \sum\nolimits_{m=3}^{4} H_{6m}^* R_m \exp(-h_m z), \tag{67}$$

$$m_{xy}^*(z) = \sum\nolimits_{m=3}^{4} H_{7m}^* R_m \exp(-h_m z), \tag{68}$$

$$m_{zy}^*(z) = \sum\nolimits_{m=3}^{4} H_{8m}^* R_m \exp(-h_m z), \tag{69}$$

where $H_{1n}^* = \frac{N_1 h_n^2 - N_2}{N_4 - a_0 h_n^2}$, $H_{2n}^* = [1 - a_0(h_n - s^2)]H_{1n}^*$, $H_{2m}^* = \frac{a_1 h_m^2 - N_5}{a_3}$, $H_{3n}^* = h_n^2 - s^2$, $H_{4n}^* = r_1 H_{3n}^* - r_2 s^2 - H_{2n}^*$, $H_{3m}^* = -isr_2 h_m$, $H_{5n}^* = r_1 H_{3n}^* - r_2 h_n^2 - H_{2n}^*$, $H_{4m}^* = isr_2 h_m$, $H_{6n}^* = -r_3 ish_n - r_4 ish_n$, $H_{5m}^* = r_3 h_m^2 + r_4 s^2 + r_5 H_{2n}^*$, $H_{7n}^* = -isr_3 h_n - isr_4 h_n$, $H_{6m}^* = r_4 h_m^2 + r_3 s^2 - r_5 H_{2n}^*$, $H_{7m}^* = \frac{is\gamma\eta_0^2 H_{2m}^*}{\rho C_0^4}$, $H_{8m}^* = \frac{-is\gamma\eta_0^2 h_m H_{2m}^*}{\rho C_0^4}$, $n = 1, 2$, $m = 3, 4$.

7 Inversion of the transforms

The transformed displacements, the conductive and thermodynamic temperatures, components of stress, the couple stress, and the micro-rotation are the functions of z and the b and s parameters of Laplace and Fourier transforms, respectively, and thus take the form $f(z, s, b)$. The methods described below are used to attain the solution of the problem (existing in the physical domain) by inverting the Fourier and Laplace transforms (see Shaw and Mukhopadhyay [30]).

8 Numerical results and discussions

The analysis is conducted for a magnesium crystal-like material. Following reference Othman et al. [24], the values of physical constants are:

$T_0 = 298K^\circ$, $\lambda = 9.4 \times 10^{10} Nm^{-2}$, $k = 1.0 \times 10^{10} Nm^{-2}$, $\mu = 4.0 \times 10^{10} Nm^{-2}$, $p_1 = 10$, $\rho = 1.74 \times 10^3 kg/m^3$, $\gamma = 0.779 \times 10^{-9} N$, $j = 0.2 \times 10^{-15} m^2$, $C_E = 1.04 \times 10^3 kgm^{-3}$, $K = 1.7 \times 10^2 Jm^{-1}s^{-1}\deg^{-1}$, $\alpha_t = 7.4033 \times 10^{-7} K^{-1}$, $t = 0.02s$, $\tau_0 = 0.1s$, $x = 0.1m$.

The roots of the characteristic equations of (38), (39), are obtained by using the Matlab program. The comparisons have been made in regards to two theories of thermoelasticity, namely; (L-S) and (CT) theories in three situations:

(i) The difference among one temperature model, classical two-temperature model, and the hyperbolic two-temperature model.

(ii) With and without gravitational field [$g = 9.8$ and $g = 0.0$].

(iii) Dual mechanical loads [Impact load and continuous load].

Figures 1-6 display the component of displacement u, the conductive temperature θ, the stress components σ_{xz}, σ_{xx}, the couple stress m_{xy} and the micro-rotation ϕ_2 distributions, respectively, for the three models: one temperature model (solid line), classical two-temperature model (large dashes line), and hyperbolic two-temperature model (dots line). In fig. 1, the thermal displacement u in opposition to distance z has been plotted for different models. It is visible that the displacement u for the classical two-temperature model is greater than that of the two other models. Fig. 2 depicts the variation of the conductive temperature and the thermodynamic temperature, respectively, in regards to distance z. From these figures we see that the classical two-temperature model has a larger value than that of the two other models. The hyperbolic two-temperature model agrees with one temperature model, where the conductive temperature and the thermodynamic temperature waves vanish before the classical two-temperature model. Figs. 3 and 4 depict the distributions of stresses σ_{xz}, σ_{xx}, in regards to the distance z, for various models. We have noticed that the stress σ_{xz}, favors the problem of boundary condition as it gets initiated from zeros, then it attains its maximum point and later falls to zeros as the distance z increases. In Figs. 5 and 6, the classical two-temperature model has a smaller value than that of the two other models. In these figures, the couple stress m_{xy} and the micro-rotation ϕ_2 have similar behaviors.

Figs. 7-12 show comparisons between the physical quantities with the distance z, at two different values of gravity (with gravity effect $g = 9.8$ and without gravity effect $g = 0.0$), concerning two theories of thermoelasticity namely: Lord and Shulman theory (LS theory), and the coupled theory (CT). Fig. 7 investigates the variation of the displacement component u, versus z. It can merely be noted that the gravity parameter displays a decreasing impact on the magnitudes of u. Fig. 8 displays the variation of the conductive temperature θ, against the distance z. It can be noted that the gravity parameter implies an increasing effect on the magnitude of θ. Figs. 9 and 10 investigate the variation of the stress components σ_{xz}, σ_{xx}, versus z. It can be seen that the gravity parameter shows a decreasing effect on the magnitudes of σ_{xz} and σ_{xx}. The stress components σ_{xz} and σ_{xx}

satisfy all the boundary conditions. Figs. 11 and 12 depict the variation of the couple stress m_{xy} and the micro-rotation ϕ_2 in regards to $z-$axis. It can be seen that the gravity parameter displays an increasing effect on the magnitude of m_{xy} and ϕ_2.

Figs. 13-18 display how the variables under consideration vary under the application of mechanical loads (impact load and continuous load) when $t = 0.02$ and $g = 9.8$. In each graph, there are four curves anticipated by the two theories L-S and CT considered in this work. These figures provide the necessary evidence for the fact that even with different values of amplitude, all the models might act similarly. Fig. 13 displays how the component of displacement u varies against the distance z. It can be noticed that in the case of continuous load, the displacement values u are comparatively small as compared to the impact load in the range $0 \leq z \leq 3.4$, whereas values remain the same for the two cases at $z \geq 3.4$. In fig. 14 the impact load has a greater effect on the conductive temperature θ in comparison to those for the continuous load. Figs. 15 and 16 study fluctuation of the stress components σ_{xz}, σ_{xx} against $z-$the axis. From these figures, the values for the components of stress σ_{xz} and σ_{xx} are affected by the two different mechanical loads. It is clearly visible that in the case of the two theories, the values for the component of tangential stress σ_{xz} initiates from a zero satisfying the boundary conditions. Figs. 17 and 18 explain that variation of the couple stress m_{xy} and the micro-rotation ϕ_2 against the distance z. It is clear that in the case of continuous loads, the magnitudes of m_{xy} and ϕ_2 are smaller in comparison to the magnitudes of impact load for $0 \leq z \leq 4$. It is observed that the couple stress and the micro-rotation began from zeros which favorable the problem boundary conditions.

9 Conclusions

In the bodywork of this paper, a problem of a "Homogeneous micropolar thermoelastic half-space" with multi-temperatures concerning the theory of generalized thermoelasticity has been proposed by Lord and Shulman. On the basis of the results obtained above, it can be concluded that:

1. It is noticed from the figures that the gravitational field plays a significant role in all the field quantities.

2. The figures successfully manifest the phenomenon of propagation at finite speeds.

3. The comparison of various theories of thermoelasticity has been carried out, *i.e.* (LS) theory and (CT) Theory.

4. It is evident from the figures that, there are significant differences among the one temperature model, classical two-temperature model, and hyperbolic two- temperature model.

5. The field quantities are very sensitive to the applied mechanical loads (continuous load and impact load).

6. An extensive spectrum of problems in thermodynamics can be efficiently dealt with using the method mentioned in this article.

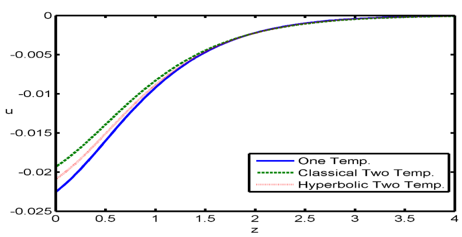

Fig. 1 Variation of thermal displacement u with horizontal distance z

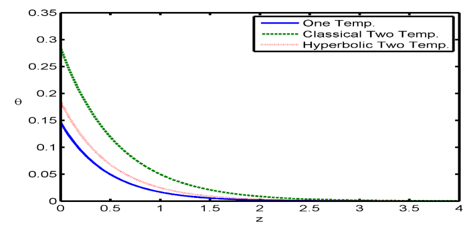

Fig. 2 Variation of conductive temperature θ with horizontal distance z

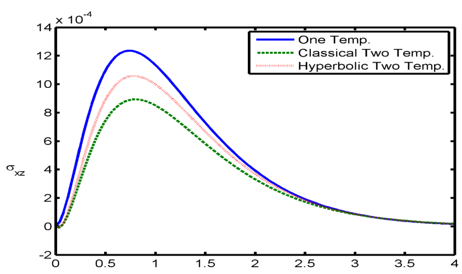

Fig. 3 Variation of normal stress σ_{xz} with horizontal distance z

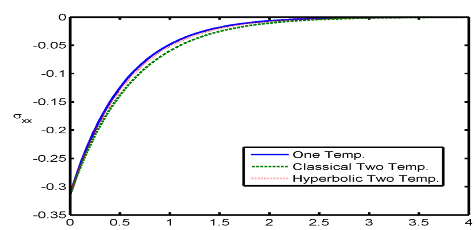

Fig. 4 Variation of tangential stress σ_{xx} with horizontal distance z

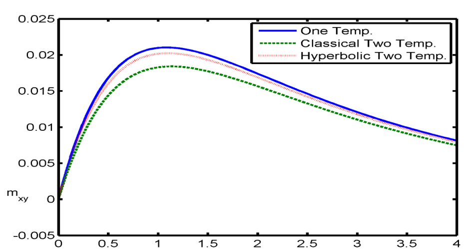

Fig. 5 Variation of couple stress m_{xy} with horizontal distance z

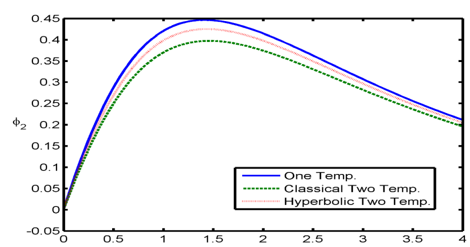

Fig. 6 Variation of micro-rotation ϕ_2 with horizontal distance z

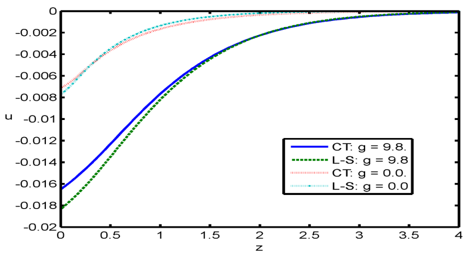

Fig. 7 Variation of thermal displacement u with horizontal distance z

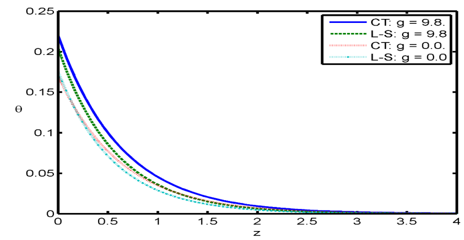

Fig. 8 Variation of conductive temperature θ with horizontal distance z

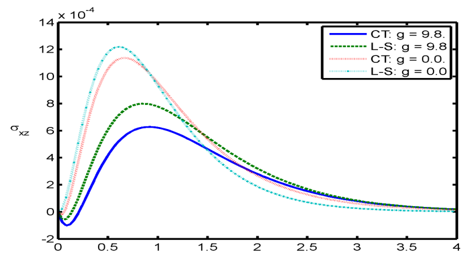

Fig. 9 Variation of tangential stress σ_{xz} with horizontal distance z

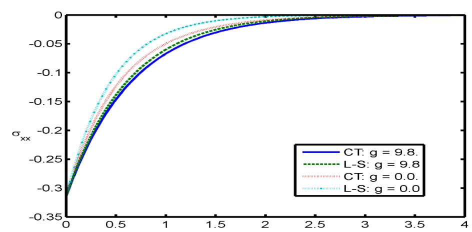

Fig. 10 Variation of normal stress σ_{xx} with horizontal distance z

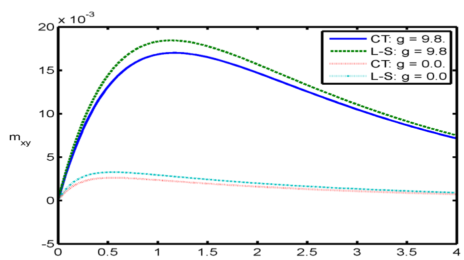

Fig. 11 Variation of couple stress m_{xy} with horizontal distance z

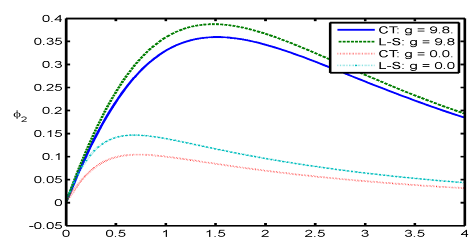

Fig. 12 Variation of micro-rotation ϕ_2 with horizontal distance z

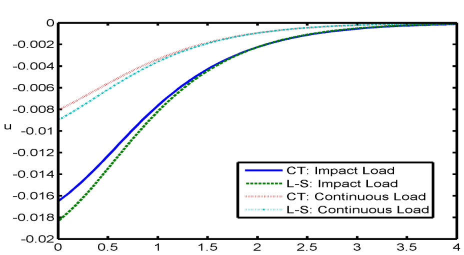

Fig. 13 Variation of thermal displacement u with horizontal distance z

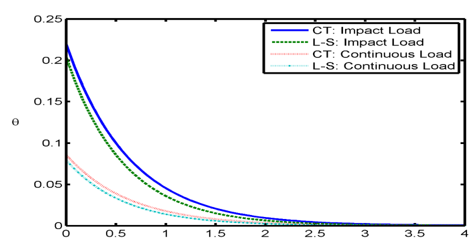

Fig. 14 Variation of conductive temperature θ with horizontal distance z

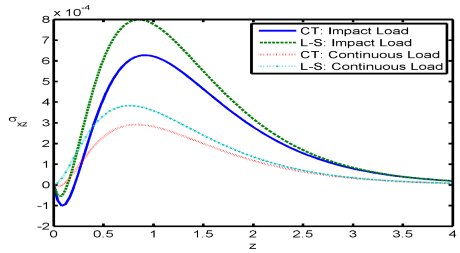

Fig. 15 Variation of normal stress σ_{xz} with horizontal distance z

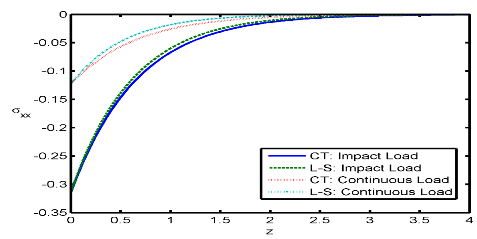

Fig. 16 Variation of tangential stress σ_{xx} with horizontal distance z

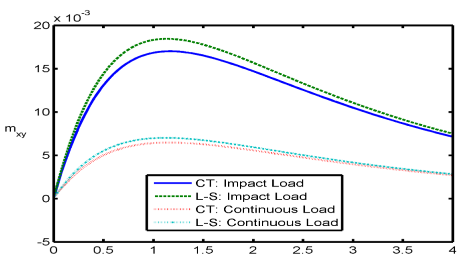

Fig. 17 Variation of couple stress m_{xy} with horizontal distance z

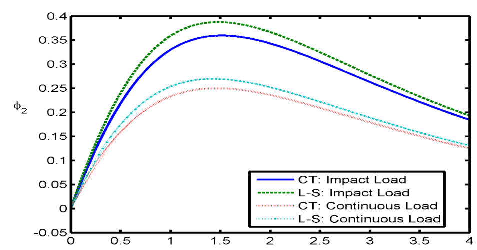

Fig. 18 Variation of micro-rotation ϕ_2 with horizontal distance z

References

[1] Abbas, I.A. and Youssef, H. M., *Two-temperature generalized thermoelasticity under ramp-type heating by finite element method*, Meccanica, **48** (2013), no. 2, 331-339.

[2] Abd-Elaziz, E. M., Marin, M. and Othman, M. I. A., *On the effect of Thomson and initial stress in a thermoporous elastic solid under G-N electromagnetic theory*, Symmetry, **11** (2019), no. 3, 413-430.

[3] Boley, B. A., *High temperature structures and materials*, Pergamon Press, Oxford, 260-315 (1964).

[4] Chen, P.J. and Williams, W.O., *A note on non-simple heat conduction*, Z. Angew. Math. Phys., **19** (1968), 969-970.

[5] Chen, P.J., Gurtin, M.E. and Williams, W.O., *On the thermodynamics of non-simple elastic materials with two temperatures*, Z. Angew. Math. Phys., **20** (1969), 107-112.

[6] El-Karamany, A.S. and Ezzat, M.A., *On the two-temperature Green-Naghdi thermoelasticity theories*, J. Therm. Stress., **34** (2011), 1207-1226.

[7] Eringen, A.C., *Foundation of micropolar thermoelasticity*, Courses and Lectures No. 23, CISM Udine Springer-Verlag, Wien and New York, (1970).

[8] Eringen, A.C., *Microcontinuum field theories I: foundations and solids*, Springer-Verlag, New York, (1999).

[9] Ezzat, M.A. and Awad, E.S., *Constitutive relations, uniqueness of solution, and thermal shock application in the linear theory of micropolar generalized thermoelasticity involving two temperatures*, J. Therm. Stress., **33** (2010), no. 3, 226-250.

[10] Green, A.E. and Lindsay, K.A., *Thermoelasticity*, J. Elasticity, **2** (1972), no. 1, 1-7.

[11] Groza, Gh., Ali Khan, S.M. and Pop, N., *Approximate solutions of boundary value problems for ODEs using Newton interpolating series*, Carpathian J. Math, **25** (2009), no. 1, 73-81.

[12] Lessen, M., *The motion of a thermoelastic solid*, Quart. Appl. Math., **15** (1957), no. 1, 105-108.

[13] Lord, H.W. and Shulman, Y., *A generalized dynamical theory of thermoelasticity*, J. Mech. Phys. Solids, **15** (1967), 299-309.

[14] Marin, M., *Contributions on uniqueness in thermoelastodynamics on bodies with voids*, Cienc. Mat. (Havana), **16** (1998), no. 2, 101-109.

[15] Marin, M., *An uniqueness result for body with voids in linear thermoelasticity*, Rend. Mat. Appl., **17** (1997), 103-113.

[16] Nowacki, W., *Couple stresses in the theory of thermoelasticity*, Bull. Pol. Acad. Sci. Tech. Sci., **14** (1966a), 129-138.

[17] Nowacki, W., *Couple stresses in the theory of thermoelasticity*, Bull. Pol. Acad. Sci. Tech. Sci., **14** (1966b), 263-272.

[18] Nowacki, W., *Couple stresses in the theory of thermoelasticity*, Bull. Pol. Acad. Sci. Tech. Sci., **14** (1966c), 801-809.

[19] Othman, M.I.A., Abd-alla, A.N. and Abd-Elaziz, E.M., *Effect of heat laser pulse on wave propagation of generalized thermoelastic micropolar medium with energy dissipation*, Ind. J. Phys., **94** (2020), 309-317.

[20] Othman, M.I.A. and Abd-Elaziz, E.M., *Effect of rotation on a micropolar magneto-thermoelastic medium with dual-phase-lag model under gravitational field*, Microsyst. Technol., **23** (2017), 4979-4987.

[21] Othman, M. I. A. and Abd-Elaziz, E. M., *Dual-phase-lag model on micropolar thermoelastic rotating medium under the effect of thermal load due to laser pulse*, Ind. J. Phys., 94 (2020), 999-1008.

[22] Othman, M.I.A. and Abouelregal, A.E., *The effect of pulsed laser radiation on a thermoviscoelastic semi-infinite solid under two-temperature theory*, Arch. Thermodyn., **38** (2017), 77-98.

[23] Othman, M.I.A., Fekry, M. and Marin, M., *Plane waves in generalized magneto-thermo-viscoelastic medium with voids under the effect of initial stress and laser pulse heating*, Struct. Eng. Mech., **73** (2020), 621-629.

[24] Othman, M.I.A., Hasona, W. M. and Abd-Elaziz, E. M., *Effect of rotation on micropolar generalized thermoelasticity with two-temperatures using a dual-phase-lag model*, Can. J. Phys., **92** (2014), 149-158.

[25] Othman, M.I.A., Hasona, W.M. and Abd-Elaziz, E.M., *The influence of thermal loading due to laser pulse on generalized micropolar thermoelastic solid with comparison of different theories*, Multidiscip. Model. Mater. Struct., **10** (2014), 328-345.

[26] Othman, M. I. A., Hasona, W. M. and Abd-Elaziz, E. M., *Effect of rotation and initial stress on generalized micropolar thermoelastic medium with three-phase-lag*, J. Comput. Theor. Nanosci., **12** (2015), 2030-2040.

[27] Othman, M.I.A., Tantawi, R.S. and Eraki, E.E.M., *Effect of rotation on a semiconducting medium with two-temperature under L-S theory*, Arch. Thermodyn., **38** (2017), 101-122.

[28] Othman, M.I.A. and Said, S.M., *Effect of diffusion and internal heat source on a two-temperature thermoelastic medium with three-phase-lag model*, Arch. Thermodyn., **39** (2018), 15-39.

[29] Pop, N., *A Finite element solution for a three-dimensional quasistatic frictional contact problem*, Rev. Roumaine des Sciences Tech. Serie Mec. Appliq, **42** (1997), 209-218.

[30] Shaw, S. and Mukhopadhyay, B., *Moving heat source response in micropolar half-space with two-temperature theory*, Contin. Mech. Thermodyn., **25** (2013), 523-535.

[31] Shanker, M.U. and Dhaliwal, R.S., *Dynamic coupled thermoelastic problem in micropolar theory*, Internat. J. Engrg. Sci., **13** (1975), 121-148.

[32] Singh, S.S. and Lianngenga, R., *Effect of micro-inertia in the propagation of waves in micropolar thermoelastic materials with voids*, Appl. Math. Model., **49** (2017), 487-497.

[33] Tauchert, T.R. Claus Jr, W.D. and Ariman, T., *The linear theory of micropolar thermoelasticity*, Internat. J. Engrg. Sci., **6** (1968), 37-47.

[34] Weiner, J.H., *A uniqueness theorem for the coupled thermoelastic problem, Quart. Appl. Math.*, **15** (1957), 102-105.

[35] Youssef, H. M., *Theory of two-temperature-generalized thermoelasticity*, IMA J. Appl. Math., **71** (2006), 383-390.