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A review of the book Diophantine m-tuples and Elliptic Curves

by Andrej Dujella

In my opinion, the present book is a monograph on a very specialized topic in number theory, namely Diophantine m-tuples. The author of the book has been working on this topic for many years. I remember that the first time I met professor Andrej Dujella in 2009 at a winter school of number theory, which took place at the University of Debrecen, he held a very interesting talk, introducing the participants in this topic.

Diophantine m-tuples are related to elliptic curves and certain Diophantine equations, the solving of which requires elliptic curves theory or the hypergeometric method.

The book contains 5 chapters. After an introductory chapter, in which the author briefly mentions the latest results (obtained by several mathematicians) on this topic, including some conjectures, the author devotes three chapters (Chapter 2, Chapter 3 and Chapter 4) to elliptic curves.

Chapter 2 introduces readers in the theory of elliptic curves. The concepts and theorems are presented very clearly. This chapter also contains interesting problems completely solved. Some of them are treated computationally using Pari and Sage softwares. A difficult problem in the theory of elliptic curves is to find the rank of an elliptic curve (over the field of rational numbers or over various algebraic number fields). The author describes the "descent via 2-isogeny" algorithm and other methods for calculating the rank of an elliptic curve, as well as the procedures implemented in Sage, Magma or Pari software packages, in this direction, for example, "mwrank" program by John Cremona, which is included in the SageMath software. The author also presents the connection with the analytic rank of an elliptic curve and the famous Birch and Swinnerton-Dyer (BSD) conjecture.

Chapter 3 deals with those elliptic curves induced by Diophantine triples. In order to study rational Diophantine sextuples are used also Edwards curves or twisted Edwards curves. A very interesting subsection of this chapter is subsection 3.6, entitled *Elliptic curves of high rank with prescribed torsion group.* Andrej Dujella (in his article [90]) is the first mathematician who has used elliptic curves induced by rational Diophantine triples to construct elliptic curves of high rank. This chapter contains many results in this direction, by Andrej Dujella, but also by other mathematicians: Gusic and Tadic, Peral, Klagsbrun, Weger, Tzanakis and others, all these results having been published in prestigious journals. The use of the elements of computational number theory is also very useful in this chapter (for example the "mwrank" program by John Cremona, for calculating the rank of some elliptic curves).

Chapter 4 deals with general methods for finding integer points on elliptic curves. The author makes a nice presentation of Pell equations, Thue equations, transformation of elliptic curves to Thue equations. The author also presents the finding of the whole integer points on some special elliptic curves, namely the Bachet-Mordell curves and the cases when many of these elliptic curves do not have integer points.

The author presents Weger's algorithm for finding integer points on an elliptic curve and Tzanakis' algorithm for finding integer points on a curve of the form $y^2 = ax^4 + bx^3 + cx^2 + d$. From a computational point of view, many functions (implemented in Magma and Sage softwares) are used in these methods: function "integral-points" (in the Sage software package), functions "IntegralPoints" and "IntegralQuarticPoints" (in Magma software). A special place in this chapter is dedicated to the Baker-Davenport theorem and the use of linear forms in logarithms in the study of Diophantine m-tuples.

Chapter 5 is dedicated to one of the generalizations of the notion of Diophantine m-tuples. This chapter contains the most recent results in this direction, obtained by Professor Andrej Dujella, alone or in collaboration with other mathematicians (see [3], [148], [149], [131], [301], [133], etc.).

In my opinion, this book is very complex, containing results from a very interesting and current research field. I think that this book deserved to be published by the Springer publishing house and I think it will be a benchmark for current research in number theory.

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