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# ON LACUNARY $\Delta^m$-STATISTICAL CONVERGENCE OF TRIPLE SEQUENCE IN INTUITIONISTIC FUZZY NORMED SPACE

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**Abstract**

In this study, we define lacunary $\Delta^m$-statistical convergence in the framework of intuitionistic fuzzy normed spaces (IFNS) for triple sequences. We prove several results for lacunary $\Delta^m$-statistical convergence of triple sequence in IFNS. We further established lacunary $\Delta^m$-statistical Cauchy sequences and provided the Cauchy convergence criterion for this novel idea of convergence.

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## 1 Introduction

Fast [11] was first to establish the concept of statistical convergence, which numerous authors have since investigated. Following the publication of Fridy [12, 13], active research on this subject was initiated. Several mathematicians have investigated the features of convergence and statistical convergence and applied them to a variety of fields, including approximation theory [3], finitely additive set functions [2], sequence space [14, 16] and statistical convergence for fuzzy numbers [1, 19].

The notion of fuzziness was provided by Zadeh [25]. A significant number of research publications built on the idea of fuzzy sets/numbers appeared in the literature and has been one of the most active area of research in many branches of sciences. Saadati and Park [20] introduced the concept of intuitionistic fuzzy

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normed space. Recently, R. Antal et.al. [1] studied the concept of $\Delta^m$-statistical convergence of double sequence in intuitionistic fuzzy normed space.

Numerous research on difference sequence spaces and their generalisations have been published in literature [7, 8, 9, 10, 23, 24]. B.C.Tripathy et.al. studied a new type of generalized Difference Cesaro Sequence Spaces [23] and new type of difference sequence spaces [24]. A.Esi studied the generalized difference sequence spaces defined by Orlicz functions [7] and strongly generalized difference $[V^\lambda, \Delta^m, p]$-summable sequence spaces defined by a sequence of moduli [8]. Later on, saveral authors studied generalized $\Delta^m$ Statistical Convergence in Probabilistic Normed Space [9] and generalized Strongly difference convergent sequences associated with multiplier sequences [10], respectively.

Here is an overview of the current work. We review the foundational definitions of the intuitionistic fuzzy normed space in Section 2. Section 3 presents lacunary $\Delta^m$-statistical convergence in intuitionistic fuzzy normed space, where we established several results that demonstrate how generalised this convergence process is. We further established Lacunary $\Delta^m$-statistical Cauchy sequences and provided the Cauchy convergence criterion for this novel idea of convergence.

## 2 Definitions and preliminaries

Here we mention some basic definitions of intuitionistic fuzzy normed space and other preliminaries.

**Definition 1.** *[22] A continuous t-norm is the mapping* $\otimes : [0,1] \times [0,1] \to [0,1]$ *such that*

1. $\otimes$ *is continuous, associative, commutative and with identity 1 ,*
2. $a_1 \otimes b_1 \leq a_2 \otimes b_2$ *whenever* $a_1 \leq a_2$ *and* $b_1 \leq b_2, \forall a_1, a_2, b_1, b_2 \in [0,1]$.

**Definition 2.** *[22] A continuous -conorm is the mapping* $\odot : [0,1] \times [0,1] \to [0,1]$ *such that*

1. $\odot$ *is continuous, associative, commutative and with identity 0 ,*
2. $a_1 \odot b_1 \leq a_2 \odot b_2$ *whenever* $a_1 \leq a_2$ *and* $b_1 \leq b_2, \forall a_1, a_2, b_1, b_2 \in [0,1]$.

**Definition 3.** *[20] An intuitionistic fuzzy normed space (IFNS) is referred to the 5-tuple* $(X, \varphi, \vartheta, \otimes, \odot)$ *with vector space* $X$*, fuzzy sets* $\varphi, \vartheta$ *on* $X \times (0, \infty)$*, continuous t-norm* $\otimes$ *and continuous t-conorm* $\odot$*, if for each* $y, z \in X$ *and* $s, t > 0$*, we have*

1. $\varphi(y,t)+\vartheta(y,t)\leq 1$
2. $\varphi(y,t)>0$ *and* $\vartheta(y,t)\leq 1$,
3. $\varphi(y,t)=1$ *and* $\vartheta(y,t)=0 \Longleftrightarrow y=0$,
4. $\varphi(\alpha y,t)=\varphi\left(y,\frac{t}{|\alpha|}\right)$ *for* $\alpha\neq 0$,
5. $\varphi(y,s)\otimes\varphi(z,t)\leq\varphi(y+z,s+t)$ *and* $\vartheta(y,s)\odot\vartheta(z,t)\leq\vartheta(y+z,s+t)$,
6. $\vartheta(y,o):(0,\infty)\to[0,1]$ *and* $\vartheta(y,o):(0,\infty)\to[0,1]$ *are continuous,*
7. $\lim_{t\to\infty}\varphi(y,t)=1$, $\lim_{t\to 0}\varphi(y,t)=0$, $\lim_{t\to\infty}\vartheta(y,t)=1$ *and* $\lim_{t\to 0}\vartheta(y,t)=0$.

Then $(\varphi,\vartheta)$ is known as intuitionistic fuzzy norm.

**Definition 4.** *[20] Let $(X,\|o\|)$ be any normed space. For every $t>0$ and $y\in X$, take $\varphi=\frac{t}{t+\|y\|}$, $\vartheta=\frac{\|y\|}{t+\|y\|}$. Also, $a\otimes b=ab$ and $a\odot b=\min\{a+b,1\}$ $\forall\, a,b\in[0,1]$. Then, a 5-tuple $(X,\varphi,\vartheta,\otimes,\odot)$ is an IFNS which satisfies the above mentioned conditions.*

**Definition 5.** *[20] Let $(X,\varphi,\vartheta,\otimes,\odot)$ be an* IFNS *with norm $(\varphi,\vartheta)$. A sequence $y=(y_k)$ in $X$ is called convergent to some $\xi\in X$ with respect to the intuitionic fuzzy norm $(\varphi,\vartheta)$ if there exists $k_0\in\mathbb{N}$ for each $\epsilon>0$ and $t>0$ such that $\varphi(yk-\xi,t)>1-\epsilon$ and $\vartheta(yk-\xi,t)<\epsilon$ for all $k\geq k_0$. It is denoted by $(\varphi,\vartheta)-\lim_{k\to\infty}y_k=\xi$.*

**Definition 6.** *[20] Let $(X,\varphi,\vartheta,\otimes,\odot)$ be an* IFNS *with norm $(\varphi,\vartheta)$. A sequence $y=(y_k)$ in $X$ is called convergent to some $\xi\in X$ with respect to the intuitionic fuzzy norm $(\varphi,\vartheta)$ if there exists $k_0\in\mathbb{N}$ for each $\epsilon>0$ and $t>0$*

$$\delta\left(\{k\in\mathbb{N}:\varphi\left(y_k-\xi,t\right)\leq 1-\epsilon \quad or \quad \vartheta\left(y_k-\xi,t\right)\geq\epsilon\}\right)=0.$$

*It is denoted by $S^{\varphi,\vartheta}-\lim_{k\to\infty}y_k=\xi$.*

A subset $E$ of the set $\mathbb{N}$ of natural numbers is said to have a "natural density" $\delta(E)$ if

$$\delta(E)=\lim_{n}\frac{1}{n}|\{k\leq n:k\in E\}|,$$

where the vertical bars denote the cardinality of the enclosed set.

The number sequence $x=(x_k)$ is said to be statistically convergent to number $l$ if for each $\epsilon>0$,

$$\lim_{n} \frac{1}{n} \left|\{k \leq n : |x_k - l| \geq \epsilon\}\right| = 0,$$

and $x$ is said to be statistically cauchy sequence if for every $\epsilon > 0$ there exists a number $N = N(\epsilon)$ such that

$$\lim_{n} \frac{1}{n} \left|\{k \leq n : |x_k - x_N| \geq \epsilon\}\right| = 0.$$

**Definition 7.** *[20] Let $(X, \varphi, \vartheta, \otimes, \odot)$ be an* IFNS *with norm $(\varphi, \vartheta)$. A double sequence $y = (y_{jk})$ in $X$ is called statistically convergent to some $\xi \in X$ with respect to the intuitionic fuzzy norm $(\varphi, \vartheta)$ if there exists $k_0 \in \mathbb{N}$ for each $\epsilon > 0$ and $t > 0$*

$$\delta\left(\{k \in \mathbb{N} : \varphi\left(y_{jk} - \xi, t\right) \leq 1 - \epsilon \text{ or } \vartheta\left(y_{jk} - \xi, t\right) \geq \epsilon\}\right) = 0.$$

*It is denoted by $S^{(\varphi,\vartheta)} - \lim_{k\to\infty} y_{jk} = \xi$.*

The function $X : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(C)$ can be used to define a triple sequence (real or complex), where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ stand for the sets of natural, real, and complex numbers, respectively. At the beginning, Sahiner et al. [21] introduced and studied the many conceptions of triple sequences and their statistical convergence. Triple sequence statistical convergence on probabilistic normed space was recently introduced by Savas and Esi [6], where as statistical convergence of triple sequences in topological groups was later introduced by Esi [5]. For further study on triple sequence spaces, we may refer to [14, 15, 16, 17].

Kizmaz [18] introduced the difference sequence space $Z(\Delta)$ as given below

$$Z(\Delta) = \{y = (y_k) : (\Delta y_k) \in Z\}$$

for $Z = \ell_\infty, c, c_0$ i.e. spaces of all bounded, convergent and null sequences respectively, where $\Delta_y = (\Delta y_k) = (y_k - y_{k+1})$. In particular, $\ell_\infty(\Delta), c(\Delta)$ and $c_0(\Delta)$ are also Banach spaces, relative to a norm induced by $\|y\|_\Delta = |y_1| + \sup_k |\Delta y_k|$.

The generalized difference sequence spaces $Z\left(\Delta^m\right)$ was introduced by [4] as follows :

$$Z\left(\Delta^m\right) = \{y = (y_k) : (\Delta^m y_k) \in Z\}$$

for $Z = \ell_\infty, c, c_0$ where $\Delta^m(y) == (\Delta^m y_k) = (\Delta_{m-1} y_k - \Delta_{m-1} y_{k+1})$. So that

$\Delta^m y_k = \sum_{r=0}^{p} (-1)^r \begin{pmatrix} m \\ r \end{pmatrix} x_{k+r}$.

The difference operator $\Delta$ on triple sequence $x_{mnl}$ is defined as :

$\Delta_{x_{mnl}} = x_{mnl} - x_{(m+1)nl} - x_{m(n+1)l} - x_{mn(l+1)} = x_{(m+1)(n+1)l} + x_{(m+1)n(l+1)} + x_{m(n+1)(l+1)} - x_{(m+1)(n+1)(l+1)}$.

The generalized difference spaces for triple sequences can be approximated as:

$$Z(\Delta^m) = \{y = (y_{jkl}) : (\Delta^m y_{jkl}) \in Z\}$$

for $Z = \ell_\infty^3, c^3, c_0^3$ where $\Delta^m(y) == (\Delta^m y_{jkl}) = \left(\Delta_{m-1} y_{jkl} - \Delta_{m-1} y_{jk,(l+1)}\right)$. So that $\Delta^m y_k = \sum_{r=0}^{p} (-1)^{r+s+u} \begin{pmatrix} m \\ r \end{pmatrix} \begin{pmatrix} m \\ s \end{pmatrix} \begin{pmatrix} m \\ u \end{pmatrix} x_{j+r,k+s,l+u}$.

**Definition 8.** *[6] The triple sequence $\theta_{j,k,l} = \{(j_r, k_s, l_t)\}$ is called the triple lacunary sequence if there exist three increasing sequences of integers such that*

$$j_o = 0, h_r = j_r - j_{r-1} \to \infty \text{ as } r \to \infty,$$

$$k_o = 0, h_s = k_s - k_{s-1} \to \infty \text{ as } s \to \infty,$$

*and*

$$I_o = 0, h_t = I_t - I_{t-1} \to \infty \text{ as } t \to \infty$$

*Let $k_{r,s,t} = j_r k_s I_t, h_{r,s,t} = h_r h_s h_t$ and $\theta_{j,k,l}$ is determined by*

$$I_{r,s,t} = \{(j,k,l) : j_{r-1} < j \leq j_r, k_{s-1} < k \leq k_s \text{ and } I_{t-1} < I \leq I_t\}$$
$$q_r = \frac{j_r}{j_{r-1}}, q_s = \frac{k_s}{k_{s-1}}, q_t = \frac{l_t}{l_{t-1}} \text{ and } q_{r,s,t} = q_r q_s q_t.$$

*Let $K \subset \mathbb{N} \times \mathbb{N} \times \mathbb{N}$. The number*

$$\delta_3^\theta = \lim_{r,s,t} \frac{1}{h_{r,s,t}} |\{(j,k,l) \in I_{r,s,t} : (j,k,l) \in K\}|$$

*is said to be the $\theta_{r,s,t}$-density of $K$, provided the limit exists.*

## 3 Triple lacunary $\Delta^m$-statistical convergence in IFNS.

In the context of intuitionistic fuzzy normed spaces for triple sequences, we define Lacunary $\Delta^m$-statistical convergence and establish certain results.

**Definition 9.** *Let $(X, \varphi, \vartheta, \otimes, \odot)$ be a IFNS with norm $(\varphi, \vartheta)$ and $\theta_{j,k,l}$ be a triple lacunary sequence. A triple sequence $y = (y_{jkl})$ in $X$ is called lacunary $\Delta^m$-statistically convergent to some $\xi \in X$ with respect to the intuitionistic fuzzy norm $(\varphi, \vartheta)$ if for each $\epsilon > 0$ and $t > 0$*

$$\delta_3^\theta \left(\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi\left(\Delta^m y_{jkl} - \xi, t\right) \leq 1 - \epsilon \quad or \quad \vartheta\left(\Delta^m y_{jkl} - \xi, t\right) \geq \epsilon\}\right) = 0 \tag{1}$$

*or equivalently*

$$\delta_3^\theta(\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi, t) > 1 - \epsilon \quad or \quad \vartheta(\Delta^m y_{jkl} - \xi, t) < \epsilon\}) = 1. \tag{1*}$$

*In this case, we write $S_{\theta_{j,k,l}}^{\varphi,\vartheta} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$ or $X_{jkl} \xrightarrow{(\varphi,\vartheta)} \xi \left(S_{\theta_{j,k,l}}\right)$ and denote the set of all $S_{\theta_{j,k,l}}$-convergent triple sequences in the intuitionistic fuzzy normed space by $S_{\theta_{j,k,l}}^{(\varphi,\vartheta)}$.*

**Definition 10.** *Let $(X, \varphi, \vartheta, \otimes, \odot)$ be a IFNS with norm $(\varphi, \vartheta)$ and $\theta_{j,k,l}$ be a triple lacunary sequence. A triple sequence $y = (y_{jkl})$ in $X$ is called lacunary $\Delta^m$-statistically Cauchy with respect to the intuitionistic fuzzy norm $(\varphi, \vartheta)$ if there exists $j_0, k_0, l_o \in \mathbb{N}$ for each $\epsilon > 0$ and $t > 0$ such that for all $j, r \geq j_0, k, s \geq k_0$ and $l, u \geq l_0$, we have*

$$\delta_3^\theta \left(\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi\left(\Delta^m y_{jkl} - \Delta^m y_{rsu}, t\right) \leq 1 - \epsilon \quad or \quad \vartheta\left(\Delta^m y_{jkl} - \Delta^m y_{rsu}, t\right) \geq \epsilon\}\right) = 0.$$

*It is denoted by $S_{jkl}^{\varphi,\vartheta} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$.*

From (1) and (1*), we have the following lemma.

**Lemma 1.** *Let $(X, \varphi, \vartheta, \otimes, \odot)$ be an* IFNS *with norm $(\varphi, \vartheta)$ and $\theta_{j,k,l}$ be a triple lacunary sequence. Then the following statements are equivalent for triple sequence $y = (y_{jkl})$ in $X$ whenever $\epsilon > 0$ and $t > 0$,*

1. $S_{\theta_{j,k,l}}^{\varphi,\vartheta} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$,
2. $\delta_3^\theta \left(\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi\left(\Delta^m y_{jkl} - \xi, t\right) > 1 - \epsilon\}\right) = \delta_3^\theta(\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} :\ \vartheta\left(\Delta^m y_{jkl} - \xi, t\right) < \epsilon\}) = 1$,
3. $\delta_3^\theta \left(\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi\left(\Delta^m y_{jkl} - \xi, t\right) \leq 1 - \epsilon\}\right) = \delta_3^\theta(\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} :\ \vartheta\left(\Delta^m y_{jkl} - \xi, t\right) \geq \epsilon\}) = 0$,
4. $S_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \varphi\left(\Delta^m y_{jkl} - \xi, t\right) = 1$ *and* $S_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \vartheta\left(\Delta^m y_{jkl} - \xi, t\right) = 0$.

**Theorem 1.** *Let $(X, \varphi, \vartheta, \otimes, \odot)$ be a* IFNS *with norm $(\varphi, \vartheta)$ and $\theta_{j,k,l}$ be a triple lacunary sequence. If $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$, then $\xi$ is unique.*

*Proof.* If possible, let $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi_1$ and $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi_2$.
For given $\epsilon \in (0,1)$ and $t > 0$, take $\alpha > 0$ such that $(1-\alpha) \otimes (1-\alpha) > 1-\epsilon$ and $\alpha \odot \alpha < \epsilon$.

Consider

$$K_{1,\varphi}(\alpha, t) = \{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi_1, t/2) \le 1-\alpha\},$$

$$K_{2,\varphi}(\alpha, t) = \{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi_2, t/2) \le 1-\alpha\},$$

$$K_{3,\vartheta}(\alpha, t) = \{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \vartheta(\Delta^m y_{jkl} - \xi_1, t/2) \ge \alpha\},$$

$$K_{4,\vartheta}(\alpha, t) = \{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \vartheta(\Delta^m y_{jkl} - \xi_2, t/2) \ge \alpha\}.$$

Using lemma 3.1, we have

$$\delta_3^\theta(K_1, \varphi(\alpha, t)) = \delta_3^\theta(K_3, \vartheta(\alpha, t)) = 0.$$

$$\delta_3^\theta(K_2, \varphi(\alpha, t)) = \delta_3^\theta(K_4, \vartheta(\alpha, t)) = 0.$$

Let $K_{\varphi,\vartheta}(\alpha, t) = [K_{1,\varphi}(\alpha, t) \bigcup K_{2,\varphi}(\alpha, t)] \bigcap [K_{3,\vartheta}(\alpha, t) \bigcup K_{4,\vartheta}(\alpha, t)]$. Clearly,

$$\delta_3^\theta K_{\varphi,\vartheta}(\alpha, t) = 0.$$

Whenever $(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} - K_{\varphi,\vartheta}(\alpha, t)$, we have two possibilities, either $(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} - [K_{1,\varphi}(\alpha, t) \bigcup K_{2,\varphi}(\alpha, t)]$ or $(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} - [K_{3,\vartheta}(\alpha, t) \bigcup K_{4,\vartheta}(\alpha, t)]$.

First, we consider $(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} - [K_{1,\varphi}(\alpha, t) \bigcup K_{2,\varphi}(\alpha, t)]$. Then

$$\begin{aligned} \varphi(\xi_1 - \xi_2, t) &\ge \varphi(\Delta^m y_{jkl} - \xi_1, t/2) \otimes \varphi(\Delta^m y_{jkl} - \xi_2, t/2) \\ &> (1-\alpha) \otimes (1-\alpha) \\ &> 1-\epsilon. \end{aligned}$$

As given $\epsilon \in (0,1)$ was arbitrary, then $\varphi(\xi_1 - \xi_2, t) = 1$ for all $t > 0$, then $\xi_1 = \xi_2$.

Similarly, if $(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} - [K_{3,\vartheta}(\alpha, t) \bigcup K_{4,\vartheta}(\alpha, t)]$

$$\begin{aligned} \vartheta(\xi_1 - \xi_2, t) &\le \vartheta(\Delta^m y_{jkl} - \xi_1, t/2) \odot \vartheta(\Delta^m y_{jkl} - \xi_2, t/2) \\ &< \alpha \odot \alpha \\ &< \epsilon. \end{aligned}$$

Since $\epsilon \in (0,1)$ was arbitrary, then $\varphi(\xi_1, \xi_2, t) = 0$ for all $t > 0$, i.e., $\xi_1 = \xi_2$. Therefore $S_{\theta_{j,k,l}}^{\varphi,\vartheta} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$ exists uniquely. □

**Theorem 2.** *Let* $(X, \varphi, \vartheta, \otimes, \odot)$ *be a* IFNS *with norm* $(\varphi, \vartheta)$ *and* $\theta_{j,k,l}$ *be a triple lacunary sequence. If* $(\varphi, \vartheta) - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$*, then* $S_{\theta_{j,k,l}}^{\varphi,\vartheta} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$*. But converse may not be true.*

*Proof.* Let $(\varphi, \vartheta) - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$. Then, there exists $j_0, k_0$ and $l_0 \in \mathbb{N}$ for given $\epsilon > 0$ and any $t > 0$ such that for all $j \geq j_0, k \geq k_0$ and $l \geq l_0$ we have $\varphi(\Delta^m y_{jkl} - \xi, t) > 1 - \epsilon$ and $\vartheta(\Delta^m y_{jkl} - \xi, t) < \epsilon$.

Further, the set $A(\epsilon, t) = \{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi, t) \leq 1 - \epsilon$ or $\vartheta(\Delta^m y_{jkl} - \xi, t) \geq \epsilon\}$, contains only finite number of elements. We know that natural density of any finite set is always zero. Therefore, $\delta_3^\theta(A(\epsilon, t)) = 0$ i.e. $S_{\theta_{j,k,l}}^{\varphi,\vartheta} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$.

□

But converse is not true, this can be justified with the example.

**Example** Let $(\mathbb{R}, ||.||)$ be the real normed space under the usual norm. Define $a \otimes b = ab$ and $a \odot b = \min\{a + b, 1\}$ $\forall a, b \in [0,1]$. Also for every $t > 0$ and all $y \in \mathbb{R}$, consider $\varphi(y, t) = \frac{t}{t+|y|}$ and $\vartheta(y, t) = \frac{|y|}{t+|y|}$. Then, clearly $(\mathbb{R}, \varphi, \vartheta, \otimes, \odot)$ is an IFNS. Define the sequence

$$\Delta^m x_{jkl} = \begin{cases} jkl, & \text{for } j_r - \left[\left|\sqrt{h_r}\right|\right] + 1 \leq j \leq j_r \\ & k_s - \left[\left|\sqrt{h_s}\right|\right] + 1 \leq k \leq k_s \\ & \text{and } l_t - \left[\left|\sqrt{h_t}\right|\right] + 1 \leq l \leq l_t \\ \xi, & \text{otherwise.} \end{cases}$$

By given $\epsilon > 0$ and $t > 0$, we obtain the below set for $\xi = 0$.

$$K(\epsilon, t) = \{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi(\Delta^m y_{jkl}, t) \leq 1 - \epsilon \quad \text{or} \quad \vartheta(\Delta^m y_{jkl}, t) \geq \epsilon\}$$

$$= \left\{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |\Delta^m y_{jkl}| \geq \frac{\epsilon t}{1 - \epsilon} > 0\right\}$$

$$= \{(j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |\Delta^m y_{jkl}| = jkl\}$$

$$= \left\{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : |\Delta^m y_{jkl}| \geq \frac{\epsilon t}{1-\epsilon} > 0\right\}$$

$$= \Big\{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : j_r - \left[\left|\sqrt{h_r}\right|\right] + 1 \leq j \leq j,$$
$$k_s - \left[\left|\sqrt{h_s}\right|\right] + 1 \leq k \leq k_s$$
$$\text{and} \quad l_t - \left[\left|\sqrt{h_t}\right|\right] + 1 \leq l \leq l_t\Big\}$$

and so, we get

$$\lim_{r,s,t} \tfrac{1}{h_r,h_s,h_t} \mid \big\{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : j_r - \left[\left|\sqrt{h_r}\right|\right] + 1 \leq j \leq j_r$$
$$k_s - \left[\left|\sqrt{h_s}\right|\right] + 1 \leq k \leq k_s$$
$$\text{and} \;\; l_t - \left[\left|\sqrt{h_t}\right|\right] + 1 \leq l \leq l_t\big\}$$
$$\leq \lim_{r,s,t} \tfrac{\sqrt{h_r}\sqrt{h_s}\sqrt{h_t}}{h_r h_s h_t} = 0.$$

Hence $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = 0$.

By the above defined sequence $(\Delta^m y_{jkl})$, we get

$$\varphi\left(\Delta^m x_{jkl}, t\right) = \begin{cases} \frac{t}{t+|jkl|}, & \text{for } _r - \left[\left|\sqrt{h_r}\right|\right] + 1 \leq j \leq j_r \\ & k_s - \left[\left|\sqrt{h_s}\right|\right] + 1 \leq k \leq k_s \\ & \text{and } l_t - \left[\left|\sqrt{h_t}\right|\right] + 1 \leq l \leq l_t \\ 0, & \text{otherwise.} \end{cases}$$

i.e $\varphi\left(\Delta^m x_{jkl}, t\right) \leq 1, \quad \forall j,k,l.$

and

$$\vartheta\left(\Delta^m x_{jkl}, t\right) = \begin{cases} \frac{|jkl|}{t+|jkl|}, & \text{for } j_r - \left[\left|\sqrt{h_r}\right|\right] + 1 \leq j \leq j_r, \\ & k_s - \left[\left|\sqrt{h_s}\right|\right] + 1 \leq k \leq k_s \\ & \text{and } l_t - \left[\left|\sqrt{h_t\ }\right|\right] + 1 \leq l \leq l_t \\ 0, & \text{otherwise.} \end{cases}$$

i.e $\vartheta\left(\Delta^m x_{jkl}, t\right) \geq 0, \quad \forall j,k,l.$

This shows that $(\varphi,\vartheta) - \lim\limits_{j,k,l\to\infty} \Delta^m y_{jkl} \neq 0$.

**Theorem 3.** *Let $(X,\varphi,\vartheta,\otimes,\odot)$ be a IFNS with norm $(\varphi,\vartheta)$ and $\theta_{j,k,l}$ be a triple lacunary sequence. Then $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi \iff$ there exists a set $P = \{(j_a,k_b,l_c) : a,b,c = 1,2,3,\ldots\} \subset \mathbb{N}\times\mathbb{N}\times\mathbb{N}$ such that $\delta(P) = 1$ and $(\varphi,\vartheta) - \lim_{j_a,k_b,l_c\to\infty} \Delta^m y_{j_a k_b l_c} = \xi$.*

*Proof.* Assume that $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$. For $t > 0$ and $\alpha \in \mathbb{N}$, we take

$M(\alpha, t) = \{(j, k, l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi, t > 1 - 1/\alpha$ and $\vartheta(\Delta^m y_{jkl} - \xi, t) < 1/\alpha\}$,

and

$K(\alpha, t) = \{(j, k, l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi, t) \leq 1 - 1/\alpha$ or $\vartheta(\Delta^m y_{jkl} - \xi, t) \geq 1/\alpha\}$.

As $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$, then $\delta_3(K(\alpha, t)) = 0$.

Also, for any $t > 0$ and $\alpha \in \mathbb{N}$, evidently we get $M(\alpha, t) \supset M(\alpha + 1, t)$, and

$$\delta_3(M(\alpha, t)) = 1, \tag{3.1}$$

For $(j, k, l) \in M(\alpha, t)$, we prove $(\varphi, \vartheta) - \lim_{j_a,k_b,l_c\to\infty} \Delta^m y_{j_a k_b l_c} = \xi$.

On the contrary, suppose that triple sequence $y = (y_{jkl})$ is not $\Delta^m$-convergent to $\xi$ for all $(j, k, l) \in M(\alpha, t)$. So, there exists some $\alpha > 0$ and $k_0 \in \mathbb{N}$ such that

$$\begin{aligned} &\varphi(\Delta^m y_{jkl} - \xi, t) \leq 1 - \rho \text{ or } \vartheta(\Delta^m y_{jkl} - \xi, t) \geq \rho\} \text{ for all } j, k, l \geq k_0 \\ &\Longrightarrow \varphi(\Delta^m y_{jkl} - \xi, t) \geq 1 - \rho \text{ and } \vartheta(\Delta^m y_{jkl} - \xi, t) \leq \rho\} \text{ for all } j, k, l \geq k_0 \end{aligned}$$

Therefore,

$\delta_3(\{(j, k, l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi, t) \geq 1 - \rho$ and $\vartheta(\Delta^m y_{jkl} - \xi, t) \leq \rho\}\}) = 0$
i.e. $\delta_3(M(\rho, t)) = 0$. Since $\rho > 1/\alpha$, then $\delta_3(M(\alpha, t)) = 0$ as $M(\alpha, t) \subset M(\rho, t)$, which is a contradiction to (3.1). This shows that there exists a set $M(\alpha, t)$ for which $\delta_3(M(\alpha, t)) = 1$ and the triple sequence $y = (y_{jkl})$ is statistically $\Delta^m$-convergent to $\xi$.

Converesely, suppose there exists a subset

$P = \{(j_a, k_b, l_c) : a, b, c = 1, 2, 3, \ldots\} \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ with $\delta_3(P) = 1$
and $(\varphi, \vartheta)-\lim_{j_a,k_b,l_c\to\infty} \Delta^m y_{j_a k_b l_c} = \xi$. i.e. for given $\rho > 0$ and any $t > 0$ we have $N_0 \in \mathbb{N}$, which gives

$$\varphi(\Delta^m y_{jkl} - \xi, t) > 1 - \rho$$

and

$$\vartheta(\Delta^m y_{jkl} - \xi, t) < \rho \text{ for all } j, k, l \geq N_0.$$

Now, let

$$\begin{aligned} & K(\rho,t) \\ = \ & \{(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} : \varphi(\Delta^m y_{jkl}-\xi,t)\le 1-\rho \text{ or } \vartheta(\Delta^m y_{jkl}-\xi,t)\ge\rho\}. \end{aligned}$$

Then,

$K(\rho,t)\subseteq\mathbb{N}-\{(j_{N_0+1},k_{N_0+1},l_{N_0+1}),\dots\}$. As $\delta_3(P)=1\Longrightarrow\delta_3(K(\alpha,t))\le 0$.

Hence, $S^{\varphi,\vartheta}_{\theta_{j,k,l}}-\lim_{j,k,l\to\infty}\Delta^m y_{jkl}=\xi$.

□

**Theorem 4.** *Let $(X,\varphi,\vartheta,\otimes,\odot)$ be a* IFNS *with norm $(\varphi,\vartheta)$ and $\theta_{j,k,l}$ be a triple lacunary sequence. Let $y=(y_{jkl})$ be any triple sequence. Then $S^{\varphi,\vartheta}_{\theta_{j,k,l}}-\lim_{j,k,l\to\infty}\Delta^m y_{jkl}=\xi\Longleftrightarrow$ there is a triple sequence $x=(x_{jkl})$ such that $(\varphi,\vartheta)-\lim_{j,k,l\to\infty}\Delta^m y_{jkl}=\xi$ and $\delta_3(\{(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\Delta^m y_{jkl}=\Delta^m x_{jkl}\})=1$*

*Proof.* Assume that $S^{\varphi,\vartheta}_{\theta_{j,k,l}}-\lim_{j,k,l\to\infty}\Delta^m y_{jkl}=\xi$. By Theorem (3.3), we set

$P=\{(j_a,k_b,l_c):a,b,c=1,2,3,\dots\}\subseteq\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ with $\delta_3(P)=1$ and $(\varphi,\vartheta)-\lim_{j,k,l\to\infty}\Delta^m y_{j_ak_bl_c}=\xi$.

Consider the sequence

$$\Delta^m x_{jkl}=\begin{cases}\Delta^m y_{jkl}, & (j,k,l)\in P\\ \xi, & \text{otherwise},\end{cases}$$

which gives the required result.

Converesely, consider $x=(x_{jkl})$ and $z=(z_{jkl})$ in $X$ with the property $(\varphi,\vartheta)$–$\lim_{j,k,l\to\infty}\Delta^m y_{jkl}=\xi$ and $\delta_3(\{(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\Delta^m yjkl=\Delta^m x_{jkl}\})=1$. Then for each $\epsilon>0$ and $t>0$, $\{(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\varphi(\Delta^m y_{jkl}-\xi,t)\le 1-\epsilon$ or $\vartheta(\Delta^m y_{jkl}-\xi,t)\ge\epsilon\}\subseteq A\cup B$, where

$A=\{(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\varphi(\Delta^m y_{jkl}-\xi,t)\le 1-\epsilon \text{ or } \vartheta(\Delta^m y_{jkl}-\xi,t)\ge\epsilon\}$;
$B=\{(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:(\Delta^m y_{jkl}\neq\Delta^m x_{jkl})\}$.

Since $(\varphi,\vartheta)-\lim_{j,k,l\to\infty}\Delta^m y_{j_ak_bl_c}=\xi$ then the set $A$ contains at most finitely many terms. Also $\delta_3(B)=0$ as $\delta_3(B^c)=1$ where

$$B^c=\{(j,k,l)\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\Delta^m y_{jkl}=\Delta^m x_{jkl}\}.$$

Therefore

$\delta_3\{(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\varphi(\Delta^m y_{jkl}-\xi,t)\le 1-\epsilon \text{ or } \vartheta(\Delta^m y_{jkl}-\xi,t)\ge\epsilon\}$. We get $S^{\varphi,\vartheta}_{\theta_{j,k,l}}-\lim_{j,k,l\to\infty}\Delta^m y_{jkl}=\xi$.

□

**Theorem 5.** *Let* $(X, \varphi, \vartheta, \otimes, \odot)$ *be a* IFNS *with norm* $(\varphi, \vartheta)$ *and* $\theta_{j,k,l}$ *be a triple lacunary sequence. Let* $y = (y_{jkl})$ *be any triple sequence. Then* $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi \iff$ *there exists two triple sequence* $z = (z_{jkl})$ *and* $x = (x_{jkl})$ *in* $X$ *such that* $\Delta^m y_{jkl} = \Delta^m z_{jkl} + \Delta^m x_{jkl}$ *for all* $(j, k, l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ *where* $(\varphi, \vartheta) - \lim_{j,k,l\to\infty} \Delta^m y_{j_a k_b l_c} = \xi$ *and* $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$.

*Proof.* Assume that $S_{\theta^{\varphi}_{j,k,l},\vartheta} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$. By Theorem (3.3), we set $P = \{(j_a, k_b, l_c) : a, b, c = 1, 2, 3, \ldots\} \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ wit $\delta_3(P) = 1$ and $(\varphi, \vartheta) - \lim_{j,k,l\to\infty} \Delta^m y_{ja} k_b l_c = \xi$.

Consider two triple sequences $z = (z_{jkl})$ and $x = (x_{jkl})$, then

$$\Delta^m z_{jkl} = \begin{cases} \Delta^m y_{jkl}, & (j, k, l) \in P \\ \xi, & \text{otherwise.} \end{cases}$$

and

$$\Delta^m x_{jkl} = \begin{cases} 0, & (j, k, l) \in P \\ \Delta^m y_{jkl} - \xi, & \text{otherwise,} \end{cases}$$

which gives the required result.

Converesely, consider $x = (x_{jkl})$ and $z = (z_{jkl})$ in $X$ with $\Delta^m y_{jkl} = \Delta^m z_{jkl} + \Delta^m x_{jkl}$ for all $(j, k, l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ where $(\varphi, \vartheta) - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$ and $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$. Then we get result using Theorem (3.4) and Theorem (3.5). □

**Theorem 6.** *A triple sequence* $y = (y_{jkl})$ *in* IFNS $(X, \varphi, \vartheta, \otimes, \odot)$ *is lacurnary* $\Delta^m$*– statistically convergent with respect to* $(\varphi, \vartheta)$ *if and only if it is lacunary* $\Delta^m$*-statistically Cauchy with respect to* $(\varphi, \vartheta)$.

*Proof.* Let $S^{\varphi,\vartheta}_{\theta_{j,k,l}} - \lim_{j,k,l\to\infty} \Delta^m y_{jkl} = \xi$. Then, for each $\epsilon > 0$ and $t > 0$, take $\alpha > 0$ such that $(1 - \alpha) \otimes (1 - \alpha) > 1 - \epsilon$ and $\alpha \odot \alpha < \epsilon$.

Let $K(\alpha, t) = \{(j, k, l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \varphi(\Delta^m y_{jkl} - \xi, t/2) \leq 1{-}\alpha$ or $\vartheta(\Delta^m y_{jkl} - \xi, t/2) \geq \alpha\}$. Therefore $\delta_3(K(\alpha, t)) = 0$ and $\delta_3([K(\alpha, t)]^c) = 1$.

Let $M(\epsilon, t) = \{(j, k, l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \varphi(\Delta^m y_{jkl} - \Delta^m y_{rsu}, t) \leq 1 - \epsilon$ or $\vartheta(\Delta^m y_{jkl} - \Delta^m y_{rsu}, t) \geq \epsilon\}$.

Now, we prove $M(\epsilon, t) = K(\epsilon, t)$, for this if $(j, k, l) \in M(\epsilon, t) = K(\epsilon, t)$. Then we get $\varphi(\Delta_p y_{jkl} - \xi, t/2) \leq 1 - \alpha$ or $\vartheta(\Delta^m y_{jkl} - \xi, t/2) \geq \alpha\}$.

Also

$$\begin{aligned}1-\epsilon \geq \varphi\left(\Delta^m y_{jkl}-\Delta^m y_{rsu}, t\right) &\geq \varphi\left(\Delta^m y_{jkl}-\xi, t/2\right) \otimes \vartheta\left(\Delta^m y_{jkl}-\xi, t/2\right)\\ &> (1-\alpha) \otimes (1-\alpha)\\ &> 1-\epsilon\end{aligned}$$

and

$$\begin{aligned}\epsilon \geq \vartheta\left(\Delta_p y_{jkl}-\Delta_p y_{rsu}, t\right) &\leq \vartheta\left(\Delta_p y_{jkl}-\xi, t/2\right) \odot \varphi\left(\Delta_p y_{jkl}-\xi, t/2\right)\\ &< \alpha \odot \alpha\\ &< \epsilon.\end{aligned}$$

which is not possible. Therefore $M(\epsilon,t) \subset K(\alpha,t)$ and $\delta_3(M(\epsilon,t)) = 0$ i.e. $y = (y_{jkl})$ is $\Delta^m$-statistically convergent with respect to $(\varphi,\vartheta)$.

Coversely, assume that $y = (y_{jkl})$ is $\Delta^m$-statiscally Cauchy with respect to $(\varphi,\vartheta)$ but not $\Delta^m$-statiscally convergent with respect to $(\varphi,\vartheta)$. Thus for $\epsilon > 0$ and $t > 0$, $\delta_3(M(\epsilon,t)) = 0$, where

$M(\epsilon,t) = \{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \varphi(\Delta^m y_{jkl}-\Delta^m y_{j_0k_0l_0}, t) \leq 1-\epsilon \quad \text{or} \quad \vartheta(\Delta^m y_{jkl}-\Delta^m y_{j_0k_0l_0}, t) \geq \epsilon\}$.

Take $\alpha > 0$ such that $(1-\alpha)\otimes(1-\alpha) > 1-\epsilon$ and $\alpha\odot\alpha < \epsilon$. Also, $\delta_3(K(\alpha,t)) = 0$, where

$$\begin{aligned}&K(\alpha,t)\\ = \ &\{(j,k,l) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \varphi(\Delta^m y_{jkl}-\xi, t/2) \geq 1-\epsilon \text{ or } \vartheta(\Delta^m y_{jkl}-\xi, t/2) < \epsilon\}.\end{aligned}$$

Now

$$\begin{aligned}\varphi\left(\Delta^m y_{jkl}-\Delta^m y_{j_0k_0l_0}, t\right) &\geq \varphi\left(\Delta^m y_{jkl}-\xi, t/2\right) \otimes \vartheta\left(\Delta^m y_{j_0k_0l_0}-\xi, t/2\right)\\ &> (1-\alpha) \otimes (1-\alpha)\\ &> 1-\epsilon\end{aligned}$$

and

$$\begin{aligned}\vartheta\left(\Delta^m y_{jkl}-\Delta^m y_{j_0k_0l_0}, t\right) &\leq \vartheta\left(\Delta^m y_{jkl}-\xi, t/2\right) \odot \varphi\left(\Delta^m y_{j_0k_0l_0}-\xi, t/2\right)\\ &< \alpha \odot \alpha\\ &< \epsilon.\end{aligned}$$

Therefore, $\delta_3\left([M(\epsilon,t)]^c\right) = 0$ i.e. $\delta_3(M(\epsilon,t)) = 1$, which is a contradiction as $y = (y_{jkl})$ is $\Delta^m$-statistically cauchy. Hence, $y = (y_{jkl})$ is $\Delta^m$-statiscally convergent with respect to $(\varphi,\vartheta)$. □

## 4 Conclusion.

In this paper we defined Lacunary $\Delta^m$-statistical convergence on intuitionistic fuzzy normed space and established certain results. The findings are more widespread than the equivalent normed spaces since every ordinary norm implies an intuitionistic fuzzy norm.

## 5 Declaration

**Conflicts of interests:** There is no conflict of interest.

**Availibility of data and materials:** This paper has no associated data.

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