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TETRA GEOMETRIC MEAN RUNGE-KUTTA ANALYSIS OF BIOECONOMIC FISHERIES MODEL

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Dedicated to Professor Radu Păltănea on the occasion of his 70th anniversary

Abstract

The optimal management and utilization of natural resources is eventually important, due to increase in the consumption of natural resources and in view of this, the main aim of fisheries is to maintain the marine organisms at levels that produces maximum sustainable yield (MSY). In this study, the one step numerical approximations schemes are developed to find the solution of such bioeconomic fisheries models whose analytical solution is not possible. In the proposed work, the explicit Runge-Kutta scheme based on fourth order tetra geometric mean is established and this method solve fishery management models of Schaefer and Gordon-Schaefer. The models analyze bioeconomic growth rate, carrying capacity, total and marginal costs and revenues. In solving, the models through proposed tetra geometric mean Runge-Kutta method, it is obtained that the method is easy to implement. The results obtained are much better as compared to other methods when compared in terms of errors and central processing unit (CPU) time.

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Key words: Fishery-Schaefer model, Gordon-Schaefer model, maximum sustainable yield, explicit Runge-Kutta method, autonomous system.

1 Introduction

The present study emphasizes on application of one-step Runge-Kutta (RK) method which is iterative explicit and based on geometric mean. The main idea of the RK schemes is to elaborate modified Euler schemes by evaluating slopes on the points other than end points over the given interval. The method so developed was second order RK-method and this was extended by Heun for third order [1]. Kutta

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developed the fourth order method popularly known as classical method, Kutta further developed fifth order method but the method encountered errors which was corrected by Nystrom [2]. Hutta [3] analyzed sixth order method on the basis of research work of Gill [4] and Merson [5]. The RK method further extended up to order seven and eight by Dormand and Mikkawy [6] and Dormand and Prince [7] termed as RK Nystrom method. The RK-method of higher order was developed by Butcher [8]. The proposed work is based on RK-method using geometric mean for incremental function and first such work based on mean is introduced by Sanugi [9] who developed explicit RK-method based on geometric mean. The nth order fuzzy initial value problem is solved by fourth order RK method using centroidal mean [10] and contra harmonic mean based RK method of fourth order is used to approximate initial value problem (IVP) [11]. The method developed in the proposed work is an extended form of the RK-method of third order based on geometric mean [12], [13] and the various techniques developed so far for solving different types of differential equations using RK-method is critically reviewed by Chauhan and Srivastava [14]. Akanbi and Wusu established explicit harmonic mean based RK-method of fourth order [15] and thus extended the third order explicit RK-method based on geometric mean [16]. A multi-derivative approach for third order explicit RK-method has been developed by Wusu and Akanbi [17]. A variety of numerical techniques have been established so far to handle a variety of differential equation models [18]–[26]. But the application of RK-technique over bioeconomic fisheries model is a new phenomenal approach and the Schaefer model under consideration in the present study is a bioeconomic model which not only works in the field of fishery management but in the various financial management schemes of other industries too. The model was first developed by Schaefer and he applied it to the fish population. While the Gordon-Schaefer model is used to predict and analyze the fishery management skills to economize the use of resources by providing employment opportunities without exploiting the biomass. The fishery model of such type was first introduced by Gordon [27]. The detailed economic analysis of fishery management has been done by Clarkin by studying research work of Gordon and Schaefer [28]-[29]. The effect of ecosystem and overexploitation of resources is studied analytically by Lauck [30] and Castilla [31]. The description of disrupted habitats at ecosystem level is given by Harrington [32] and Pikitch [33]. Powers and Monk [34] describe that the successful management of fishery can be done by including ecosystem attributes Schaefer model so far is solved by Green's theorem only. The rest of the sections of this paper are structured as per follows: in Section 2 and 3 the general descriptions of fourth order explicit RK-method its algorithm and convergence is given respectively. The section 4 describes the Schaefer model and section 5 covers numerical illustration along with graphical and tabular comparison in section 6. The paper ends up with the section 7 of conclusion.

2 Fourth order tetra geometric mean Runge-Kutta method (TGMRK)

The method of RK is a one step method which require only solution at x_n to evaluate the solution at x_{n+1}. The more general abstract form of the method is

$$y_{n+1} = y_n + h(x_n, y_n; h) \tag{1}$$

The s-stage RK-method for the solution of IVP

$$y' = f(x, y), y(0) = y_0 \tag{2}$$

is

$$y_{n+1} = y_n + h\sum_{i=1}^{s} b_i k_i \tag{3}$$

where

$$k_i = f(x_n + c_i h, y_n + h\sum_{j=1}^{s} a_{ij}k_j, i = 1, 2, \ldots, s.$$

The proposed method has number of stages $s = 4$, the incremental function is the fourth root of the slopes

$$K_1, K_2, K_3, K_4$$

so the fourth stage geometric mean based explicit RK-method is

$$y_{n+1} = y_n + \sqrt[4]{(K_1.K_2.K_3.K_4)}$$

where slopes are given as

$$\begin{aligned}
K_1 &= hf && (4)\\
K_2 &= hf + ff_yh^2a_{21} + 1/2f^2f_{yy}h^3a_{21}^2 + 1/6f^3f_{yyy}h^4a_{21}^3 + O(h^4) && (5)\\
K_3 &= hf + fh^2f_y(a_{31} + a_{32}) + h^3(ff_y^2a_{21}a_{32} + f^2f_yy(a_{31} + a_{32})^2) &&\\
&\quad +h^4(f^2f_yf_{yy}(1/2a_{21}^2a_{32} + a_{21}a_{32}(a_{31} + a_{32})) &&\\
&\quad +1/6f^3f_{yyy}(a_{31} + a_{32})^3) + O(h^4) && (6)\\
K_4 &= hf + hf^2f_y(a_{41} + a_{42} + a_{43}) + h^3\Big[ff_y^2(a_{21}a_{42} + a_{31}a_{43} + a_{32}a_{43}) &&\\
&\quad +f^2f_{yy}((a_{41}^2)/2 + (a_{42}^2)/2 + (a_{43}^2)/2 + a_{41}a_{42} + a_{42}a_{43} + a_{41}a_{43})\Big] &&\\
&\quad +h^4\Big[ff_y^2a_{21}a_{32}a_{43} + f^2f_yf_{yy}(1/2(a_{21}^2a_{42} + a_{31}^2a_{43} + a_{32}^2a_{43}) &&\\
&\quad +a_{21}a_{42}(a_{41} + a_{42} + a_{43} + a_{32}a_{43}(a_{41} + a_{42} + a_{43}) &&\\
&\quad +a_{31}a_{43}(a_{41} + a_{42} + a_{43} + a_{32})) &&\\
&\quad +1/6f^3f_{yyy}(a_{41} + a_{42} + a_{43})\Big] + O(h^4) && (7)
\end{aligned}$$

Now fourth root is given by

$$\sqrt[4]{(K_1K_2K_3K_4)} = hf\sqrt[4]{(1+\omega_1+\omega_2+\omega_3+\omega_4)} \tag{8}$$

where

$$\begin{aligned}
\omega_1 &= ff_y(a_{41}+a_{42}+a_{43})+1+hf_{yy}(a_{21}+a_{31}+a_{32}),\\
\omega_2 &= hff_yf_{yy}(a_{41}+a_{42}+a_{43})(a_{21}+a_{31}+a_{32}),\\
\omega_3 &= h^2(f_y^2(a_{21}a_{42}+a_{31}a_{43}+a_{32}a_{43})+ff_{yy}((a_{41}^2)/2+(a_{42}^2)/2+(a_{43}^2)/2\\
&\quad +a_{41}a_{42}+a_{42}a_{43}+a_{41}a_{43})+a_{41}a_{42}+f_y^2(a_{21}a_31+2a_{21}a_32)\\
&\quad +(1/2)(a_{21}^2+ff_{yy}(a_31+a_32)^2)+ff_y^3(a_{21}a_31+2a_{21}a_{32})\\
&\quad +(1/2)(a_{21}^2+f^2f_{yy}f_y(a_{31}+a_{32})^2)(a_{41}+a_{42}+a_{43})),\\
\omega_4 &= h^3((f_y^2a_{21}a_{32}a_{43}+ff_yf_{yyy}((1/2)(a_{21}^2a_{42}+a_{31}^2a_{43}+a_{32}^2a_{43})\\
&\quad +a_{21}a_{42}(a_{41}+a_{42}+a_{43})+a_{32}a_{43}(a_{41}+a_{42}+a_{43})\\
&\quad +a_{31}a_{43}(a_{41}+a_{42}+a_{43}+a_{32}))+(1/6)f^3f_{yyy}(a_{41}+a_{42}+a_{43})\\
&\quad +(a_{21}+a_{31}+a_{32})(f_y^2f_{yy}(a_{21}a_{42}+a_31a_{43}+a_{32}a_{43})\\
&\quad +ff_{yy}^2((a_{41}^2)/2+(a_{42}^2)/2+(a_{43}^2)/2+a_{41}a_{42}+a_{42}a_{43}+a_{41}a_{43})).
\end{aligned}$$

The solution of the IVP is obtained in Taylor's series expansion on comparing the corresponding powers of h Taylor's series for comparing powers is:

$$\begin{aligned}
\phi_T(y;h) &= \sum_{r=0}^{\infty}(h^{r+1})/(r+1)!)(d/dy)^r f(y)\\
&= hf+(1/2)h^2ff_y+(1/6)h^3(f^2f_{yy}+ff_y^2)\\
&\quad +(1/24)h^4(f^3f_{yyy}+4f^2f_yf_{yy}+ff_y^3)
\end{aligned} \tag{9}$$

The solution obtained on solving the system of equation after comparison is: $a_{21}=1.633, a_{31}=-4.865, a_{32}=3.232, a_{41}=0.796, a_{42}=1.451179, a_{43}=-1.24718$

3 Convergence test of the proposed TGMRK algorithm

Lemma 1. *Consider theory of 'Existence and Uniqueness' is fulfilled by IVP (1), henceforth the increment function ϕ_{TG} will satisfy the Lipschitz provision for variable y which is the independent variable.*

Proof. With respect to y taking P as the Lipschitz constant for $f(y)$. Now

$$K_1(y_n) = hf(y_n) \tag{10}$$

implies

$$|K_1(y_n)-K_1(z_n)| = |hf(y_n)-hf(z_n)| < hP|y_n-z_n| \tag{11}$$

In the same way,

$$\begin{aligned} & hP|K_2(y_n) - K_2(z_n)| \\ = \ & |hf(y_n + a_{21}K_1) - hf(z_n + a_{21}K_1)| \\ < \ & hP|y_n - z_n| + h|a_{21}||K_1(y_n) - K_1(z_n)| \\ < \ & hP|y_n - z_n|h + h^2P|a_{21}||K_3(y_n) - K_3(z_n)| \end{aligned} \tag{12}$$

and

$$\begin{aligned} = \ & |hf(y_n + a_{31}K_1 + a_{32}K_2) - hf(z_n + a_{31}K_1 + a_{32}K_2)| \\ < \ & hP|y_n - z_n| + hP|a_{31}||K_1(y_n) - K_1(z_n)| \\ & +hP|a_{32}||K_2(y_n) - K_2(z_n)| \qquad (13) \\ < \ & P|y_n - z_n|h + h^2P(|a_{31}| + |a_{32}|) \\ & +h^3P^2|a_{21}a_{32}||K_4(y_n) - K_4(z_n)| \\ = \ & |hf(y_n + a_{41}K_1 + a_{42}K_2 + a_{43}K_3) - hf(Z_n + a_{41}K_1 \\ & +a_{42}K_2 + a_{43}K_3)| \\ < \ & hL|(y_n + a_{41}K_1 + a_{42}K_2 + a_{43}K_3) \\ & -(Z_n + a_{41}K_1 + a_{42}K_2 + a_{43}K_3)| \\ = \ & hL[|(y_n - Z_n) + a_{41}(K_1(y_n) - K_1(Z_n)) + a_{42}(K_2(y_n) - K_2(Z_n)) \\ & +a_{43}(K_3(y_n) - K_3(Z_n))|] \\ \leq \ & hL[|(y_n - Z_n)| + |a_{41}||K_1(y_n) - K_1(Z_n)| + |a_{42}||K_2(y_n) - K_2(Z_n)| \\ & +|a_{43}||K_3(y_n) - K_3(Z_n)|] \\ = \ & hL[|(y_n - Z_n)| + |a_{41}|hL|(y_n - Z_n)| + |a_{42}||((y_n - Z_n)hL \\ & +h^2L^2|a_{21}| + |a_{43}||(y_n - Z_n)|(hL + h^2L^2|a_{31}|(hL + h^2L^2|a_{31}| \\ & +|a_{32}|(h^3L^3 + h^4L^4))||a_{21}|] \\ = \ & |(y_n - Z_n)|[hL + |a_{41}|h^2L^2 + |a_{42}||(h^2L^2 + h^3L^3|a_{21}| \\ & +|a_{43}|h^2L^2 + h^3L^3|a_{31}| + |a_{32}|h^2L^2 + h^3L^3))||a_{21}|. \end{aligned} \tag{14}$$

So, for the incremental function

$$\phi_{TG} = (K_1K_2K_3K_4)$$

we have

$$\begin{aligned} & |\phi_{TG}(y_n; h) - \phi_{TG}(z_n; h)| \\ = \ & |(K_1(y_n)K_2(y_n)K_3(y_n)K_4(y_n)) - (K_1(z_n)K_2(z_n)K_3(z_n)K_4(y_n))| \end{aligned} \tag{15}$$

The application of mean value theorem it gives

$$|\phi_{TG}(y_n; h) - \phi_{TG}(z_n; h)| < P|y_n - z_n| \tag{16}$$

where

$$P = [hL + |a_{41}|h^2L^2 + |a_{42}||(h^2L^2 + h^3L^3|a_{21}| + |a_{43}|h^2L^2 + h^3L^3|a_{31}| + |a_{32}|(h^2L^2 + h^3L^3))||a_{21}|$$

is the constant called as Lipschitz constant. This verifies the convergence property of the proposed TGMRK algorithm.

□

4 Bioeconomic fisheries Schaefer model

Among Various renewable resources fishery is one of the prime example that human being is exploiting since a long time, the Schaefer model and Gordon-Schaefer model are widely used fishery model. The model is named after the biologist Schaefer which is the type of logistic model. If rate of harvesting or removal is h(t) then population growth is given by differential equation,

$$dn/dt = f(n) - h(t), \tag{17}$$

The Schaefer model is

$$dn/dt = kn(1 - (n/K)), \tag{18}$$

n(t) = size of the population,

$$k > 0$$

is intrinsic growth rate per unit,
K= carrying capacity.
Further the model is modified by Shah and Sharma [35] to the form

$$dn/dt = kn(1 - (n/K)^{(\alpha-1)}) - E, \tag{19}$$

E = effort per unit catch, α is real positive number $\alpha > 1$.
Choice of α determine types of model:

(i) Gordon-Schaefer Model, $\alpha = 2$.

(ii) Pelta -Tomlinson model, $\alpha = 3$.

(iii) Pelta-Tomlinson model , $\alpha = 4$.

The rate of catching fish is proportionate to the populace n of the fish and this rate of catching fish is given by En, where E is a positive constant.

For $\alpha = 2$, we get the Gordon-Schaefer model.

$$dn/dt = kn(1 - n/K) - En \tag{20}$$

In catch and effort, fisheries more focus is given on catch per unit effort (CPUE) in which the measurement of stock of certain species is done in indirect way. This approach helps in finding the true abundance of target species. If CPUE is unchanged then this shows sustainable harvesting and if CPUE is decreasing then there is overexploitation. In this paper, the proposed method solves the Schaefer model and the Gordon Schaefer model.

5 Illustrations

Problem 1. Biologist stocked a lake with 500 fishes and estimated the carrying capacity to be 5300. The number of fish doubled in the 1st year. Assume that the size of the fish population satisfy the logistic equation

$$dP/dt = kP(1 - P/K)$$

find the solution of Schaefer model.

Solution: Here we have: $P(0) = 500$, $P(1) = 1000$, $K = 5300$, now we find k by

$$dP/dt = kP(1 - P/K)$$

such that $P/(5300 - P) = C_1 e^k t$ $C_1 = 5/48$ and $k = 0.803149$.

The tabular comparison of absolute errors by TGMRK method and other fourth order methods is presented in Table 1 and in Table 2 the central processing unit (CPU) time of the proposed TGMRK method is presented.

x	CHM [11]	Harmonic [15]	TGMRK
1	12.5248	8.42356	1.68605
2	14.2224	11.6979	4.09095
3	15.2452	13.991	6.07831
4	17.0264	15.519	9.12346
5	21.2173	18.7014	12.2693
6	23.9083	20.0161	13.0616
7	25.1457	23.4549	15.9076
8	26.2045	25.8092	16.5466
9	28.7448	27.8334	18.9517

Table 1: Comparison of Absolute Errors of TGMRK Method with other Fourth Order Methods for Problem 1

The method of fourth order TGMRK so developed manage well for the fishery model to maintain the level of organism at maximum sustainable yield (MSY). The method is more suitable for advanced fishery model as compared to other available method, as it is cost efficient, generates less errors and fulfils condition of efficiency. The method requires less computation cost and storage as compared to other fourth order methods taken into consideration.

x	TGMRK method Time (Seconds)
1	3.41
3	11.23
6	18.75
9	30.87

Table 2: CPU Time of FGMRK Method for h = 0.0001 for Problem 1

Problem 2. Solve the Gordon-Schaefer model dn/dt=kn(1-n/K)-E,K¿0 for k=0.5, K=3500, E=42.8571, y(0)=500.

Solution: The problem is solved using proposed TGMRK Method and other fourth order methods, absolute errors are computed by considering RK-4 method as the benchmark method. The comparative errors are shown in the Table 3 and the CPU time of the proposed TGMRK method is shown in the Table 4.

x	CHM [11]	Harmonic [15]	TGMRK
1	380769	1.265773	0.380769
2	5.877458	3.148769	0.877458
3	9.256482	5.325629	1.256482
4	10.97803	7.139753	2.178026
5	10.07146	7.952396	2.371462
6	11.0188	7.586658	2.618802
7	13.7122	8.388903	3.322204
8	15.30795	10.89412	3.80799
9	16.37642	12.50285	4.512504

Table 3: Comparison of Absolute Errors of TGMRK Method with other Fourth Order Methods for Problem 2

x	TGMRK method Time (Seconds)
1	3.83
3	10.07
6	19.83
9	31.35

Table 4: CPU Time of TGMRK Method for h = 0.0001 for Problem 2

The TGMRK method is performing much better as compared to other methods under consideration. The error is compared graphically and in tabular form, which is minimum when compared with other methods of fourth order under consideration. The solution is approached faster by proposed method as compared to other methods. The step size is reduced and corresponding errors are inspected which are also reducing with respect to step size, in a pattern required for determination of order of the method. This shows that the method is of order 4.

6 Comparative analysis

The graphical comparison of absolute errors by TGMRK method and other fourth order methods for problem 1 and 2 are represented in Figure 1 respectively. The proposed TGMRK method is performing much better as compared to other methods under consideration. The graph shows that the error by TGMRK method is minimum when compared with other methods of fourth order under consideration. The solution is approached faster by proposed method as compared to other methods.

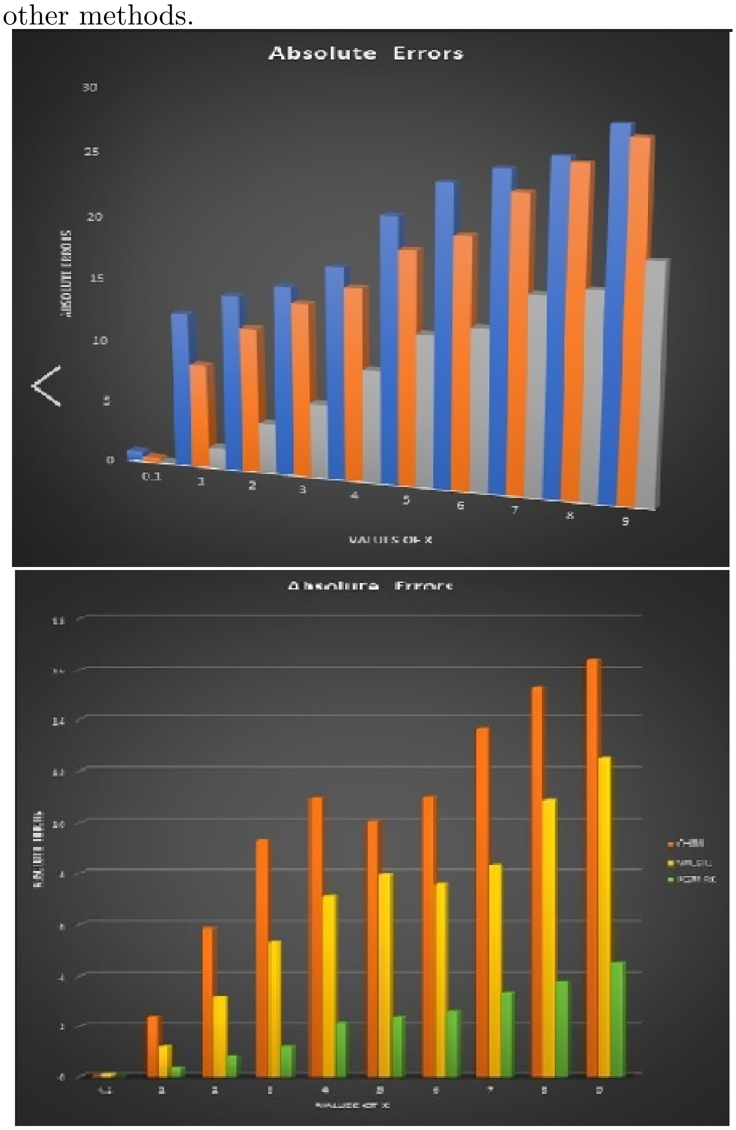

Figure 1: Absolute Error Comparison of TGMRK Method with Other Fourth Order Methods for Problem 1 and Problem 2 Respectively

7 Conclusion

In this paper, the problems based on bio economic model are solved using contra harmonic method and Wusu et.al. method and proposed TGMRK method. The results are compared in terms of absolute errors. CPU time is also evaluated for the numerical problem into consideration. The proposed method is found more compatible with respect to other methods. The TGMRK method is convergent and has more accuracy. The method has wide range of applications in the real world.

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