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PLANE STRAIN OF ISOTROPIC MICROPOLAR BODIES WITH PORES

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Dedicated to Professor Radu Păltănea on the occasion of his 70th anniversary

Abstract

The aim of this work is defined by the study of the plane deformation for micropolar isotropic materials in equilibrium theory, where, in addition to displacement and absolute temperature, the particles of the mentioned materials have pores and microrotations.The determined solution to the field equations helps us to study the effect of heat sources and pores on the deformation of the body, which deformation is studied later.

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1 Introduction

In the first part of the work are introduced the field equations (1-11) for an isotropic micropolar medium, from the theory of thermoelasticity, equations that can be found in [1-4] and also in [8]. A broader description of the theory of porous media is made by Cowin and Nunziato in [5-6]. For a good understanding of the work, also in the second section are the notations used. The equilibrium equations (22-25) from [7] later help us to obtain the solution of the field equations. This achievement is found in detail in the third section. Also in this part of the paper, the obtained theorem helps us to study the influences of heat supply moments and pores.

In the last part, we consider a cylindrical hole in an elastic space containing and, also, the domain $B = \{x : x_1^2 + x_2^2 > r_1^2, x_3 \in \mathbb{R}\}, (r_1 > 0)$, ocuppied by an elastic material with inner structure. This material undergoes a plane strain parallel to the x_1x_2- plane. To obtain the deformation, the functions $\theta, \varphi, u_\alpha$ and

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ϕ_α were determined, more precisely the solutions of the formed system, which were denoted by V, W, Q and U.

Regarding the results related to microstructured media, there are many results, of which I mention a few:[9-13]. Also, some studies that address the plane deformation of certain bodies, in various contexts, can be found in [14-16] and a more extensive presentation in [17].

2 Basic equations

In three-dimensional space we consider a region B, occupied at one time by a certain body, considered to be bordered by the parts of the smooth surface. Let $O_{x_{k(k=1,2,3)}}$ be a system of orthogonal axes to which we refer.

Before entering field equations, we add the following notations used:
$-n=$ external normal at ∂D,
$-u=$ displacement field over B,
$-\delta_{ij}=$ Kronecker delta,
$-\lambda, \mu =$ *Lamé* constants,
$-\sigma=$ linear thermal expansion,
$-k=$ thermal conductivity,
$-\alpha, \gamma, \epsilon, \xi, \zeta, a, b, c, d, I$ =constitutive moduli for the theory at hand,
Now we can introduce the following equations.

-The consitutive equations:

$$T_{ij} = \lambda u_{k,k} + \mu(u_{i,j} + u_{j,i}) + k(u_{i,j} + \epsilon_{ijk}\phi_k) + \xi\varphi\delta_{ij} - (3\lambda + 2\mu + k)\sigma\theta\delta_{ij}, \quad (1)$$

$$M_{ij} = \alpha\phi_{k,k}\delta_{ij} + \gamma\phi_{j,i} + \epsilon\phi_{i,j} + \xi\varphi\delta_{ij}, \quad (2)$$

$$g = -\xi\mu_{k,k} - a\varphi - b\theta, \quad (3)$$

$$h_i = d\varphi_i, \quad (4)$$

$$\rho\eta = (3\lambda + 2\mu + k)\sigma u_{k,k} - b\varphi + c\theta, \quad (5)$$

$$r\dot{q}_{,i} = k\theta_i - q_i. \quad (6)$$

We denote by θ the absolute temperature. T_{ij} and M_{ij} are the stress tensor and couple stress tensor, g is the instrinsic equilibrated body force and φ represents the change in volume fraction.

-The geometrical relations:

$$e_{ij} = u_{i,j} + \epsilon_{ijk}\phi_k, \qquad \psi_{ij} = \phi_{i,j}, \quad (7)$$

where ϵ_{ijk} is the Ricci symbol.

-The balance laws in local forms (in the case of equilibrium):

$$T_{ji,j} + \rho f_i = 0, \quad (8)$$

$$M_{ji,j} - \epsilon_{ijk} t_{jk} + \rho G_i = 0, \tag{9}$$

$$h_{i,i} + g + \rho L = 0, \tag{10}$$

$$q_{i,i} + \rho r = 0. \tag{11}$$

where G_i is the body couple, L is the extrinsic body force and r is the internal heat source.

The surface force vector T_i, the surface couple force M_i, the heat flux q and the change in volume fraction moment vector N_i at a regulat point of ∂B are defined by:

$$T_i = T_{ji} n_j, \quad M_i = M_{ji} n_j, \; q = q_j n_j, \; N_i = h_{ji} n_j. \tag{12}$$

Knowing that the elastic potential is a positive quadratic form, we will consider that the elastic moduli satisfy the relations imposed by it:

$$3\lambda + 2\mu > 0, \mu > 0, \varepsilon > 0, 3\lambda + 2\mu > 3k. \tag{13}$$

Next, we will consider the B region as the inside of a straight cylinder, where Σ is the open cross section, Π is the lateral boundary and L is the boundary of Σ. In choosing the Cartesian system, we take in to account that the generators of B is parallel with the x_3-axis.

Let's introduce the data wich describe the plane deformation of B, parallel to $x_1 x_2$- plane:

$$u_\alpha = u_\alpha(x_1, x_2), \;\; u_3 = 0, \;\; \phi_\alpha = \phi_\alpha(x_1, x_2), \;\; \varphi = \varphi(x_1, x_2), \;\; \theta = \theta(x_1, x_2), \tag{14}$$

unde $(x_1, x_2) \in \Sigma$.

From constitutive equations, local forms of balance laws and from (11) it follows that $e_{ij}, \psi_{ij}, T_{ij}, M_{ij}, g, h_i, \rho\eta \; and \; r \; \dot{q_{,i}}$ are independent of x_3.

Deformation tensors $e_{ij} \; and \; \psi_{ij}$ are defined by means of geometric equations,

$$e_{\alpha,\beta} = u_{\alpha,\beta} + \epsilon_{\alpha\beta\rho} \phi_\rho, \quad \psi_{\alpha\beta} = \phi_{\alpha,\beta}. \tag{15}$$

The non-zero dependent constitutive variables are $T_{\alpha\beta}, T_{33}, M_{\alpha\beta}, h_\alpha, r\dot{q_{,\alpha}}$. Furthermore:

$$T_{\alpha\beta} = \lambda u_{\rho,\rho} \delta\alpha\beta + \mu(u_{\alpha,\beta} + u_{\beta,\alpha}) + k u_{\alpha,\beta} \phi_\rho + \xi\varphi\delta_{\alpha\beta} - (2\lambda + 2\mu + k)\nu\theta\delta_{\alpha\beta}, \tag{16}$$

$$M_{\alpha\beta} = -\alpha\phi_{\rho,\rho}\delta_{\alpha\beta} - \gamma\phi_{\beta,\alpha} + \epsilon_{\alpha,\beta} + \xi\varphi\delta_{\alpha\beta}, \tag{17}$$

$$g = -\xi u_{\alpha,\alpha} - \psi\phi_{\alpha,\alpha} - a\varphi - b\theta, \tag{18}$$

$$h_\alpha = d\varphi_\alpha, \tag{19}$$

$$\rho\eta = (3\lambda + 2\mu + k)\sigma u_{\alpha,\alpha} - b\varphi + c\theta, \tag{20}$$

$$r\dot{q_{,\alpha}} = k\theta_\alpha - q_\alpha. \tag{21}$$

Therefore, we will consider that body loads are independent of x_3, and $f_3 = 0$. So, the equilibrium equations are reduced to:

$$T_{\alpha\beta,\beta} + \rho f_\alpha = 0, \tag{22}$$

$$M_{\alpha\beta,\beta} + \rho G_\alpha = 0, \tag{23}$$
$$h_{\alpha,\alpha} + g + \rho L = 0, \tag{24}$$
$$q_{\alpha,\alpha} + \rho S = 0. \tag{25}$$

The relation (12), at a regular points of Π, becomes:

$$T_\alpha = T_{\beta\alpha} n_\beta, \quad M_\alpha = M_{\beta\alpha} n_\alpha, T_3 = 0, q = q_\alpha n_\alpha, \quad N_i = h_{\beta i} n_\beta, \quad on\ L, \tag{26}$$

where, $n_\alpha = cos(n_x, x_\alpha)$, where we denote with n_x the unit vector of the outer normal to L.

According to geometric equations, constitutive equations, and equilibrium equations, we must comply with the boundary conditions. In the case of the first boundary value problem, the boundary conditions are:

$$u_\alpha = \tilde{u}_\alpha, \quad \phi_\alpha = \tilde{\phi_\alpha}, \quad \theta = \tilde{\theta}, \quad \varphi = \tilde{\varphi} \quad on\ L, \tag{27}$$

where $\tilde{u}_\alpha, \ \tilde{\phi}, \tilde{\theta}$ and $\tilde{\varphi}$ are prescribed functions. In the case of the second boundary value problem, the boundary conditions are:

$$T_{\beta\alpha} n_\beta = \tilde{T}_\alpha, \ M_{\beta\alpha} n_\beta = \tilde{M}_\alpha, \tilde{q} = q_\beta n_\beta, \quad h_{\alpha i} n_\alpha = \tilde{N}_i, \ onL, \tag{28}$$

where the given functions $\tilde{T}_\alpha, \ \tilde{M}_\alpha, \ \tilde{q} \ and \ \tilde{N}_j$ are independent of x_3.

From (1-10), results that $u_\alpha, \phi_\alpha, \theta, \ and \ \varphi$ satisfy the equations:

$$(\lambda + \mu) u_{\rho,\rho\alpha} + (\mu + k) u_{\alpha,\rho\rho} + \xi \varphi_{,\alpha} - (3\lambda + 2\mu + k)\sigma\theta_{,\alpha} = -\rho f_\alpha, \tag{29}$$
$$(\alpha + \gamma)\phi_{\rho,\rho\alpha} + \varepsilon \phi_{\alpha,\rho\rho} + \zeta \varphi_{,\alpha} - 2k\phi_\alpha = -\rho G_\alpha, \tag{30}$$
$$k\Delta\theta = -\rho S, \tag{31}$$
$$d\varphi_{\rho,\rho} - \xi u_{\rho,\rho} - \zeta \phi_{\rho,\rho} - a\varphi - b\theta = -\rho L, \qquad on\ \Sigma. \tag{32}$$

To obtain the relation (9) we start from (1), from where we deduce that

$$T_{\beta\alpha,\beta} = \lambda u_{\rho,\rho\alpha} + \mu(u_{\beta,\alpha\beta} + u_{\alpha,\beta\beta}) + k(u_{\beta,\alpha\beta} + \varepsilon_{\beta\alpha\rho}\phi_{\rho,\beta}) + \zeta\varphi_{,\beta}\delta_{\beta\alpha} - (3\lambda + 2\mu + k)\sigma\theta_{,\beta}\delta_{\beta\alpha},$$

Obvious $\varepsilon_{\beta\alpha\rho}\phi_{\rho,\beta} = 0$, so the above relation becomes:

$$T_{\beta\alpha,\beta} = \lambda u_{\rho,\rho\alpha} + \mu(u_{\beta,\alpha\beta} + u_{\alpha,\beta\beta}) + k u_{\beta,\alpha\beta} + \zeta\varphi_{,\alpha} - (3\lambda + 2\mu + k)\sigma\theta_{,\alpha}.$$

Next, using relation (8) we obtain relation (29). Analogously we get (30), (31), (32).

The first boundary value problem, involves finding the $u_\alpha, \phi_\alpha, \theta, \varphi$ functions, which satisfy the above equations, on Σ and the boundary conditions (25). Obviously, from the constitutive equations and from (13), we can express the boundary conditions (26) in terms of the functions $u_\alpha, \phi_\alpha, \theta, \varphi$.

In the case of equilibrium theories, we will divide this problem into two, the first including finding the functions θ and ϕ_α and then the functions u_α and φ. It is convenient not to separate the equations of system B, in the study of certain problems. So, throughout this paper, we assume that:

$$k > 0, \ d_2 > 0, \ d_6 > 0, \ kd_2 - d_1 d_3 > 0, \ d_4 + d_5 + d_6 > 0. \tag{33}$$

It is important to note that the restrictions imposed by the Clausius-Duhem inequality on the constituent coefficients, (Grof, 1969), are taken into account in the conditions above.

3 Solution for field equations

We start by introducing a few notations, such as:

$$c_1 = \lambda + 2\mu + k, \tag{34}$$

$$c_2 = d + \frac{-a + d_2}{\Delta}, \tag{35}$$

$$m_1 = \Big(\frac{2k}{h}\Big)^{\frac{1}{2}}, where\ h = \alpha + \gamma + \varepsilon, \tag{36}$$

$$m_2 = \Big(d_2/c_2\Big)^{\frac{1}{2}}, \tag{37}$$

$$m_3 = \Big(\frac{a}{d_6} + \Delta\Big)^{\frac{1}{2}}, \tag{38}$$

$$\kappa_1 = -c_2(3\lambda + 2\mu + k)\sigma - c_1 b\zeta, \tag{39}$$

$$\kappa_2 = -c_2(3\lambda + 2\mu + k), \tag{40}$$

$$\kappa_3 = 0. \tag{41}$$

From (2.12) results that m_1^2, m_2^2, and $m_3^2 > 0$. We introduce the operators:

$$C_1 = c_1 h\Delta(\Delta - m_1^2), \tag{42}$$

$$C_2 = kc_2\Delta(\Delta - m_2^2), \tag{43}$$

$$C_3 = d_6(\Delta - m_3^2), \tag{44}$$

$$B_1 = hc_1(\lambda + \mu)(\Delta - m_1^2), \tag{45}$$

$$B_2 = h\sigma c_1(3\lambda + 2\mu + k)(\Delta - m_1^2)(c_2\Delta - d_2), \tag{46}$$

$$B_3 = 0. \tag{47}$$

Theorem 1. *Let's consider the functions*

$$u_\alpha = -c_1 C_1 \Gamma_\alpha + B_1 \Gamma_{\rho,\rho\alpha} - B_2 f_{,\alpha} - C_3 B_3 g_{\rho,\rho\alpha}, \tag{48}$$

$$\phi_\alpha = c_1^2 \mu \Delta \psi_\alpha + c_1(\kappa_1 \Delta - \kappa_2)\Delta l_\alpha + k\zeta c_1 C_1 \Delta\Delta C_3 g_{\rho,\rho\alpha}, \tag{49}$$

$$\theta = -c_1(c_2\Delta - d_2)C_1 l, \tag{50}$$

$$\varphi_\alpha = c_1 C_1 C_2 g - c_1[k(d_4 - d_5)\Delta C_1 g_{\rho,\rho} - c_1 d_3 C_1 l, \tag{51}$$

where the fields $\Gamma_\alpha, \psi_\alpha \in C^6(\Sigma), l \in C^8(\Sigma)$, *and* $g \in C^{10}(\Sigma)$, *satisfy the equations:*

$$(\mu + k)\Delta c_1 C_1 \Gamma_\alpha = \rho f_\alpha; \tag{52}$$

$$\mu c_1 C_1 \psi_\alpha = \rho G_\alpha; \tag{53}$$

$$c_1 C_1 C_2 l = \rho S; \tag{54}$$

$$c_1 C_1 C_2 C_3 g = -\rho L. \tag{55}$$

Proof. $(\lambda+\mu)u_{\rho,\rho\alpha}+(\mu+k)u_{\alpha,\rho\rho}+\xi\phi_{,\alpha}-(3\lambda+2\mu+k)\sigma\tau_\alpha=-\rho l_\alpha-(\mu+k)\Delta c_1C_1\Gamma_\alpha+[(\lambda+2\mu+k)\Delta B_1-(\lambda+\mu)c_1C_1]\Gamma_{\rho,\rho\alpha}-C_3B_3[(\lambda+\mu)\Delta+\mu+k]g_{\rho,\rho\alpha}+(-\xi c_1^2\mu\Delta)\psi_{,\alpha}+[B_2\Delta(k-\lambda)+\xi c_1(\kappa_1\Delta-\kappa_2)\Delta]l_{,\alpha}+k\xi\zeta c_1C_1C_3\Delta\Delta\Delta g_{,\alpha}+(3\lambda+2\mu+k)\sigma c_1C_1(c_2\Delta-d_2)l_\alpha=\rho f_\alpha$

$<=> (\mu+k)\Delta c_1C_1\Gamma_\alpha=\rho f_\alpha$.

We do the same for the other equations. More specifically if we used the equations (52-55), we get what we want. □

4 Consequences of heat supply moments and pores

With the aim of studying the influences of the heat supply moments and pores on deformation, we will use the solution obtained in the theorem presented in the previous section. Therefore, let's consider that

$$\rho f_\alpha=0,\ \rho G_\alpha=0,\rho S=\delta(x-y),\rho L=0.$$

where $y(y_\alpha)$ is a fixed point, and δ is the Dirac measure .

Taking into account this assumption, the relations (52-55) are satisfied if we consider $\Gamma_\alpha=0,\psi_\alpha=0,l=\omega\ and\ g=0$. The ω function is a solution for the equation:

$$\Delta\Delta(\Delta-m_1^2)(\Delta-m_2^2)\omega=\gamma\delta(x-y), \tag{56}$$

where we use the γ notation for $(\varepsilon kc_1^2c_2)^{-1}$.

From those previously considered it follows that we obtain from the relations (52-55), the functions $u_\alpha^{(1)}(x,y),\phi_\alpha^{(1)}(x,y),\theta^{(1)}(x,y)$ and $\varphi^{(1)}(x,y)$.Therefore,

$$u_\alpha^{(1)}(x,y)=-B_2\omega_{,\alpha}; \tag{57}$$

$$\phi_\alpha^{(1)}(x,y)=c_1(\kappa_1\Delta-\kappa_2)\Delta\omega_\alpha; \tag{58}$$

$$\theta^{(1)}(x,y)=-c_1(c_2\Delta-d_2)C_1\omega; \tag{59}$$

$$\varphi^{(1)}(x,y)=-c_1bC_1\omega. \tag{60}$$

Next, we shall have the following considerations:

$$*\ \ m_1,m_2,m_3\ are\ distinct,$$

$$*\ \ \omega_s,(s=1,2,3,4), functions\ that\ satisfy\ the\ following\ equations:$$

$$\Delta\omega_1=M,\ \Delta\Delta\omega_2=M,\ (\Delta-m_1^2)\omega_3=M,\ (\Delta-m_2^2)\omega_4=M,\ where\ M\ is\ a\ given\ function.$$

Therefore, we can formulate the solution of the equation $\Delta\Delta(\Delta-m_1^2)\Delta-m_2^2)\omega=M$, as follows:

$$\omega=\sum_{s=1}^{4}z_s\omega_s,$$

where the constants $z_s, (s = 1, 2, 3, 4)$, are given by:

$$z_1 = \frac{m_1^2 + m_2^2}{m_1^4 m_2^4},\ z_2 = \frac{1}{m_1^2 m_2^2},\ z_3 = \frac{1}{m_1^4(m_1^2 - m_2^2)},\ z_4 = -\frac{1}{m_2^4(m_1^2 - m_2^2)}. \quad (61)$$

For $M = \delta(x - y), \omega_s, (s = 1, 2, 3, 4)$, are given by:

$$\omega_1 = \frac{1}{2\pi}\ln r,\ \omega_2 = \frac{1}{8\pi}r^2\ln r,\ \omega_3 = -\frac{1}{2\pi}K_0(m_1 r),\ \omega_4 = -\frac{1}{2\pi}K_0(m_2 r), r = [(x_1-y_1)^2+(x_2-y_2^2)]^{\frac{1}{2}}, \quad (62)$$

where we used the notation K_0 for the modified Bessel function of order zero. Therefore, for equation (50), we have the following solution:

$$\omega = \frac{\gamma}{2\pi}[z_1\ln r + \frac{1}{4}z_2 r^2\ln r - z_3K_0(m_1 r) - z_4K_0(m_2 r)]. \quad (63)$$

The displacement and the microrotation are introduced by the functions $u_\alpha^{(1)}$ and $\phi_\alpha^{(1)}$. In what follow, we will focus on the consequences of pores. Thus we assum that

$$\rho f_\alpha = 0,\ \rho G_\alpha = 0, \rho S = 0, \rho L = \delta_{\alpha\beta}\delta(x - y),$$

where we have β fixed. So, we will have $\Gamma_\alpha = 0, \psi_\alpha = 0, l = 0$ and $g = \delta_{\alpha\beta}\Omega$. In this case, from (52-55), it follows that Ω is a solution of the following equation:

$$\Delta\Delta(\Delta - m_1^2)(\Delta - m_2^2)(\Delta - m_3^2)\Omega = \gamma_1\delta(x - y), \quad (64)$$

where $\gamma_1 = (k\varepsilon d_6 c_1^2 c_2)^{-1}$. Therefore, we get from (52-55) the functions:

$$u_\alpha^{(1+\beta)}(x, y),\quad \phi_\alpha^{(1+\beta)}(x, y),\quad \theta^{(1+\beta)}(x, y),\quad \varphi^{(1+\beta)}(x, y).$$

5 Plane deformation

Let us consider a cylindrical hole contained in an elastic space and the domain $B = \{x : x_1^2 + x_2^2 > r_1^2, x_3 \in \mathbb{R}\}, (r_1 > 0)$, ocuppied by an elastic material with inner structure. This material will undergo a plan strain parallel to the x_1x_2-plane. Knowing these, the domain $\sum$ is defined by $\sum = \{x : x_1^2+x_2^2 > r_1^2, x_3 = 0\}$. Furthermore, we will assume that body loads are absent and the hole surface is free of surface forces.

The problem we shall study involves determining the functions θ,φ, u_α and ϕ_α.

Obviously, these functions must satisfy the following equations:

$$(\lambda + \mu)u_{\rho,\rho\alpha} + (\mu + k)u_{\alpha,\rho\rho} + \xi\varphi_{,\alpha} - (3\lambda + 2\mu + k)\sigma\theta_{,\alpha} = 0, \quad (65)$$

$$(\alpha + \gamma)\phi_{\rho,\rho\alpha} + \varepsilon\phi_{\alpha,\rho\rho} + \zeta\varphi_{,\alpha} - 2k\phi_\alpha = 0, \quad (66)$$

$$k\Delta\theta = 0, \quad (67)$$

$$d\varphi_{\rho,\rho} - \xi u_{\rho,\rho} - \zeta\phi_{\rho,\rho} - a\varphi - b\theta = 0, \qquad \textit{on } \Sigma. \quad (68)$$

Knowing that the heat flow into the body is produced by keeping the surface of the hole at a constant temperature θ^*, we add the boundary conditions to the equations (66) and (67):

$$\theta = \theta^*, \quad \varphi = \varphi_\alpha n_\alpha \; for \; r = r_1, \tag{69}$$

and, also, we add the following boundary conditions:

$$T_{\beta\alpha,\beta} n_\beta = 0, \; M_{\beta\alpha} n_\beta = 0 \; for \; r = r_1 \tag{70}$$

We consider the solution to have the form $\theta = V(r), \varphi = W(r)$, $u_\alpha = x_\alpha U(r)$ and $\phi_\alpha = Q(r)$ where $r = (x_1^2 + x_2^2)^{\frac{1}{2}}$. Therefore, they must satisfy the corresponding equations:

$$(\lambda + \mu) x_\alpha r^2 U + (\mu + k) x_\alpha r^2 U + \xi r W - (3\lambda + 2\mu + k)\sigma r V = 0, \tag{71}$$

$$(\alpha + \gamma) r^2 Q + \varepsilon r^2 Q + \zeta r W - 2kQ = 0, \tag{72}$$

$$k\Delta V = 0, \tag{73}$$

$$d r^2 W - \xi x_\alpha r U - \zeta r Q - aW - bV = 0, \qquad on \; \Sigma. \tag{74}$$

From relation (73) we obtain:

$$x_\alpha r U = \frac{1}{\lambda + 2\mu + k}\Big[(3\lambda + 2\mu + k)\sigma V - \xi W\Big] + N_1, \tag{75}$$

where N_1 is an arbitrary constant. Or, we can rewrite this relation as

$$(r^2 U)' = \frac{1}{c_1}\Big[(3\lambda + 2\mu + k)\sigma V - \xi W] r + r N_1. \tag{76}$$

Also, for the remaining relations, (74-76), we obtain the following form.

$$(\Delta - m_1^2) Q = -\frac{\zeta r}{h} W, \tag{77}$$

$$k\Delta V = 0, \tag{78}$$

$$(\Delta - m_2^2 + \tau_2 \xi) W = \tau_1 V + \tau_3 Q + \tau_2 N_1, \tag{79}$$

where for simplification we have used the notations:

$$\tau_1 = \frac{\xi(3\lambda + 2\mu + k)\sigma + b c_1}{c_1 c_2}, \quad \tau_2 = \frac{\xi}{c_1 c_2}, \quad \tau_3 = \frac{\zeta r}{c_2}. \tag{80}$$

Next, we obtain from the relation (8) the form of the function V. Therefore,

$$V = C_1 + B_1 \ln r, \tag{81}$$

where C_1 and B_1 are arbitrary constants. Also, from relation (79), we obtain:

$$\tau_3 Q = \frac{\tau_4}{m_1^2} W - \tau_4 C_2 - \tau_4 B_2 \ln r, \tag{82}$$

where we noted with $\tau_4 = \tau_3^2 \frac{c_2}{g}$.

We will further subtitute $\tau_3 Q$ and $\tau_1 V$ in the relation (81),

$$(\Delta - m_1^2 + \tau_2 \xi)W = \tau_1 C_1 + \tau_1 B_1 \ln r + \frac{\tau_4}{m_1^2} W - \tau_4 C_2 - \tau_4 B_2 lnr + \tau_2 N_1, \quad (83)$$

and in the and we will get:

$$W = \tau_1 \Big(\frac{C_1 + B_1 \ln r}{m_1^2} - B_1 \Delta \ln r \Big) + \tau_4 \Big(\frac{C_2 + B_2 \ln r}{m_1^2} - B_2 \Delta \ln r \Big) - \quad (84)$$

$$- \frac{\tau_2}{m_1^2} N_1 + N_3 k_0(m, r).$$

We therefore obtained the functions V, W and Q in relations (83), (84) and (86). The function U, is immediately determined by replacing V, W and Q in the relation (78).

Conclusions

Taking into account what has been obtained previously, this work has as main objective the achievement of solutions V, W, Q and U, which implies the plane deformation for micropolar isotropic materials in equilibrium theory. Using the equilibrium equations, we deduce the solution of the field equations, which later help us to study the effect of heat supply moments and pores in an elastic space. And finally we deduce what we set out to do, the plane deformation.

References

[1] Eringen, A.C., *Theory of Micropolar Fluids*, Journal of Mathematics and Mechanics **16** (1966), 1-18.

[2] Eringen, A.C *Theory of Micropolar elasticity. In Fracture* (Edited by H. Leibowitz), Vol II, Academic Press, New York, 622 (1968).

[3] Eringen, A.C., *Theory of Micropolar Elasticity. In: Microcontinuum Field Theories*. Springer, New York, NY,(1999).

[4] Fudulu, I.M., Marin, M., *On a variational principle and continuous dependece result for a micropolar izotropic body*, Springer, 2022.

[5] Nunziato, J.W. and Cowin, S.C., *A Nonlinear Theory of Elastic Materials with Voids*, Arch. Ration. Mech. Anal. **72** (1979), 175-201.

[6] Cowin, S.C. and Nunziato, J.W., *Linear elastic materials with voids*, J. Elasticity **13** (1983), 125-147.

[7] Passarella, F., *Some results In micropolar thermoelasticity*, Mechanics Research Communications, **23** (1996), no. 4, 349-357.

[8] Scutaru, M.L, Vlase, S.,Marin, M., Modrea, A., *New analytical method based on dynamic response of planar mechanical elastic systems*, Bound. Value Probl., **2020** (2020), no.1, Art. 104.

[9] Marin, M., Vlase, S., Fudulu, I.M., and Precup, G., *On instability in the theory of dipolar bodies with two-temperatures*, Carpathian Journal of Mathematics, **38** (2022), no. 2, 459-468.

[10] Marin, M., Vlase, S., Fudulu, I.M., and Precup, G. *Effect of voids and internal state variables in elasticity of porous bodies with dipolar structure*, Mathematics 9 (2021), no. 21, Article number 2741.

[11] Marin, M., Fudulu, I.M., and Vlase, S., *On some qualitative results in thermodynamics of Cosserat bodies*, Bound. Value Probl. **2022** (2022), Article number 69.

[12] Marin, M., *Generalized solutions in elasticity of micropolar bodies with voids*, Revista de la Academia Canaria de Ciencias, 8(1), 101-106, 1996.

[13] Marin, M., *Contributions on uniqueness in thermoelastodynamics on bodies with voids*, Ciencias matemáticas (Havana), **16** (1998), no. 2, 101-109.

[14] Ieşan, D., Quintanilla, R., *On a theory of thermoelastic materials with a double porosity structure*, Journal of Thermal Stresses **37** (2014), no. 9, 1017-1036.

[15] Saccomandi, G., *On inhomogeneous deformations in finite thermoelasticity*, IMA Journal of Applied Mathematics , **63** (1999), no. 2, 131–148.

[16] Ailawalia, P., Sachdeva, S.K., Pathania, D.S., *Plane strain deformation in a thermoelastic microelongated solid with internal heat source*, International Journal of Applied Mechanics and Engineering, **20** (2015), no. 4, 717-735.

[17] Nowacki, W., *Thermoelasticity*, Pergamon Press, 1986.