η-RICCI SOLITONS ON 3- DIMENSIONAL KENMOTSU MANIFOLDS

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Abstract

The purpose of this paper is to study η-Ricci solitons on 3-dimensional Kenmotsu manifolds. First, we prove that an η-Ricci soliton on a 3-dimensional Kenmotsu manifold is an η-Einstein manifold. Besides these, we consider η-Ricci solitons on 3 -dimensional Kenmotsu manifolds with Ricci tensor of Codazzi type and cyclic parallel Ricci tensor. Next, we study conformally flat and φ-Ricci symmetric η-Ricci soliton on 3-dimensional Kenmotsu manifolds. Finally, we construct an example to prove the existence of η-Ricci soliton on 3-dimensional Kenmotsu manifold and verify some results.

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1 Introduction

In 1982, Hamilton [21] introduced the notion of the Ricci flow to find a canonical metric on a smooth manifold. The Ricci flow is an evolution equation for metrics on a Riemannian manifold

$$\frac{\partial}{\partial t} g_{ij}(t) = -2R_{ij.} \tag{1}$$

A Ricci soliton is a natural generalization of an Einstein metric and is defined on a Riemannian manifold (M, g) [7]. A Ricci soliton is a triple (g, V, λ) with g, a Riemannian metric, V a vector field and λ a real scalar such that

$$\pounds_V g + 2S + 2\lambda g = 0, \tag{2}$$

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where S is a Ricci tensor of M and $\pounds_V$ denotes the Lie derivative operator along the vector field V. The Ricci soliton is said to be shrinking, steady and expanding accordingly as λ is negative, zero and positive, respectively [22]. Metrics satisfying (2) are interesting and useful in physics and are often referred as quasi-Einstein [9, 10]. Compact Ricci solitons are the fixed points of the Ricci flow $\frac{\partial}{\partial t}g = -2S$ projected from the space of metrics onto its quotient modulo diffeomorphisms and scalings, and often arise blow-up limits for the Ricci flow on compact manifolds. Theoretical physicists have also been looking into the equation of Ricci soliton in relation with string theory. The initial contributions in this direction are due to Friedan [18], who discusses some aspects of it. Ricci solitons were introduced in Riemannian geometry [21], as the self-similar solutions of the Ricci flow, and play an important role in understanding its singularities. Baird and Danielo [1], Calin and Crasmareanu [8], Ghosh [19], and Nagaraja and Premalatha [26] have extensively studied Ricci solitons in contact metric manifolds. Wang and Liu [29] devoted their work to the Ricci solitons on three dimensional η-Einstein almost Kenmotsu manifolds. Ricci solitons have been studied in many contexts by several authors such as [12, 13, 14, 16, 17, 23, 31].

As a generalization of Ricci soliton, the notion of η-Ricci soliton was introduced by J. T. Cho and M. Kimura [11], which was treated by C. Calin and M. Crasmareanu on Hopf hypersurfaces in complex space forms [7]. An η-Ricci soliton is a tuple (g, V, λ, μ), where V is a vector field on M, and λ , μ constants and g is a Riemannian (or pseudo-Riemannian) metric satisfying the equation

$$\pounds_V g + 2S + 2\lambda g + 2\mu\eta \otimes \eta = 0, \tag{3}$$

where S is the Ricci tensor associated to g.

In this connection, we mention the works of Blaga [3, 4] and Prakasha et al. [27] on η-Ricci solitons. Basu, et al. [2] generated some results on almost conformal Ricci soliton and η-Ricci soliton on 3-dimensional (ε, δ)-trans-Sasakian manifold. Recently, P. Majhi, U. C. De and D. Kar [25] studied η-Ricci solitons on Sasakian 3-Manifolds.

Motivated by the above mentioned works, in this frame-work, we make an effort to study η-Ricci solitons on 3-dimensional Kenmotsu manifolds.

A. Gray [20] introduced the notion of cyclic parallel Ricci tensor and Ricci tensor of Codazzi type. A Riemannian manifold is said to have cyclic parallel Ricci tensor if its Ricci tensor S of type $(0, 2)$ is non-zero and satisfies the condition

$$(\nabla_X S)(Y, Z) + (\nabla_Y S)(Z, X) + (\nabla_Z S)(X, Y) = 0. \tag{4}$$

Again a Riemannian manifold is said to have Ricci tensor of Codazzi type if its Ricci tensor S of type $(0, 2)$ is non-zero and satisfies the following condition

$$(\nabla_X S)(Y, Z) = (\nabla_Y S)(X, Z). \tag{5}$$

The present paper is organized as follows:

Section 2, contains some preliminary results of 3-dimensional Kenmotsu manifolds and we prove that an η-Ricci soliton on a 3-dimensional Kenmotsu manifold

is an η-Einstein manifold. Moreover, we consider η-Ricci solitons on 3-dimensional Kenmotsu manifolds with Ricci tensor of Codazzi type and cyclic parallel Ricci tensor. Next, we study conformally flat and φ-Ricci symmetric η-Ricci soliton on 3-dimensional Kenmotsu manifolds. Finally, we construct an example to prove the existence of η-Ricci solitons on 3-dimensional Kenmotsu manifolds and verify some results.

2 Preliminaries

Let M be an n-dimensional almost contact Riemannian manifold with the almost contact metric structure (φ, ξ, η, g), that is, φ is a $(1,1)$ tensor field, ξ is the structure vector field, η is a 1-form and g is the Riemannian metric. It is well-known that (φ, ξ, η, g) satisfy [5, 6]

$$\varphi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \varphi\xi = 0, \quad \eta(\varphi X) = 0, \tag{6}$$

$$g(X, \xi) = \eta(X), \tag{7}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{8}$$

for any vector fields X, Y on M.

Moreover,

$$(\nabla_X \varphi)Y = -g(X, \varphi Y)\xi - \eta(Y)\varphi(X), \tag{9}$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{10}$$

where ∇ denotes the Riemannian connection of g, then $(M, \varphi, \xi, \eta, g)$ is called an almost Kenmotsu manifold.

In a Kenmotsu manifold [24], the following relations hold:

$$(\nabla_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y), \tag{11}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{12}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{13}$$

for any vector fields X, Y and Z on M, where $R(X, Y)Z$ is the Riemannian curvature tensor and S is the Ricci tensor.

In a 3-dimensional Riemannian manifold the curvature tensor is given by

$$\begin{aligned} R(X, Y)Z &= g(Y, Z)QX - g(X, Z)QY + S(Y, Z)X - S(X, Z)Y \\ &\quad - \frac{r}{2}[g(Y, Z)X - g(X, Z)Y], \end{aligned} \tag{14}$$

where Q is the Ricci operator, i.e., $g(QX, Y) = S(X, Y)$ and r is the scalar curvature of the manifold.

It is known that the Ricci tensor of a 3-dimensional Kenmotsu manifold is given by [15]

$$S(X, Y) = \frac{1}{2}[(r+2)g(X, Y) - (r+6)\eta(X)\eta(Y)]. \tag{15}$$

Hence, from above equation, we can state that a 3-dimensional Riemannian manifold is an η-Einstein manifold. A 3-dimensional Riemannian manifold is a manifold of constant negative curvature if and only if the scalar curvature $r = -6$.

Proposition 1. *An η-Ricci soliton on a 3-dimensional Kenmotsu manifold is an η-Einstein manifold.*

Proof. Assume that the 3-dimensional Kenmotsu manifold admits an η-Ricci soliton (g, ξ, λ, μ). Then relation (3) yields

$$(\pounds_\xi g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) + 2\mu\eta(X)\eta(Y) = 0, \qquad (16)$$

or equivalently,

$$2S(X, Y) = -(\pounds_\xi g)(X, Y) - 2\lambda g(X, Y) - 2\mu\eta(X)\eta(Y), \qquad (17)$$

for all smooth vector fields $X, Y \in \Gamma(TM)$.

$$(\pounds_\xi g)(X, Y) = g(\nabla_X \xi, Y) + g(X, \nabla_Y \xi), \qquad (18)$$

by virtue of (6), we have

$$(\pounds_\xi g)(X, Y) = 2[g(X, Y) - \eta(X)\eta(Y)]. \qquad (19)$$

Using (19) in (17), we get

$$S(X, Y) = -(\lambda + 1)g(X, Y) - (\mu - 1)\eta(X)\eta(Y). \qquad (20)$$

Thus, we conclude that η-Ricci soliton (g, ξ, λ, μ) on 3-dimensional Kenmotsu manifolds is an η-Einstein manifold. This completes the proof. $\square$

Proposition 2. *For an η-Ricci soliton on a 3-dimensional Kenmotsu manifold we have $\lambda + \mu = 2$.*

Proof. The Ricci tensor of a 3-dimensional Kenmotsu manifold is given by [15]

$$S(X, Y) = \frac{1}{2}[(r + 2)g(X, Y) - (r + 6)\eta(X)\eta(Y)], \qquad (21)$$

where r is the scalar curvature. Comparing the above equation with $S(X, Y) = -(\lambda + 1)g(X, Y) - (\mu - 1)\eta(X)\eta(Y)$, we get $\lambda = -\frac{1}{2}(r + 4)$ and $\mu = \frac{1}{2}(r + 8)$.

From which it follows that

$$\lambda + \mu = 2. \qquad (22)$$

This completes the proof. $\square$

3 η-Ricci Soliton on 3-dimensional Kenmotsu manifolds with Ricci tensor of Codazzi type

This section is devoted to studying η-Ricci soliton on a 3-dimensional Kenmotsu manifold with Ricci tensor of Codazzi type. Therefore, taking the covariant differentiation of (20) with respect to Z, we get

$$
\begin{aligned}
(\nabla_Z S)(X,Y) &= -(\mu-1)[(\nabla_Z \eta)(X)\eta(Y) + \eta(X)(\nabla_Z \eta)(Y)], \\
(\nabla_Z S)(X,Y) &= (1-\mu)[g(\varphi X, \varphi Z)\eta(Y) + g(\varphi Y, \varphi Z)\eta(X)].
\end{aligned}
\tag{23}
$$

By hypothesis the Ricci tensor S is of Codazzi type. Then

$$
(\nabla_Z S)(X,Y) = (\nabla_Y S)(Z,X).
\tag{24}
$$

By virtue of (23) and (24), we get

$$
(1-\mu)[g(\varphi X, \varphi Z)\eta(Y)+g(\varphi Y, \varphi Z)\eta(X)] = (1-\mu)[g(\varphi Z, \varphi Y)\eta(X)+g(\varphi X, \varphi Y)\eta(Z)].
\tag{25}
$$

Substituting $Z = \xi$ in the above equation, we have

$$
(1-\mu)g(\varphi X, \varphi Y) = 0,
\tag{26}
$$

for any X, Y on M, it follows $\mu = 1$. From relation (22), we get $\lambda = 1$. Therefore, a 3-dimensional Kenmotsu manifold with Ricci tensor of Codazzi type admits an η-Ricci soliton.

Thus, we conclude the following:

Theorem 1. *A 3-dimensional Kenmotsu manifold with Ricci tensor of Codazzi type admits an η-Ricci soliton with $\lambda = 1$ and $\mu = 1$.*

4 η-Ricci soliton on 3-dimensional Kenmotsu manifolds with cyclic parallel Ricci tensor

In this section, we consider η-Ricci soliton on 3-dimensional Kenmotsu manifolds with cyclic parallel Ricci tensor. Therefore

$$
(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0,
\tag{27}
$$

for all smooth vector fields $X, Y, Z \in \Gamma(TM)$. Using (20) in (27), we have

$$
\begin{aligned}
(1-\mu)[&g(\varphi Y, \varphi X)\eta(Z) + g(\varphi Z, \varphi X)\eta(Y) \\
&+g(\varphi Z, \varphi Y)\eta(X) + g(\varphi X, \varphi Y)\eta(Z) \\
&+g(\varphi X, \varphi Z)\eta(Y) + g(\varphi Y, \varphi Z)\eta(X)] \\
=~ &0.
\end{aligned}
\tag{28}
$$

Substituting $X = \xi$ in (28), we find

$$(1 - \mu)g(\varphi Y, \varphi Z) = 0, \qquad (29)$$

for any Y, Z on M, it follows $\mu = 1$. From relation (22), we get $\lambda = 1$. Therefore, a 3-dimensional Kenmotsu manifold with cyclic parallel Ricci tensor admits an η-Ricci soliton.

Thus, we conclude the following:

Theorem 2. *A* 3-*dimensional Kenmotsu manifold with cyclic parallel Ricci tensor admits an η-Ricci soliton with $\lambda = 1$ and $\mu = 1$.*

5 φ-Ricci Symmetric η-Ricci soliton on 3-dimensional Kenmotsu manifolds

This section is devoted to the study of φ-Ricci Symmetric η-Ricci soliton on 3-dimensional Kenmotsu manifolds. A Kenmotsu manifold is said to be φ-Ricci symmetric [28] if

$$\varphi^2(\nabla_X Q)Y = 0, \qquad (30)$$

for all smooth vector fields X, Y on M.

The Ricci tensor for an η-Ricci soliton on 3-dimensional Kenmotsu manifold is given by

$$S(X, Y) = -(\lambda + 1)g(X, Y) - (\mu - 1)\eta(X)\eta(Y). \qquad (31)$$

Then, it follows that

$$QX = -(\lambda + 1)X - (\mu - 1)\eta(X)\xi, \qquad (32)$$

for all smooth vector fields X on M. Taking covariant derivative of (32), we obtain

$$(\nabla_X Q)Y = (\mu - 1)g(X, Y)\xi + 2(\mu - 1)\eta(X)\eta(Y)\xi + (\mu - 1)\eta(Y)X. \qquad (33)$$

Applying φ^2 on both sides of the above equation, we have

$$\varphi^2(\nabla_X Q)Y = (\mu - 1)\eta(Y)\varphi^2 X. \qquad (34)$$

By virtue of (30) and (34), we have $\mu = 1$. From relation (22), we have $\lambda = 1$. Therefore, a φ-Ricci symmetric 3-dimensional Kenmotsu manifold admits an η-Ricci soliton.

Thus, we can state the following:

Theorem 3. *A φ-Ricci symmetric* 3-*dimensional Kenmotsu manifold admits an η-Ricci soliton with $\lambda = 1$ and $\mu = 1$.*

6 Conformally flat η-Ricci soliton on 3-dimensional Kenmotsu manifolds

This section is devoted to the study of conformally flat η-Ricci soliton on 3-dimensional Kenmotsu manifolds. Therefore [30]

$$(\nabla_X S)(Y, Z) - (\nabla_Y S)(X, Z) = \frac{1}{4}[g(Y, Z)dr(X) - g(X, Z)dr(Y)]. \qquad (35)$$

for all smooth vector fields X, Y on M.

Applying (20) in (35), we obtain

$$\begin{aligned}
(1 - \mu)&[g(\varphi Y, \varphi X)\eta(Z) + g(\varphi Z, \varphi X)\eta(Y) \\
&-g(\varphi X, \varphi Y)\eta(Z) - g(\varphi Z, \varphi Y)\eta(X)] \\
= \quad &\frac{1}{4}[g(Y, Z)dr(X) - g(X, Z)dr(Y)].
\end{aligned} \qquad (36)$$

Substituting $X = \xi$ in (36), we have

$$(1 - \mu)g(\varphi Z, \varphi Y) = \frac{1}{4}\eta(Z)dr(Y). \qquad (37)$$

It follows that

$$(1 - \mu)[X - \eta(X)\xi] = \frac{1}{4}dr(Y)\xi. \qquad (38)$$

This implies

$$(1 - \mu)\varphi X = 0. \qquad (39)$$

From the above equation, we have $\mu = 1$. From relation (22), we obtain $\lambda = 1$. Therefore, a conformally flat 3-dimensional Kenmotsu manifold admits an η-Ricci soliton.

Thus, we are in a position to state the following:

Theorem 4. *A conformally flat* 3-*dimensional Kenmotsu manifold admits an* η-*Ricci soliton with* $\lambda = 1$ *and* $\mu = 1$.

7 Example of existence of η-Ricci solitons on 3 - dimensional Kenmotsu manifolds

We consider the 3-dimensional manifold $M = \{(x, y, z) \in R^3 : z \neq 0\}$, where (x, y, z) are the standard coordinates in R^3. We choose the vector fields

$$e_1 = z\frac{\partial}{\partial x}, e_2 = z\frac{\partial}{\partial y} \text{ and } e_3 = -z\frac{\partial}{\partial z},$$

which are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$g(e_1, e_2) = g(e_2, e_3) = g(e_3, e_1) = 0,$$

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1.$$

Let η be the 1- form defined by $\eta(Z) = g(Z, e_3)$ for any vector field Z on M. We define the $(1,1)$ tensor field φ as $\varphi(e_1) = e_2, \varphi(e_2) = -e_1$ and $\varphi(e_3) = 0$.

Then using the linearity of φ and g, we have

$$\eta(e_3) = 1, \varphi^2 Z = -Z + \eta(Z)e_3,$$

$$g(\varphi Z, \varphi W) = g(Z, W) - \eta(Z)\eta(W),$$

for any vector fields Z, W on M. Thus, for $e_3 = \xi$, the structure (φ, ξ, η, g) defines an almost contact metric structure on M.

Let ∇ be the Levi-Civita connection with respect to metric g and R be the curvature tensor of metric g. Then, we have

$$[e_1, e_2] = 0, \quad [e_1, e_3] = e_1 \text{ and } \quad [e_2, e_3] = e_2.$$

The Riemannian connection ∇ of the metric tensor g is given by Koszul's formula which is

$$
\begin{aligned}
2g(\nabla_X Y, Z) &= Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) \\
&\quad - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]).
\end{aligned}
$$

Using Koszul's formula, we can easily calculate

$$\nabla_{e_1} e_3 = e_1, \quad \nabla_{e_1} e_2 = 0, \quad \nabla_{e_1} e_1 = -e_3,$$

$$\nabla_{e_2} e_3 = e_2, \quad \nabla_{e_2} e_2 = -e_3, \quad \nabla_{e_2} e_1 = 0,$$

$$\nabla_{e_3} e_3 = 0, \quad \nabla_{e_3} e_2 = 0, \quad \nabla_{e_3} e_1 = 0.$$

From the above, it follows that the manifold satisfying $\nabla_X \xi = X - \eta(X)\xi$, for $e_3 = \xi, (\varphi, \xi, \eta, g)$ is a Kenmotsu structure on M. Consequently $M^3(\varphi, \xi, \eta, g)$ is a Kenmotsu manifold.

Also, the Riemannian curvature tensor R is given by

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z.$$

With the help of the above results, we can verify the following results:

$$R(e_1, e_2)e_2 = -e_1, \quad R(e_1, e_3)e_3 = -e_1, \quad R(e_2, e_1)e_1 = -e_2,$$

$$R(e_2, e_3)e_3 = -e_2, \quad R(e_3, e_1)e_1 = -e_3, \quad R(e_3, e_2)e_2 = -e_3.$$

Then, the Ricci tensor S is given by

$$S(e_1, e_1) = -2, \quad S(e_2, e_2) = -2 \text{ and } \quad S(e_3, e_3) = -2.$$

From (25), we obtain $S(e_1, e_1) = -(1+\lambda), S(e_2, e_2) = -(1+\lambda)$ and $S(e_{3,} e_3) = -(\lambda + \mu)$, therefore $\lambda = 1$ and $\mu = 1$.

Thus, the data (g, ξ, λ, μ) for $\lambda = 1$ and $\mu = 1$ defines an η-Ricci soliton. Also, the Ricci tensor is of Codazzi type and cyclic parallel. Hence, Theorem 1 and Theorem 2 are verified. Again, if the manifold is φ-Ricci symmetric, then Theorem 3 is verified.

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