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TRANSFORMATION FORMULAS OF INCOMPLETE HYPERGEOMETRIC FUNCTIONS VIA FRACTIONAL CALCULUS OPERATORS

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Abstract

The desire for present article is to derive from the application of fractional calculus operators a transformation that expresses a potentially useful incomplete hypergeometric function in various forms of a countable sum of lesser-order functions. Often listed are numerous (known or new) specific cases and implications of the findings described herein.

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1 Introduction

Karlsson [1] given an expression considering the generalized hypergeometric function ${}_rF_s(.)$ including positive integral differences between some numerator together with denominator quantities in view of a limited number pertaining to functions in the lesser order, notably

$$
{}_rF_s\left[\begin{array}{c} b_1+m_1,\cdots,b_n+m_n,a_{n+1},\cdots,a_r; \\ \\ b_1,\cdots,b_n,b_{n+1},\cdots,b_s; \end{array} z\right]=\sum_{j_1=0}^{m_1}\cdots\sum_{j_n=0}^{m_n}C\left(j_1,\cdots,j_n\right)
$$

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$$z^{J_n}\ {}_{r-n}F_{s-n}\left[\begin{array}{c} a_{n+1}+J_n,\cdots,a_r+J_n; \\ \\ b_{n+1}+J_n,\cdots,b_s+J_n; \end{array} z\right], \tag{1}$$

where

$$C\left(j_1,\cdots,j_n\right)=\binom{m_1}{j_1}\cdots\binom{m_n}{j_n}$$
$$\cdot\frac{(b_2+m_2)_{J_1}\cdots(b_n+m_n)_{J_{n-1}}(a_{n+1})_{J_n}\cdots(a_r)_{J_n}}{(b_1)_{J_1}\cdots(b_n)_{J_n}(b_{n+1})_{J_n}\cdots(b_s)_{J_n}}, \tag{2}$$

and

$$J_n=j_1+\cdots+j_n, \tag{3}$$

$m_i(i=1,\cdots,n)$ have being positive integers, so that $n\leq$min(r,s), fix up with that $r\leq s$ ($|z|<1$, if $r=s+1$), together with no one of the denominator parameters is a negative integer or zero.

Afterward, Raina [5] provided the subsequent extension of (1) to the situation where the discrepancies between the numerator and denominator parameters $m_i(i=1,\cdots,n)$ are arbitrary, such as:

$${}_rF_s\left[\begin{array}{c} b_1+\varsigma_1,\cdots,b_n+\varsigma_n,a_{n+1},\cdots,a_r; \\ \\ b_1,\cdots,b_n,b_{n+1},\cdots,b_s; \end{array} z\right]=\sum_{j_1=0}^{\infty}\cdots\sum_{j_n=0}^{\infty}$$

$$\cdot\binom{\varsigma_1}{j_1}\cdots\binom{\varsigma_n}{j_n}\frac{z^{J_n}(b_2+\varsigma_2)_{J_1}\cdots(b_n+\varsigma_n)_{J_{n-1}}(a_{n+1})_{J_n}\cdots(a_r)_{J_n}}{(b_1)_{J_1}\cdots(b_n)_{J_n}(b_{n+1})_{J_n}\cdots(b_s)_{J_n}}$$

$${}_{r-n}F_{s-n}\left[\begin{array}{c} a_{n+1}+J_n,\cdots,a_r+J_n; \\ \\ b_{n+1}+J_n,\cdots,b_s+J_n; \end{array} z\right], \tag{4}$$

provided that $r\leq s$ ($|z|<1$, if $r=s+1$), and n is any positive integer such as $n\leq$ min(r,s), where J_n is defined through (3), and $\Re(b_i+\varsigma_i)>0$ for $i=1,\cdots,n$.

Srivastava et al. [9], recently made known and analyzed certain theoretical concepts and properties of a genuinely useful class of the preceding two generalized incomplete hypergeometric functions characterized below:

$${}_r\Gamma_s\left[\begin{array}{r}(\tau_1,x),\tau_2,\cdots,\tau_r; \\ \varsigma_1,\cdots,\varsigma_s\,;\end{array} z\right]:=\sum_{n=0}^{\infty}\frac{[\tau_1;x]_n(\tau_2)_n\cdots(\tau_r)_n}{(\varsigma_1)_n\cdots(\varsigma_s)_n}\frac{z^n}{n!} \tag{5}$$

and

$$ {}_r\gamma_s \left[\begin{matrix} (\tau_1, x), \tau_2, \cdots, \tau_r; \\ \varsigma_1, \cdots, \varsigma_s; \end{matrix} \; z \right] := \sum_{n=0}^{\infty} \frac{(\tau_1; x)_n (\tau_2)_n \cdots (\tau_r)_n}{(\varsigma_1)_n \cdots (\varsigma_s)_n} \frac{z^n}{n!}, \tag{6} $$

where $(\tau_1; x)_n$ as well as $[\tau_1; x]_n$ have become an important generalization of the $(\vartheta)_n$ Pochhammer formula, the incomplete Pochhammer symbol is described as follows:

$$ [\vartheta; x]_n := \frac{\Gamma(\vartheta + n, x)}{\Gamma(\vartheta)} \quad (\vartheta, n \in C; x \geq 0) \tag{7} $$

and

$$ (\vartheta; x)_n := \frac{\gamma(\vartheta + n, x)}{\Gamma(\vartheta)} \quad (\vartheta, n \in C; x \geq 0). \tag{8} $$

Both incomplete definitions for Pochhammer $[\vartheta; x]_n$ as well as $(\vartheta; x)_n$ fulfill the preceding relation of decomposition:

$$ [\vartheta; x]_n + (\vartheta; x)_n = (\vartheta)_n \quad (\vartheta, n \in C; x \geq 0). \tag{9} $$

Further, Srivastava [8] using mathematical induction, derived the following reduction formula:

$$ {}_{r+1}\Gamma_s \left[\begin{matrix} (a_0, x), b_1 + m_1, \cdots, b_n + m_n, a_{n+1}, \cdots, a_r; \\ \\ b_1, \cdots, b_n, b_{n+1}, \cdots, b_s; \end{matrix} \quad z \right] = \sum_{j_1=0}^{m_1} \cdots \sum_{j_n=0}^{m_n} $$

$$ \cdot \binom{m_1}{j_1} \cdots \binom{m_n}{j_n} \frac{z^{J_n} (b_2 + m_2)_{J_1} \cdots (b_n + m_n)_{J_{n-1}} (a_{n+1})_{J_n} \cdots (a_r)_{J_n}}{(b_1)_{J_1} \cdots (b_n)_{J_n} (b_{n+1})_{J_n} \cdots (b_s)_{J_n}} $$

$$ {}_{r+1-n}\Gamma_{s-n} \left[\begin{matrix} (a_0 + J_n, x), a_{n+1} + J_n, \cdots, a_r + J_n; \\ \\ b_{n+1} + J_n, \cdots, b_s + J_n; \end{matrix} \quad z \right] \tag{10} $$

and

$$ {}_{r+1}\gamma_s \left[\begin{matrix} (a_0, x), b_1 + m_1, \cdots, b_n + m_n, a_{n+1}, \cdots, a_r; \\ \\ b_1, \cdots, b_n, b_{n+1}, \cdots, b_s; \end{matrix} \quad z \right] = \sum_{j_1=0}^{m_1} \cdots \sum_{j_n=0}^{m_n} $$

$$ \cdot \binom{m_1}{j_1} \cdots \binom{m_n}{j_n} \frac{z^{J_n} (b_2 + m_2)_{J_1} \cdots (b_n + m_n)_{J_{n-1}} (a_{n+1})_{J_n} \cdots (a_r)_{J_n}}{(b_1)_{J_1} \cdots (b_n)_{J_n} (b_{n+1})_{J_n} \cdots (b_s)_{J_n}} $$

$$ {}_{r+1-n}\gamma_{s-n} \left[\begin{matrix} (a_0 + J_n, x), a_{n+1} + J_n, \cdots, a_r + J_n; \\ \\ b_{n+1} + J_n, \cdots, b_s + J_n; \end{matrix} \quad z \right], \tag{11} $$

provided that $r+1 \leq s$ ($|z|<1$, if $r=s+1$), and n is any positive integer such that $n \leq \min(r,s)$, where the notation J_n is defined through (3), with m_i are positive integer for $i=1,\cdots,n$.

Persuaded necessarily by the illustrated potential for utilizations of the generalized essentially hypergeometric functions in numerous differing area of numerical, physical, building and measurable sciences (be apprised of, for particulars, [9] and the references specify with in), Now, in this paper our aim is to derive general reduction formula for the generalized incomplete hypergeometric functions, by employing the concept of fractional calculus operators, especially the Leibnitz rule. We moreover consider different (known or modern) uncommon cases and results of the comes about which are determined in this paper.

2 Reduction formula for the generalized incomplete hypergeometric functions

In pursuance of any bounded arrangement of real and complex numbers $\{A_n\}$, characterize a function $f(z)$ by means of the power series

$$f(z)=\sum_{n=0}^{\infty} A_n z^n, \qquad |z|<\Re. \tag{12}$$

A broadly utilized essential equation within the fractional calculus is the formula

$$D_z^{\varsigma}\left\{z^{\tau-1}\right\}=\frac{\Gamma(\tau)}{\Gamma(\tau-\varsigma)} z^{\tau-\varsigma-1} \qquad \Re(\tau)>0, \tag{13}$$

substantial for all values of ς. Moreover, for subjective ς, the Leibniz formula is shown by [2], as:

$$D_z^{\varsigma}\{u(z)v(z)\}=\sum_{n=0}^{\infty}\binom{\varsigma}{n} D_z^{\varsigma-n}\{u(z)\} D_z^{n}\{v(z)\}, \tag{14}$$

where $u(z)$ and $v(z)$ are function of the frame $z^{a-1}h(z)$ & $z^{b-1}k(z)$, respectively, where $h(z)$ and $k(z)$ are analytic in the disc $|z|<\rho$, given that $\Re(a)>0$ and $\Re(a+b)>0$. Now, both side multiplied by $z^{\tau+\varsigma-1}$ of (12), we get

$$z^{\tau+\varsigma-1} f(z)=\sum_{n=0}^{\infty} A_n z^{\tau+\varsigma+n-1}, \tag{15}$$

and also operate D_z^{ς} on both the side, after that (15) in conjuction with (13) and (14) present with

$$\sum_{n=0}^{\infty}\frac{(\tau+\varsigma)_n}{(\tau)_n} A_n\, z^n=\sum_{n=0}^{\infty}\binom{\varsigma}{n}\frac{z^n}{(\tau)_n} D_z^{n}\{f(z)\}, \tag{16}$$

provided that $\Re(\tau+\varsigma) > 0$, the subjective arrangement of real or complex numbers is bounded, and $|z| < \Re$. The fractional derivative operator D_z^{ς} can be connected term astute to the correct side of the series of (14) in see of result said in a later monograph of Srivastava and Manocha [10], under the hypotheses surrounding the above equation (15).
Our fundamental comes about in this segment are contained in Theorem 1 and 2 underneath as:

Theorem 1. *The subsequent reduction procedure hold true for the generalized incomplete hypergeometric functions:*

$$
{}_{r+1}\Gamma_s \left[\begin{array}{l} (a_0,x), b_1+\varsigma_1, \cdots, b_n+\varsigma_n, a_{n+1}, \cdots, a_r; \\ \\ b_1, \cdots, b_n, b_{n+1}, \cdots, b_s; \end{array} z \right] = \sum_{j_1=0}^{\infty} \cdots \sum_{j_n=0}^{\infty}
$$

$$
\cdot \binom{\varsigma_1}{j_1} \cdots \binom{\varsigma_n}{j_n} \frac{z^{J_n}(b_2+\varsigma_2)_{J_1} \cdots (b_n+\varsigma_n)_{J_{n-1}} (a_{n+1})_{J_n} \cdots (a_r)_{J_n}}{(b_1)_{J_1} \cdots (b_n)_{J_n} (b_{n+1})_{J_n} \cdots (b_s)_{J_n}}
$$

$$
{}_{r+1-n}\Gamma_{s-n} \left[\begin{array}{l} (a_0+J_n,x), a_{n+1}+J_n, \cdots, a_r+J_n; \\ \\ b_{n+1}+J_n, \cdots, b_s+J_n; \end{array} z \right], \qquad (17)
$$

provided that $r \leq s$ *(*$|z| < 1$*, if* $r = s+1$*), and* n *is any positive integer such as* $n \leq \ min(r,s)$*, where* J_n *is lay down by* (3)*, and* $\Re(b_i+\varsigma_i) > 0$ *for* $i = 1, \cdots, n$*.*

Proof. To prove the formula (17), we make use of the result (16). In case, if we specialize the self-assertive sequence $\{A_n\}$, by letting

$$
A_n = \frac{[a_0,x]_n \prod_{i=2}^{r}(a_i)_n}{\prod_{i=2}^{s}(b_i)_n n!} \qquad n \geq 0, \qquad (18)
$$

then (15) is follow (on little adjustment of parameters) to give up the result

$$
{}_{r+1}\Gamma_s \left[\begin{array}{l} (a_0,x), b_1+\varsigma_1, a_2, \cdots, a_r; \\ \\ b_1, \cdots, b_s; \end{array} z \right] = \sum_{j_1=0}^{\infty} \binom{\varsigma_1}{j_1} \frac{z^{J_1}(a_2)_{J_1} \cdots (a_r)_{J_1}}{(b_2)_{J_1} \cdots (b_s)_{J_1}}
$$

$$
{}_{r}\Gamma_{s-1} \left[\begin{array}{l} (a_0+J_1,x), a_2+J_1, \cdots, a_r+J_1; \\ \\ b_2+J_1, \cdots, b_s+J_1; \end{array} z \right], \qquad (19)
$$

provided that $r \leq s$ (and $|z| < 1$, if $r = s+1$), $\Re(b_i+\varsigma_i > 0$, and no denominator parameter is zero or a negative integer.
The repetitive implementation of (19) to each of its R.H.S. when $a_i = b_i+\varsigma_i$ $(i = 2, \cdots, n)$, yields the intended outcome (17).
This supplements the Theorem's proof. □

Theorem 2. *For generalized incomplete hypergeometric functions the preceeding reduction formulae remain true:*

$$ {}_{r+1}\gamma_s \left[\begin{array}{c} (a_0,x), b_1+\varsigma_1,\cdots,b_n+\varsigma_n, a_{n+1},\cdots,a_r; \\ \\ b_1,\cdots,b_n,b_{n+1},\cdots,b_s; \end{array} z \right] = \sum_{j_1=0}^{\infty}\cdots\sum_{j_n=0}^{\infty} $$

$$ \cdot \left(\begin{array}{c} \varsigma_1 \\ j_1 \end{array} \right) \cdots \left(\begin{array}{c} \varsigma_n \\ j_n \end{array} \right) \frac{z^{J_n}(b_2+\varsigma_2)_{J_1}\cdots(b_n+\varsigma_n)_{J_{n-1}}(a_{n+1})_{J_n}\cdots(a_r)_{J_n}}{(b_1)_{J_1}\cdots(b_n)_{J_n}(b_{n+1})_{J_n}\cdots(b_s)_{J_n}} $$

$$ {}_{r+1-n}\gamma_{s-n} \left[\begin{array}{c} (a_0+J_n,x), a_{n+1}+J_n,\cdots,a_r+J_n; \\ \\ b_{n+1}+J_n,\cdots,b_s+J_n; \end{array} z \right], \quad (20) $$

provided that $r \leq s$ $(|z|<1,$ *if* $r=s+1)$*, and* n *is a positive integer such as* $n \leq$ *min*(r,s)*, where* J_n *is provided by* (3)*, and* $\Re(b_i+\varsigma_i)>0$ *for* $i=1,\cdots,n$*.*

Proof. We can easily prove Theorem 2 in a similar line of proof of Theorem 1. □

Remark 1: The results (17) and (20) immediately reduces to (10) and (11) respectively if the specific parameters $\varsigma_i (i=1,\cdots,n)$ are substituted by the positive integers $m_i (i=1,\cdots,n)$. It must be expressed that perhaps (3) is exactly a certain result obtained in advance by Srivastava ([8], p. 123, eqn. 16) by taking after a distinctive line of approach utilizing basically the Chu?Vandermonde summation formula ([7], p. 243, Entry (II.4)).

3 A further generalized reduction formulas

We have established another representation of our main results by using of generalized Leibniz rule.

Theorem 3. *Let* $r \leq s$*,* $r=s+1$*,* $|z|<1$*,* $\omega_i (i=1,\cdots,n)$ *are arbitrary complex numbers, and* $0<\zeta_1\leq 1,\ (i=1,\cdots,n)$*, then the following result holds:*

$$ {}_{r+1}\Gamma_s \left[\begin{array}{c} (a_0,x), b_1+\varsigma_1,\cdots,b_n+\varsigma_n, a_{n+1},\cdots,a_r; \\ \\ b_1,\cdots,b_n,b_{n+1},\cdots,b_s; \end{array} z \right] $$

$$ = \prod_{i=1}^{n} \left\{ \sum_{j_i=-\infty}^{\infty} \zeta_i \left(\begin{array}{c} \varsigma_i \\ \zeta_i\ j_i+\omega_i \end{array} \right) \frac{1}{(b_i)_{\zeta_i\ j_i+\omega_i}\Gamma(1-\zeta_i\ j_i-\omega_i)} \right\} $$

$$ \cdot {}_{r+1}\Gamma_s \left[\begin{array}{c} (a_0,x),1,\cdots,1,a_{n+1},\cdots,a_r; \\ \\ 1-\zeta_1\ j_1-\omega_1,\cdots,1-\zeta_n\ j_n-\omega_n, b_{n+1},\cdots,b_s; \end{array} z \right]. \quad (21) $$

Proof. In order to prove the theorem, we consider Osler's generalized Leibniz rule [4]:

$$D_z^{\tau}\left\{u(z)v(z)\right\} = \sum_{n=-\infty}^{\infty} \zeta \left(\begin{array}{c} \tau \\ \zeta n+\omega \end{array} \right) D_z^{\tau-\zeta n-\omega}\left\{u(z)\right\} D_z^{\zeta n+\omega}\left\{v(z)\right\}, \qquad (22)$$

where $0 < \zeta \leq 1$, τ and ω are arbitrary (real and complex) numbers. At this point of view of established specific case of a fractional derivative formula due to Raina and Koul [6] (see moreover [3]), it is possible to achieve the following result effectively:

$$ {}_{r+1}\Gamma_s \left[\begin{array}{cc} (a_0,x), b_1+\varsigma_1, a_2, \cdots, a_r; & \\ & z \\ b_1,\cdots,b_s; & \end{array} \right] = \sum_{j_1=-\infty}^{\infty} \zeta_1 \left(\begin{array}{c} \varsigma_1 \\ \zeta_1\ j_1+\omega_1 \end{array} \right)$$

$$\cdot \frac{1}{(b_1)_{\zeta_1\ j_1+\omega_1}\Gamma(1-\zeta_1\ j_1-\omega_1)} {}_{r+1}\Gamma_s \left[\begin{array}{cc} (a_0,x), 1, a_2,\cdots,a_r; & \\ & z \\ 1-\zeta_1\ j_1-\omega_1, b_2,\cdots,b_s; & \end{array} \right] \qquad (23)$$

provided that $r \leq s (r = s+1$, if $|z| < 1)$, $0 < \zeta_1 \leq 1$, ω_1 is any arbitrary complex number, and none of the denometer parameters is a negative integer or zero.

When $\zeta_1 = 1$ and $\omega_1 = 0$, (23) does not clearly appear to be reducible to (19). However, a slight rearrangements on the right side of the resulting equation, or else, utilizing the basic identity

$$\frac{1}{\Gamma(1-j)} {}_{r+1}\Gamma_s \left[\begin{array}{cc} (a_0,x), 1, a_2,\cdots,a_r; & \\ & z \\ 1-j, b_2,\cdots,b_s; & \end{array} \right] = \frac{(a_0)_j (a_2)_j \cdots (a_r)_j}{(b_2)_j \cdots (b_s)_j} z^j$$

$$ {}_{r}\Gamma_{s-1} \left[\begin{array}{cc} (a_0+j,x), a_2+j,\cdots,a_r+j; & \\ & z \\ b_2+j,\cdots,b_s+j; & \end{array} \right] \qquad (24)$$

makes (23) (when $\zeta_1 = 1$, $\omega_1 = 0$), reducible to (19). The repeated application of (23) for each $a_i = b_i + \varsigma_i$ $(i = 2,\cdots,n)$ yields the result (21). □

Theorem 4. *Let* $r \leq s$, $r = s+1$, $|z| < 1$, $\omega_i (i = 1,\cdots,n)$ *are arbitrary complex numbers, and* $0 < \zeta_1 \leq 1$, $(i = 1,\cdots,n)$, *then the following result holds:*

$$ {}_{r+1}\gamma_s \left[\begin{array}{cc} (a_0,x), b_1+\varsigma_1,\cdots,b_n+\varsigma_n, a_{n+1},\cdots,a_r; & \\ & z \\ b_1,\cdots,b_n,b_{n+1},\cdots,b_s; & \end{array} \right]$$

$$= \prod_{i=1}^{n} \left\{ \sum_{j_i=-\infty}^{\infty} \zeta_i \left(\begin{array}{c} \varsigma_i \\ \zeta_i\ j_i+\omega_i \end{array} \right) \frac{1}{(b_i)_{\zeta_i\ j_i+\omega_i}\Gamma(1-\zeta_i\ j_i-\omega_i)} \right\}$$

$$\cdot {}_{r+1}\gamma_s \left[\begin{matrix} (a_0, x), 1, \cdots, 1, a_{n+1}, \cdots, a_r; \\ \\ 1-\zeta_1\ j_1-\omega_1, \cdots, 1-\zeta_n\ j_n-\omega_n, b_{n+1}, \cdots, b_s; \end{matrix} \quad z \right]. \tag{25}$$

Proof. In similar manner of proof of Theorem 3, hence it is given without proof. □

4 Special cases and concluding remark

In this section, we consider certain corresponding results for theorems as consequences of our key findings by choosing the parameters appropriately. For this reason, if we set $x = 0$ and replace r by $r-1$, Theorem 1 yields to the noted reduction formulas for generalized hypergeometric functions, given earlier by Raina [5]. Moreover, the result (17) immediately reduces to (1) (see, [1]), if we set $x = 0$, replace r with $r-1$ and the specific parameters $\varsigma_i (i = 1, \cdots, n)$ are substituted by the positive integers $m_i (i = 1, \cdots, n)$.

In our present study, with that of the aid of the incomplete functions theory, we have investigated certain transformation formulas that expresses a potentially useful incomplete hypergeometric function in various forms of a countable sum of lesser-order functions. The incomplete hypergeometric functions generalize hypergeometric functions and many other special functions emerging in the sciences of mathematics, physics and engineering. Having regard to this assumption, several additional corollaries and outcomes of the general findings stated by Theorems 1 to 4 can be deduced in an equivalent way.

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