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ON FORWARD ITERATED HAUSDORFFNESS AND DEVELOPMENT OF EMBRYO FROM ZYGOTE IN BITOPOLOGICAL DYNAMICAL SYSTEMS

Santanu ACHARJEE[*,1], Kabindra GOSWAMI[2] and Hemanta Kumar SARMAH[3]

Abstract

Topological dynamical system is an area of dynamical system to investigate dynamical properties in terms of a topological space. Nada and Zohny [Nada, S.I. and Zohny, H., *An application of relative topology in biology*, Chaos, Solitons and Fractals. **42** (2009), 202–204] applied topological dynamical system to explore the development process of an embryo from the zygote until birth and made three conjectures. In this paper, we disprove conjecture 3 of Nada and Zohny [Nada, S.I. and Zohny, H., *An application of relative topology in biology*, Chaos, Solitons and Fractals. **42** (2009), 202–204] by applying some of our mathematical results of bitopological dynamical system. Also, we introduce forward iterated Hausdorff space, backward iterated Hausdorff space, pairwise iterated Hausdorff space and establish relations between them in bitopological dynamical system. We formulate the function that represents cell division (specially, mitosis) and using this function we show that in the development process of a human baby from the zygote until its birth, there is a stage where the developing stage is forward iterated Hausdorff.

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1 Introduction

Topological dynamical system is a branch of dynamical system which has been extensively studied by many researchers. Using results of topological dynamical

[1*] *Corresponding author*, Department of Mathematics, *Gauhati University*, Assam, e-mail: sacharjee326@gmail.com, sacharjee326@gauhati.ac.in
[2]Department of Mathematics, *Gauhati University*, Assam, e-mail: kabindragoswami@gmail.com
[3]Department of Mathematics, *Gauhati University*, Assam, e-mail: hsarmah@gauhati.ac.in

system; Nada and Zohny [16] explored the development process of an embryo from the zygote until birth and they made three conjectures related to it. But, there are limitations of topological dynamical system because of the involvement of only one topology in the entire mathematical process. Bitopological dynamical system is a new branch of dynamical system which has recently been introduced by Acharjee et al. [3] to overcome the drawback of topological dynamical system. Moreover, Acharjee et al. [3] disproved conjecture 1. of Nada and Zohny [16] by using some results of bitopological dynamical system and introduced some new ideas like transitive map, pairwise iterated compactness, etc. Thus, it motivates us to develop new results along with applications.

Kelly [14] introduced the concept of bitopological space. Later, bitopological space was extensively studied by many researchers of various branches. Pervin [18] introduced the concept of pairwise continuity in a bitopological space. Kelly [14] introduced the concept of pairwise Hausdorffness in a bitopological space. After that many researchers ([21], [13]) studied the concept of pairwise Hausdorffness because of its fundamental importance in the theory of bitopological space. For recent theoretical works in bitopological space, one may refer to Acharjee and Tripathy [6], Acharjee et al. [4], Acharjee et al. [5] and many others. Recently, bitopological space has been applied to many areas of science and social science, viz. medical science [23], economics ([7], [10]), computer science [9], etc. Recently, Acharjee et al. [2] disproved conjecture 2 of Nada and Zohny [16] and in another paper [1], they introduced entropy in bitopological dynamical system.

Mitosis is a basic process of cell division in which one cell divides producing two daughter cells. Recently, many researchers ([15], [11], [12]) have studied the mitosis process and cell division from various perspectives. This paper is divided into three sections. In the preliminary section, we recall some existing definitions of bitopological space as well as bitopological dynamical system. In the next section, we introduce forward iterated Hausdorff space, backward iterated Hausdorff space and pairwise iterated Hausdorff space; which generalize the notion of pairwise Hausdorffness in a bitopological dynamical system. Also, we establish some relationships among these generalizations. Then, we show that during the development process of a human embryo from zygote until birth, the developing stage after gastrulation is forward iterated Hausdorff. Finally, we disprove conjecture 3 of Nada and Zohny [16] by introducing three limits in the development process of a human embryo from the zygote till birth. One may refer to [22] for explanations on embryology.

2 Preliminary definitions

In this section, we recall some existing definitions of bitopological space and bitopological dynamical system.

Definition 1. *[14] A quasi-pseudo-metric on a set X is a non-negative real-valued function $p(,)$ on the product $X \times X$ such that:*

(i) $p(x, x) = 0$, *where* $x \in X$,
(ii) $p(x, z) \leq p(x, y) + p(y, z)$, *where* $x, y, z \in X$.

Definition 2. *[14] Let* $p(,)$ *be a quasi-pseudo-metric on* X *and let* $q(,)$ *be defined by* $q(x, y) = p(y, x)$, *where* $x, y \in X$. *Then,* $q(,)$ *is also a quasi-pseudo-metric on* X. *We say that* $p(,)$ *and* $q(,)$ *are conjugate, and denote the set* X *with the structure by* (X, p, q).
If $p(,)$ *is a quasi-pseudo-metric on a set* X, *then the open* p*-sphere with centre* x *and radius* $\epsilon > 0$ *is the set* $S_p(x, \epsilon) = \{y : p(x, y) < \epsilon\}$. *The collection of all open* p*-spheres forms a base for a topology. Similarly,* $q(,)$ *determines a topology for* X. *We shall denote the topology determined by* $p(,)$ *by* τ_1 *and the topology that of* $q(,)$ *by* τ_2.

Definition 3. *[14] A space* X *on which are defined two (arbitrary) topologies* τ_1 *and* τ_2 *is called a bitopological space and denoted by* (X, τ_1, τ_2).

Definition 4. *[19] A function* f *from a bitopological space* (X, τ_1, τ_2) *into a bitopological space* (Y, ψ_1, ψ_2) *is said to be pairwise continuous (respectively, a pairwise homeomorphism) iff the induced functions* $f : (X, \tau_1) \to (Y, \psi_1)$ *and* $f : (X, \tau_2) \to (Y, \psi_2)$ *are continuous (respectively, homeomorphisms).*

Pervin [18] called this map a continuous map. However, we call this map as pairwise continuous map, due to Reilly [19].

Definition 5. *[14] A bitopological space* (X, τ_1, τ_2) *is said to be pairwise Hausdorff if for each two distinct points* x *and* y, *there are a* τ_1*-neighbourhood* U *of* x *and a* τ_2*-neighbourhood* V *of* y *such that* $U \cap V = \phi$.

Definition 6. *[8] A set* $A \subset X$ *is* $+$*invariant when* $f(A) \subset A$ *and* A *is* $-$*invariant when* $A \subset f(A)$. A *is called invariant when* $f(A) = A$.

We call a map $f : X \to X$ *as* $+$*invariant if for all* $A \subset X$, $f(A) \subset A$ *and* $-$*invariant when* $A \subset f(A)$. *The map* f *is invariant when* $f(A) = A$, *for all* $A \subset X$.

Definition 7. *[3] Let* (X, τ_1, τ_2) *be a bitopological space. A bitopological dynamical system is a pair* (X, f), *where* (X, τ_1, τ_2) *is a bitopological space and* $f : X \to X$ *is a pairwise continuous map. The dynamics is obtained by iterating the map.*
The forward orbit of a point $x \in X$ *under* f *is defined as* $O_+(x) = \{f^n(x) : n \in \mathbb{N}\}$, *where* f^n *denotes the* n^{th} *iteration of the map* f. *If* f *is a homeomorphism, then the backward orbit of* x *is the set* $O_-(x) = \{f^{-n}(x) : n \in \mathbb{N}\}$ *and the full orbit of* x *(or simply orbit of* x*) is the set* $O(x) = \{f^n(x) : n \in \mathbb{Z}\}$.

Here, homeomorphism of f *indicates homeomorphism of the function* $f : (X, \tau_i) \to (X, \tau_i)$ *separately for all* $i \in \{1, 2\}$ *as it is clear in terms of bitopological space. Thus, we can consider pairwise homeomorphism equivalently.*

3 Main results

In this section, our main aim is to introduce pairwise iterated Hausdorffness and some associated results. Moreover, we show that during the development of a human embryo from the zygote until birth, the developing stage after gastrulation is forward iterated Hausdorff. Let $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$ denote the set of non-negative integers, the set of integers and the set of real numbers respectively.

Pairwise Hausdorffness is a basic concept in bitopological space. So, we define some generalized versions of pairwise Hausdorffness in bitopological dynamical system with respect to iteration.

Definition 8. *Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \to X$ is a pairwise continuous map. We call (X, f) as $(m, n)-$forward iterated Hausdorff if for any two distinct points x and y of X, there exist $m, n \in \mathbb{N}$, $U \in \tau_1$ and $V \in \tau_2$, such that $f^m(x) \in U$, $f^n(y) \in V$ and $U \cap V = \phi$.*

Definition 9. *Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \to X$ is a pairwise continuous map. We call (X, f) as $(m, n)-$backward iterated Hausdorff if for any two distinct points x and y of X, there exist $m, n \in \mathbb{N}$, $U \in \tau_1$ and $V \in \tau_2$, such that $f^{-m}(x) \in U$, $f^{-n}(y) \in V$ and $U \cap V = \phi$.*

Definition 10. *Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \to X$ is a pairwise continuous map. We call (X, f) as $(m, n)-$pairwise iterated Hausdorff if for any two distinct points x and y of X, there exist $m, n \in \mathbb{Z}$, $U \in \tau_1$ and $V \in \tau_2$, such that $f^m(x) \in U$, $f^n(y) \in V$ and $U \cap V = \phi$*

.

The following theorem establishes the relation between pairwise Hausdorff, forward iterated Hausdorff, backward iterated Hausdorff and pairwise iterated Hausdorff without considering any extra condition.

Theorem 1. *Let (X, f) be a bitopological dynamical system, where (X, τ_1, τ_2) is a bitopological space and $f : X \to X$ is a pairwise continuous map. Then, the following conditions are equivalent:*
(i) (X, τ_1, τ_2) is pairwise Hausdorff,
(ii) (X, f) is $(0, 0)$-forward iterated Hausdorff,
(iii) (X, f) is $(0, 0)$-backward iterated Hausdorff,
(iv) (X, f) is $(0, 0)$-pairwise iterated Hausdorff.

It is to be noted that (m, n)-forward iterated Hausdorffness of the bitopological dynamical system (X, f) may not imply pairwise Hausdorffness of the bitopological space (X, τ_1, τ_2) for $m \neq 0$ or $n \neq 0$. It can be seen from the following example.

We procure some parts of the following example from Acharjee et al. [3].

Example 1. Let us consider the bitopological space $(\mathbb{R}, \tau_1, \tau_2)$, where τ_1 is the left hand topology and τ_2 is the right hand topology. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x - 1$. Now, for any open sets $U = (-\infty, a) \in \tau_1$ and

$V = (b, \infty) \in \tau_2$, where $a, b \in \mathbb{R}$, we have $f^{-1}(U) = (-\infty, a+1)$, which is τ_1-open and $f^{-1}(V) = (b+1, \infty)$, which is τ_2-open. Thus, f is a pairwise continuous map. Here, $(\mathbb{R}, \tau_1, \tau_2)$ is not pairwise Hausdorff [20] as if $x, y \in \mathbb{R}$ and $x > y$ then every τ_1-open set containing x contains y, and every τ_2-open set containing y also contains x. But, $(\mathbb{R}, f)$ is (m, n)-forward iterated Hausdorff; since for $x, y \in \mathbb{R}$ and $x > y$, there are $m = \lceil x - y \rceil + 3, n = 0$, $U = (-\infty, y-2) \in \tau_1$ and $V = (y-1, \infty) \in \tau_2$ such that $f^m(x) \in U$, $f^n(y) \in V$ and $U \cap V = \phi$. Also, if $x < y$, then there are $m = 1, n = 0$, $U = (-\infty, x) \in \tau_1$ and $V = (x, \infty) \in \tau_2$ such that $f^m(x) \in U$, $f^n(y) \in V$ and $U \cap V = \phi$. Here, $\lceil\ \rceil$ is the ceiling function.

Now, we get the following result for a $-$invariant map.

Theorem 2. *Let (X, f) be a bitopological dynamical system, where f is a $-$invariant map. If (X, f) is (m, n)-forward iterated Hausdorff, where either $m \neq 0$ or $n \neq 0$, then (X, τ_1, τ_2) is pairwise Hausdorff.*

Proof. Since (X, f) is (m, n)-forward iterated Hausdorff, so for any two distinct points x and y of X, there exist $m, n \in \mathbb{N}$, $U \in \tau_1$ and $V \in \tau_2$, where either $m \neq 0$ or $n \neq 0$ such that $f^m(x) \in U$, $f^n(y) \in V$ and $U \cap V = \phi$. Now, $f^m(x) \in U \implies x \in f^{-m}(U) \subset U$ since f is $-$invariant. Similarly, we get $y \in V$. Thus, for any two distinct points x and y of X, there exist $U \in \tau_1$ and $V \in \tau_2$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Hence, (X, τ_1, τ_2) is pairwise Hausdorff. □

Now, we get the following result for a $+$invariant map.

Theorem 3. *Let (X, f) be a bitopological dynamical system, where f is a $+$invariant map. If (X, τ_1, τ_2) is pairwise Hausdorff and f is a pairwise open map, then (X, f) is (m, n)-forward iterated Hausdorff.*

Proof. Since (X, τ_1, τ_2) is pairwise Hausdorff, so for any two distinct points x and y of X, then there exist $U \in \tau_1$ and $V \in \tau_2$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Now, for $m, n \in \mathbb{N}$, $f^m(x) \in f^m(U) \subset U$ and $f^n(y) \in f^n(V) \subset V$ as f is $+$invariant. Thus, there exist $m, n \in \mathbb{N}$ such that $f^m(x) \in U$, $f^n(y) \in V$ and $U \cap V = \phi$. Thus, (X, f) is (m, n)-forward iterated Hausdorff. □

Corollary 1. *Let (X, f) be a bitopological dynamical system, where f is a $+$invariant map. If (X, τ_1, τ_2) is pairwise Hausdorff and f is a pairwise open map, then the set $\{(m, n) : m, n \in \mathbb{N}$ and (X, f) is (m, n)-forward iterated Hausdorff$\}$ is infinite.*

Now, we recall some biological terms and use some of the above results in the growth process of an embryo from the zygote to till its birth.

Definition 11. *[22] Mitosis is the process whereby one cell divides giving rise to two daughter cells each with 46 chromosomes.*

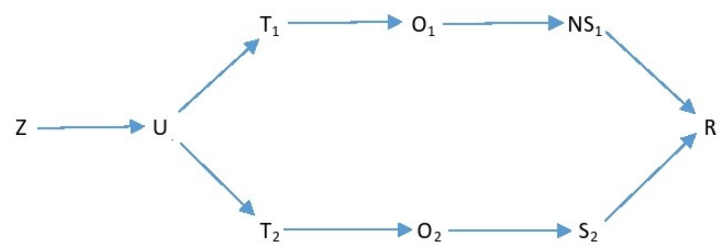

Figure 1: Here, Z- the zygote, U- development of the zygote just before gastrulation, T_1- Neural tissues, T_2- Non-neural tissues, O_1- Neural organs, O_2- Non-neural organs, NS_1- Neural organ systems, S_2- Non-neural organ systems and R- the baby at the time of birth.

Definition 12. *[22] Gastrulation is the process of forming the three primary germ layers from the epiblast involving movement of cells through the primitive streak to form endoderm and mesoderm.*

According to [17], fertilization is a complex multi-step process which completes in 24 hours. The sperm of a male meets an ovum of a female and then forms a zygote. Once fertilization takes place, there are quick changes at the cellular level of the zygote. The zygote is a single cell, and it undergoes mitosis to create many cells. According to [11], cell division is a continuous process. Now, we define a function that represents cell division (mitosis).

In the process of mitosis, one cell divides giving rise to two daughter cells. Also, initially there is only one cell, the zygote, from which the whole organism (the baby) develops. So, initially at the 0^{th} step; there is only one cell present, the zygote. Let x_i^j be the j^{th} cell of i^{th} step. For eg. x_0^1 represents the zygote. Then, the zygote undergoes first mitosis to produce two daughter cells namely x_1^{11} and x_1^{12}. Then, these two daughter cells again produce four daughter cells, namely x_2^{111}, x_2^{112}, x_2^{121} and x_2^{122}. The process continues to form many cells. Let, R be the whole organism (the baby) at the time of birth.
We define $h : R \to R$ by $h(x_i^j) = \{x_i^{j1}, x_i^{j2}\}$, where x_i^j is the mother cell, and x_i^{j1}, x_i^{j2} are two daughter cells.

Now, we consider the bitopological space (R, τ_1, τ_2) as defined by Acharjee et al. [3], where
$\tau_1 = \{(\phi, \tau_1(t_0)), (U_1, \tau_1(t_1)), (U_2, \tau_1(t_2)), ..., (U_m, \tau_1(t_m)), (R, \tau_1(T))\}$ and
$\tau_2 = \{(\phi, \tau_2(t_0)), (V_1, \tau_2(t_1)), (V_2, \tau_2(t_2)), ..., (V_n, \tau_2(t_n)), (R, \tau_2(T))\}$,
where $\phi = Z = U_0 = V_0$, since initially there is only the zygote from which the whole organism develops. $U_n = X$ is the brain together with the central nervous system of the whole organism and $V_n = Y$ is the other body parts of the whole organism except the brain and the central nervous system. Also, U_1, U_2,... represent different development stages of the brain and the central nervous system; while V_1, V_2,... represent different development stages of the other body parts except the brain and the central nervous system. Here, t_0 is the time of fertilization and T is the time of birth without indicating the stages of the

growth. In notional sense; $(U_i, \tau_j(t_i))$ indicates that to reach the stage U_i at the time of growth under topology τ_j; the required time is $\tau_j(t_i)$ where $j \in \{1, 2\}$. It is important to note that before gastrulation $U_i = V_i$. Here, X and Y together forms the whole organism, the baby say R, i.e. $X \cup Y = R$. Figure 1. represents it which is due to Acharjee et al. [2].

Now, inverse image of any τ_1-open set U which is the collection of brain and central nervous system cells under the map h is the set of mother cells, namely M, from where the cells of the set U are developed. This set of mother cells again belongs to τ_1 as M is the developmental stage of the brain and the central nervous system before U. Similar argument follows for any τ_2-open set V. Thus, $h : (R, \tau_1, \tau_2) \to (R, \tau_1, \tau_2)$ is a pairwise continuous map. Hence, (R, h) forms a bitopological dynamical system.

Now, since after the gastrulation, the development process of the brain and the central nervous system and development process of other body parts without brain and the central nervous system separates from each other, so we consider the postgastrulation part of the whole organism as R^*.

Let $x, y \in R^*$ be any two cells after gastrulation such that x is a brain or central nervous system cell and y is a cell of other body parts except the brain and the central nervous system. Then, obviously, x is a part of the developmental stages of the brain and the central nervous system; while y is a part of the developmental stages of other body parts without the brain and the central nervous system. So, $x \in U$ and $y \in V$, where $U \in \tau_1^*$ and $V \in \tau_2^*$. Here, τ_1^* and τ_2^* are relative topologies of R^*. Also, $U \cap V = \phi$ since U consists of cells of the brain or the central nervous system and V consists of the cells of other body parts except the brain and the central nervous system. Hence, for $x \neq y \in R^*$, there exist $U \in \tau_1^*$ and $V \in \tau_2^*$ such that $x \in U$, $y \in V$ and $U \cap V = \phi$. Thus, the subspace $(R^*, \tau_1^*, \tau_2^*)$ of (R, τ_1, τ_2) is pairwise Hausdorff in the sense that two brain and central nervous system cells are not distinguished from each other (and also two cells of other body parts except the brain and the central nervous system are not distinguished from each other).

Again, let $U \in \tau_1^*$. Then, U consists of the brain and the central nervous system cells. These cells undergo mitosis to produce their daughter cells, which are again cells of the brain and the central nervous system. The collection of these daughter cells, say U_1, is nothing but the developmental stages of the brain and the central nervous system after U. Thus, $h^*(U) = U_1$, where h^* is the restriction of the map h on R^* i.e. $h^* = h|_{R^*}$. So, the mapping $h^* : (R^*, \tau_1^*) \to (R^*, \tau_1^*)$ is an open map. Similarly, the mapping $h^* : (R^*, \tau_2^*) \to (R^*, \tau_2^*)$ is also an open map. Thus, the mapping $h^* : (R^*, \tau_1^*, \tau_2^*) \to (R^*, \tau_1^*, \tau_2^*)$ is a pairwise open map.

Now, let $x, y \in R^*$ be any two cells after gastrulation such that x is a brain cell or central nervous system cell and y is a cell of other body parts except the brain and the central nervous system. Then, $x \in U$ and $y \in V$, for some $U \in \tau_1^*$ and $V \in \tau_2^*$. It gives $h^*(x) \in h^*(U)$ and $h^*(y) \in h^*(V)$. Since h^* is an open map, so $h^*(U) = U_1 \in \tau_1^*$ and $h^*(V) = V_1 \in \tau_2^*$. Also, $U_1 \cap V_1 = \phi$ because U_1 consists of cells of the brain or the central nervous system and V_1 consists of the cells of other body parts except the brain and the central nervous system.

Thus, for $x \neq y \in R^*$, there exist $m = 1$, $n = 1$, $U_1 \in \tau_1^*$ and $V_1 \in \tau_2^*$ such that $(h^*)^m(x) \in U_1$, $(h^*)^n(y) \in V_1$ and $U_1 \cap V_1 = \phi$. Thus, the bitopological dynamical system (R^*, h^*) is forward iterated Hausdorff in the sense that two brain and central nervous system cells are not distinguished from each other (and also two cells of other body parts except the brain and the central nervous system are not distinguished from each other).

We have found that the growth process of an organism from the zygote, after gastrulation, is both pairwise Hausdorff and forward iterated Hausdorff. Hence, we are predicting theoretically that after the gastrulation; no structural relationship can be found between the growth of the brain together with the central nervous system and the other body parts except the brain and the central nervous system. But, there may be some functional relationships between these two growth processes. In this case, we are expecting evidences based on experiments to find exact relationships between growth of the brain together with the central nervous system and growth of the other body parts except the brain and the central nervous system.

4 On conjecture 3 of Nada and Zohny

In this section, we disprove conjecture 3 of Nada and Zohny [16] by using our results [3] of bitopological dynamical system.

According to [17], when fertilization takes place, a zygote is formed. The zygote is a single cell, and it undergoes mitosis to create many cells. According to [11], cell division is a continuous process. So, the zygote differentiates to increase its volume as time goes on. Thus, only growth happens in the development of a baby from the zygote [22]. So, thinking of a decreasing chain $H_1 \supset H_2 \supset ... \supset H_\infty$ with $\xi_1 \supset \xi_2 \supset ... \supset \xi_i \longrightarrow \phi$ as of Nada and Zohny [16] practically gives us nothing.

Recently, Acharjee et al. [3] showed that the growth process of a human baby from the zygote till its birth is a bitopological dynamical system. According to [3], during the development process of a human baby from the zygote; the brain together with the central nervous system and development process of the other body parts except the brain with the central nervous system occurs parallelly. Based on the theories of [3], we show that there exist three limits in the growth of a human baby from the zygote till its birth which disprove conjecture 3 of Nada and Zohny [16].

We can rewrite the two topologies on R, which is due to Acharjee et al. [3], as given below.
$\tau_1 = \{(\phi, \tau_1(t_0)), (W_1, \tau_1(t_1^{/})), (W_2, \tau_1(t_2^{/})), ..., (W_i, \tau_1(t_i^{/})), (U_1, \tau_1(t_1)), ...,$
$(U_m, \tau_1(t_m)), (R, \tau_1(T))\}$ and
$\tau_2 = \{(\phi, \tau_2(t_0)), (W_1, \tau_2(t_1^{/})), (W_2, \tau_2(t_2^{/})), ..., (W_i, \tau_2(t_i^{/})), (V_1, \tau_2(t_1)), ...,$
$(V_n, \tau_2(t_n)), (R, \tau_2(T))\}$.
Here, $\phi = Z = W_0$, $U_m = X$ is the brain together with the central nervous system of the whole organism and $V_n = Y$ is the other body parts of the whole organism

except the brain and the central nervous system. Also, U_1, U_2,... represent different development stages of the brain and the central nervous system; while V_1, V_2,... represent different development stages of the other body parts except the brain and the central nervous system. Here, t_0 is the time of fertilization, T is the time of birth and $t_i^/ = t$ is the time of gastrulation without indicating the stages of the growth. In notional sense; $(U_i, \tau_j(t_i))$ indicates that to reach the stage U_i at the time of growth under topology τ_j; the required time is $\tau_j(t_i)$ where $j \in \{1, 2\}$. Similarly, $(W_i, \tau_j(t_i^/))$ indicates that to reach the stage W_i at the time of growth under topology τ_j; the required time is $\tau_j(t_i^/)$ where $j \in \{1, 2\}$. Also, W_1, W_2,... represent different development stages of the organism before gastrulation and $W_i = W$ is the state of the embryo just before gastrulation. It is important to note that after gastrulation, the development process of the brain together with the central nervous system and development process of the other body parts except the brain and the central nervous system separates from each other. Later, X and Y together form the whole organism R i.e. $X \cup Y = R$.

Here, $Z = W_0 \subset W_1 \subset W_2 \subset ... \subset W_i \subset U_1 \subset U_2 \subset ... \subset U_m \subset R$ and $Z = W_0 \subset W_1 \subset W_2 \subset ... \subset W_i \subset V_1 \subset V_2 \subset ... \subset V_n \subset R$. Also, $\tau_1(t_0) < \tau_1(t_1^/) < \tau_1(t_2^/) < ... < \tau_1(t_m) < \tau_1(T)$ and $\tau_2(t_0) < \tau_2(t_1^/) < \tau_2(t_2^/) < ... < \tau_2(t_n) < \tau_2(T)$.

Now, we explore two limits after gastrulation, as $\lim\limits_{k\to\infty} U_k$ and $\lim\limits_{k\to\infty} V_k$. Here, first one determines the brain and the central nervous system X at the time of birth; while the second one determines the other body parts except the brain and the central nervous system Y at the time of birth i.e. $\lim\limits_{k\to\infty} U_k = X$ and $\lim\limits_{k\to\infty} V_k = Y$. Before the gastrulation, we can explore another limit; $\lim\limits_{k\to\infty} W_k = W$; which determines the state of developing embryo just before gastrulation.

Hence, we disprove conjecture 3 of Nada and Zohny [16] with the help of our theories. In future, it is our prior duty to study these three limits and their possible relationships for several practical purposes.

5 Conclusion

In this paper, we introduced concepts of forward iterated Hausdorff, pairwise iterated Hausdorff, etc. in a bitopological dynamical system and we established some relationships between them. Also, we showed that the development process of a human baby from the zygote till its birth can be represented by a bitopological dynamical system after defining a function that represents the mitosis process. Finally, we found that after gastrulation, the growth process of a human baby from the zygote till its birth is forward iterated Hausdorff. Later, we disproved conjecture 3 of Nada and Zohny [16] by exploring three limits with suitable mathematical and medical evidences. We firmly believe that if the relationships between these three limits can be found in future from experiments, then the medical treatments can be given to stop any congenital malformation just by studying the brain development or the body development in prenatal stage. To make this a reality, we need a deep research in human embryogenesis through the process of mitosis and bitopological dynamical system. Moreover, we predicted theoretically that after the gastrulation; no structural relationship can be found between the growth process of the brain together with the central nervous system and the growth process of the other body parts except the brain and the central nervous system. But, it will need experimental evidences with keen observations in every femtosecond regarding the birth of the baby from the zygote to till its birth.

Conflict of interest: The authors declare that there is no conflict of interest.

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