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A NOTE ON ALMOST CONTACT METRIC HYPERSURFACES OF NEARLY KÄHLERIAN 6-SPHERE

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Abstract

It is proved that the almost contact metric structure induced on a totally geodesic hypersurface of the nearly Kahlerian six-dimensional sphere S^6 is necessarily nearly cosymplectic.

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Key words: almost contact metric structure, nearly cosymplectic structure, nearly Kählerian manifold, almost Hermitian manifold, six-dimensional sphere, totally geodesic hypersurface, type number.

1 Introduction

The existence of 3-vector cross products on Cayley algebra gives a set of important examples of almost Hermitian manifolds [14]. As it is well known, every 3-vector cross product on Cayley algebra induces an almost Hermitian structure on its six-dimensional oriented submanifold [5], [14], [23], [24]. Such almost Hermitian structures (in particular, nearly Kählerian structures) were studied by a number of authors. For example, a complete classification of nearly Kählerian [23] and Kahlerian [24] structures on six-dimensional submanifolds of the octave algebra has been obtained by V.F. Kirichenko. M. Banaru has studied Hermitian structures (i.e. integrable almost Hermitian structures) on six-dimensional submanifolds of the octonions algebra [5], [6], [7]. We also note that the six-dimensional sphere S^6 with a canonical nearly Kählerian structure was also considered by such remarkable geometers as N. Ejiri [10], A. Gray [14], [15], [16], Haizhong Li and Guoxin Wei [18], [19], H. Hashimoto [20], [21], [22], K. Sekigawa [29], L. Vranchen [30] and others.

It is known that almost contact metric structures are induced on oriented hypersurfaces of almost Hermitian manifolds. In the present note, almost contact

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metric structures on totally geodesic hypersurfaces of nearly Kählerian six-sphere are considered.

In [4], it has been proved that the type number of a nearly cosymplectic hypersurface in a nearly Kählerian manifold is at most one. In the recent article [3], it has been proved that if the type number of an oriented hypersurface of the nearly Kählerian six-dimensional sphere S^6 is equal to 1, then the induced almost contact metric structure on this hypersurface is necessarily nearly cosymplectic. In this paper, some additional results on almost contact metric hypersurfaces of nearly Kählerian six-sphere are given. Namely, we shall consider almost contact metric structure on 0-type hypersurfaces (i.e. totally geodesic hypersurfaces) of the nearly Kählerian six-dimensional sphere. We shall show that this almost contact metric structure is also nearly cosymplectic. The main result is the following.

Theorem 1. *The almost contact metric structure induced on an oriented totally geodesic hypersurface of the nearly Kählerian six-dimensional sphere S^6 is necessarily nearly cosymplectic.*

Taking into account the above mentioned results from [3] and [4], we completely resolve the problem of small type number hypersurfaces of nearly Kählerian six-sphere.

2 Preliminaries

Let us consider an almost Hermitian manifold, i.e. a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n}. All considered manifolds, tensor fields and similar objects are assumed to be of the class C^∞.

The specification of an almost Hermitian structure on a manifold is equivalent to the setting of a G-structure, where G is the unitary group $U(n)$ [6], [25]. Its elements are the frames adapted to the structure (A-frames). They look as follows:

$$(p, \varepsilon_1, \ldots, \varepsilon_n,\ \varepsilon_{\hat{1}}, \ldots, \varepsilon_{\hat{n}}),$$

where ε_a are the eigenvectors corresponding to the eigenvalue $i = \sqrt{-1}$, and $\varepsilon_{\hat{a}}$ are the eigenvectors corresponding to the eigenvalue $-i$. Here the index a ranges from 1 to n, and we state $\hat{a} = a + n$.

Therefore, the matrixes of the operator of the almost complex structure and of the Riemannian metric written in an A-frame look as follows, respectively:

$$\left(J^k_j\right) = \left(\begin{array}{c|c} iI_n & 0 \\ \hline 0 & -iI_n \end{array}\right); \quad (g_{kj}) = \left(\begin{array}{c|c} 0 & I_n \\ \hline I_n & 0 \end{array}\right),$$

where I_n is the identity matrix; $k, j = 1, \ldots, 2n$.

We recall that the fundamental form (or Kählerian form) of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY\rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

By direct computing it is easy to obtain that in A-frame the fundamental form matrix looks as follows:

$$(F_{kj}) = \left(\begin{array}{c|c} 0 & iI_n \\ \hline -iI_n & 0 \end{array}\right).$$

The first group of the Cartan structural equations of an almost Hermitian manifold written in an A-frame looks as follows [1], [25]:

$$\begin{aligned} d\,\omega^a &= \omega^a_b \wedge \omega^b \ + \ B^{ab}{}_c\,\omega^c \wedge \omega_b \ + \ B^{abc}\,\omega_b \wedge \omega_c; \\ d\,\omega_a &= -\omega^b_a \wedge \omega_b \ + \ B_{ab}{}^c\,\omega_c \wedge \omega^b \ + \ B_{abc}\,\omega^b \wedge \omega^c, \end{aligned} \tag{1}$$

where

$$\begin{aligned} B^{ab}{}_c \ &= \ -\frac{i}{2}\,J^a_{\hat{b},\,c};\ B_{ab}{}^c \ = \ \frac{i}{2}\,J^{\hat{a}}_{b,\,\hat{c}}; \\ B^{abc} \ &= \ \frac{i}{2}\,J^a_{[\hat{b},\,\hat{c}]};\ B_{abc} \ = \ -\frac{i}{2}\,J^{\hat{a}}_{[b,\,c]}. \end{aligned}$$

The systems of functions $\{B^{ab}_c\}$, $\{B^c_{ab}\}$, $\{B^{abc}\}$, $\{B_{abc}\}$ are the components of the Kirichenko tensors of M^{2n} [2], [8], [25], $a, b, c = 1, \dots, n$, $\hat{a} = a + n$.

An almost Hermitian manifold is called nearly Kählerian (or W_1-manifold in Gray-Hervella notation [17]), if

$$\nabla_X\ (\,F\,)\ (X, Y) = 0,$$

where $X, Y \in \aleph(M^{2n})$.

We recall also that an almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi, \xi, \eta, g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\,\cdot\rangle$ is the Riemannian metric [9], [25], [28]. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta \circ \Phi = 0,\ \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X, \Phi Y\rangle = \langle X, Y\rangle - \eta\,(X)\,\eta\,(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on N. As an example of an almost contact metric structure we can consider the cosymplectic structure that is characterized by the following condition:

$$\nabla\eta = 0, \quad \nabla\Phi = 0,$$

where ∇ is the Levi-Civita connection of the metric. It has been proved that the manifold, admitting the cosymplectic structure, is locally equivalent to the product $M \times R$, where M is a Kählerian manifold [25].

An almost contact metric structure (Φ, ξ, η, g) is called nearly cosymplectic, if the following condition is fulfilled [11], [25]:

$$\nabla_X(\Phi)\,Y + \nabla_Y(\Phi)\,X = 0,\ X, Y \in \aleph(N).$$

We note that the nearly cosymplectic structures have many remarkable properties and play an important role in contact geometry. We mark out a number of articles by H. Endo on the geometry of nearly cosymplectic manifolds (see, for instance, [11], [12], [13]) as well as the recent work by E.V. Kusova on this subject [27].

At the end of this section, note that when we give a Riemannian manifold and its submanifold (in particular, its hypersurface) the rank of determined second fundamental form is called the type number [26].

3 The proof of Theorem 1

Let us use the first group of Cartan structural equations of an almost contact metric structure on an oriented hypersurface N^{2n-1} of an almost Hermitian manifold M^{2n} [4], [7]:

$$\begin{gathered}
d\,\omega^a = \omega^a_b \wedge \omega^b + B^{ab}_c\,\omega^c \wedge \omega_b + B^{abc}\,\omega_b \wedge \omega_c + \\
+ (\sqrt{2}\,B^{an}_b + i\sigma^a_b)\,\omega^b \wedge \omega + (-\sqrt{2}\,\tilde{B}^{nab} - \frac{1}{\sqrt{2}}B^{ab}_n - \frac{1}{\sqrt{2}}\tilde{B}^{abn} + i\,\sigma^{ab})\,\omega_b \wedge \omega; \\
d\,\omega_a = -\omega^b_a \wedge \omega_b + B^c_{ab}\,\omega_c \wedge \omega^b + B_{abc}\,\omega^b \wedge \omega^c + \\
+ (\sqrt{2}\,B^b_{an} - i\sigma^b_a)\,\omega_b \wedge \omega + (-\sqrt{2}\,\tilde{B}_{nab} - \frac{1}{\sqrt{2}}\tilde{B}_{abn} - \frac{1}{\sqrt{2}}B^n_{ab} - i\,\sigma_{ab})\,\omega^b \wedge \omega; \quad (2) \\
d\,\omega = \sqrt{2}\,B_{nab}\,\omega^a \wedge \omega^b + \sqrt{2}\,B^{nab}\,\omega_a \wedge \omega_b + \\
+ \left(\sqrt{2}\,B^{na}_b - \sqrt{2}\,B^a_{nb} - 2i\,\sigma^a_b\right)\,\omega^b \wedge \omega_a + \\
+ \left(\tilde{B}_{nbn} + B^n_{nb} + i\,\sigma_{nb}\right)\omega \wedge \omega^b + \left(\tilde{B}^{nbn} + B^{nb}_n - i\,\sigma^b_n\right)\omega \wedge \omega_b,
\end{gathered}$$

where

$$\tilde{B}^{abc} = \frac{i}{2}\,J^a_{\hat{b},\,\hat{c}};\ \tilde{B}_{abc} = -\frac{i}{2}\,J^{\hat{a}}_{b,\,c}$$

and σ is the second fundamental form of the immersion of N into M^{2n}. We also render concrete the structural equations (1) of a six-dimensional almost Hermitian submanifold of Cayley algebra [5], [6], [8]:

$$d\omega^a = \omega^a_b \wedge \omega^b + \frac{1}{\sqrt{2}}\varepsilon^{abh}D_{hc}\omega^c \wedge \omega_b + \frac{1}{\sqrt{2}}\varepsilon^{ah[b}D^{c]}_h\omega_b \wedge \omega_c; \quad (3)$$

$$d\omega_a = -\omega^b_a \wedge \omega_b + \frac{1}{\sqrt{2}}\varepsilon_{abh}D^{hc}\omega_c \wedge \omega^b + \frac{1}{\sqrt{2}}\varepsilon_{ah[b}D^h_{c]}\omega^b \wedge \omega^c.$$

Here $\varepsilon_{abc} = \varepsilon^{123}_{abc}$, $\varepsilon^{abc} = \varepsilon^{abc}_{123}$ are the components of the third-order Kronecher tensor [25];

$$D^{hc} = D_{\hat{h}\hat{c}}, \quad D^c_h = D_{h\hat{c}}, \quad D^h_c = D_{\hat{h}c};$$
$$D_{cj} = \mp T^8_{cj} + iT^7_{cj}, \quad D_{\hat{c}j} = \mp T^8_{\hat{c}j} - iT^7_{\hat{c}j},$$

where $\left\{T^{\varphi}_{kj}\right\}$ are the components of the configuration tensor (in Gray-Kirichenko notation [24]); $\varphi = 7, 8$; $a, b, c, d, g, h = 1, 2, 3$; $\hat{a} = a + 3$; $k, j = 1, 2, 3, 4, 5, 6$.

Comparing these equations with (1), we get the expressions for the Kirichenko tensors of six-dimensional almost Hermitian submanifolds of Cayley algebra (in particular, for the nearly Kählerian six-dimensional sphere S^6):

$$B^{ab}_c = \frac{1}{\sqrt{2}}\varepsilon^{abh} D_{hc}\,;\; B^c_{ab} = \frac{1}{\sqrt{2}}\varepsilon_{abh} D^{hc}\,;$$

$$B^{abc} = \frac{1}{\sqrt{2}}\varepsilon^{ah[b} D^{c]}_h\,;\; B_{abc} = \frac{1}{\sqrt{2}}\varepsilon_{ah[b} D^h_{c]}\,.$$

Knowing that the Kirichenko tensors ${B^{ab}}_c$ and ${B_{ab}}^c$ of the nearly Kählerian six-sphere vanish [8], we rewrite these structural equations as follows:

$$\begin{gathered} d\,\omega^\alpha \,=\, \omega^\alpha_\beta \wedge \omega^\beta \;+\, B^{\alpha\beta\gamma}\,\omega_\beta \wedge \omega_\gamma + \\ +\; i\sigma^\alpha_\beta\,\omega^\beta \wedge \omega \,+\, (-\sqrt{2}\,\tilde{B}^{n\alpha\beta} \;-\; \frac{1}{\sqrt{2}}\tilde{B}^{\alpha\beta n} + i\,\sigma^{\alpha\beta})\,\omega_\beta \wedge \omega; \\ d\,\omega_\alpha \,=\, -\omega^\beta_\alpha \wedge \omega_\beta \;+\, B_{\alpha\beta\gamma}\,\omega^\beta \wedge \omega^\gamma - \\ -\; i\sigma^\beta_\alpha\,\omega_\beta \wedge \omega \,+\, (-\sqrt{2}\,\tilde{B}_{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}_{\alpha\beta n} - i\,\sigma_{a\beta})\,\omega^\beta \wedge \omega; \\ d\,\omega \,=\, \sqrt{2}\,B_{n\alpha\beta}\,\omega^\alpha \wedge \omega^\beta \,+\, \sqrt{2}\,B^{n\alpha\beta}\,\omega_\alpha \wedge \omega_\beta - \\ -2i\,\sigma^\alpha_\beta\,\omega^\beta \,\wedge\, \omega_\alpha \,+\, \left(\tilde{B}_{n\beta n} \,+\, i\,\sigma_{n\beta}\right)\,\omega \wedge \omega^\beta \,+\, \left(\tilde{B}^{n\beta n} \,-\, i\,\sigma^\beta_n\right)\,\omega \wedge \omega_\beta\;. \end{gathered} \tag{4}$$

On the other hand, we obtain the more precise structural equations of the nearly Kählerian structure on the six-sphere:

$$\begin{gathered} d\omega^a = \omega^a_b \wedge \omega^b + \mu\,\varepsilon^{acb}\omega_b \wedge \omega_c; \\ d\omega_a = -\omega^b_a \wedge \omega_b + \bar{\mu}\varepsilon_{acb}\omega^b \wedge \omega^c. \end{gathered} \tag{5}$$

If an almost contact metric hypersurface of a nearly Kählerian manifold is totally geodesic (or its type number is equal to zero), then the matrix of its second fundamental form vanishes:

$$(\sigma_{ps}) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},\; p, s = 1,\, 2,\, 3,\, 4,\, 5\;.$$

That is why we get the following Cartan structural equations of an almost contact metric structure on an oriented totally geodesic hypersurface of nearly Kählerian six-sphere:

$$d\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta} + B^{\alpha\beta\gamma}\,\omega_{\beta} \wedge \omega_{\gamma} + (-\sqrt{2}\,\tilde{B}^{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}^{\alpha\beta n})\,\omega_{\beta} \wedge \omega;$$

$$d\omega_{\alpha} = -\omega^{\beta}_{\alpha} \wedge \omega_{\beta} + B_{\alpha\beta\gamma}\,\omega^{\beta} \wedge \omega^{\gamma} + (-\sqrt{2}\,\tilde{B}_{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}_{\alpha\beta n})\,\omega^{\beta} \wedge \omega;$$

$$d\omega = \sqrt{2}\,B_{n\alpha\beta}\,\omega^{\alpha} \wedge \omega^{\beta} + \sqrt{2}\,B^{n\alpha\beta}\,\omega_{\alpha} \wedge \omega_{\beta} + \\ + \left(\tilde{B}_{n\beta n}\right)\,\omega \wedge \omega^{\beta} + \left(\tilde{B}^{n\beta n}\right)\,\omega \wedge \omega_{\beta} \quad .$$

These equations perfectly correspond to well-known Cartan structural equations of the nearly cosymplectic structure [3], [4], [25], [27]:

$$d\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta} + H^{\alpha\beta\gamma}\,\omega_{\beta} \wedge \omega_{\gamma} + H^{\alpha\beta}\,\omega_{\beta} \wedge \omega;$$

$$d\omega_{\alpha} = -\omega^{\beta}_{\alpha} \wedge \omega_{\beta} + H_{\alpha\beta\gamma}\,\omega^{\beta} \wedge \omega^{\gamma} + H_{\alpha\beta}\,\omega^{\beta} \wedge \omega;$$

$$d\omega = -\frac{2}{3}G_{\alpha\beta}\,\omega^{\alpha} \wedge \omega^{\beta} - \frac{2}{3}G^{\alpha\beta}\,\omega_{\alpha} \wedge \omega_{\beta}\,.$$

So, the almost contact metric structure on an oriented totally geodesic hypersurface of nearly Kählerian six-dimensional sphere is nearly cosymplectic, and our Theorem 1 is completely proved.

In [4], it has been proved that the type number of an oriented hypersurface of the nearly Kählerian manifold is at most one. As it was mentioned, paper [3] contains the following result on 1-type number hypersurface of the nearly Kählerian six-sphere.

Theorem 2. *The type number of an oriented hypersurface of the nearly Kahlerian six-dimensional sphere is equal to 1 if and only if the induced almost contact metric structure on this hypersurface is nearly cosymplectic. [3]*

That is why we can generalize the both Theorems in the following statement.

Theorem 3. *If t is the type number of an oriented hypersurface of the nearly Kählerian six-dimensional sphere S^6, then the condition $t \le 1$ holds if and only if the induced almost contact metric structure on this hypersurface is nearly cosymplectic.*

We remark that there is no information about almost contact metric structures on hypersurfaces of S^6 with type number $t \ge 2$. That is why at the end of this paper we pose the following open problem.

Problem. Find a characterization of the almost contact metric structure on a 2-type or 3-type hypersurface in the nearly Kählerian six-dimensional sphere S^6. In particular, can the almost contact metric structure on such a hypersurface be Kenmotsu or Sasakian?

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