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COEFFICIENT INEQUALITY FOR A GENERALIZED SUBCLASS OF ANALYTIC FUNCTIONS

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Abstract

For $\alpha(0 \leq \alpha < 1)$, $\eta(0 \leq \eta \leq 1)$, $\beta(0 < \beta \leq 1)$, let $H^m_{\lambda,\mu}(\alpha;\beta,\eta)$ be the class of normalised functions defined in the unit disk $\mathcal{U}$ by

$$\Re\left(\frac{(1-\eta)z(D^m_{\lambda,\mu}f(z))' + \eta z(D^{m+1}_{\lambda,\mu}f(z))'}{(1-\eta)D^m_{\lambda,\mu}g(z) + \eta D^{m+1}_{\lambda,\mu}g(z)}\right) > 0 \tag{1}$$

where $D^m_{\lambda,\mu}$ is a linear multiplier differential operator and $g \in P^m_{\lambda,\mu}(\alpha;\beta,\eta)$ the class of normalized functions defined in the unit disk $\mathcal{U}$ by

$$\left|\arg\left(\frac{(1-\eta)z(D^m_{\lambda,\mu}f(z))' + \eta z(D^{m+1}_{\lambda,\mu}f(z))'}{(1-\eta)D^m_{\lambda,\mu}f(z) + \eta D^{m+1}_{\lambda,\mu}f(z)} - \alpha\right)\right| < \frac{\pi}{2}\beta \tag{2}$$

For $f \in H^m_{\lambda,\mu}(\alpha;\beta,\eta)$ and given by $f(z) = z + a_2z^2 + a_3z^3 + \ldots$, a sharp upper bound is obtained for $\left|a_3 - \xi a_2^2\right|$ when $A_2\xi \geq A_1^2$ where A_1 and A_2 are given below.

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1 Indroduction

Let $\mathcal{A}$ denote the family of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{3}$$

which are analytic in the open unit disk $\mathcal{U} = \{z : |z| < 1\}$. Further, let $\mathcal{S}$ denote the class of functions which are univalent in $\mathcal{U}$. A function $f(z)$ belonging to $\mathcal{A}$ is said to be strongly starlike of order β and type α in $\mathcal{U}$, and denoted by $\overline{S}^*(\alpha;\beta)$ if it satisfies

$$\left|\arg\left(\frac{zf'(z)}{f(z)} - \alpha\right)\right| < \frac{\pi}{2}\beta \quad (z \in \mathcal{U}) \tag{4}$$

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for some $\alpha(0 \leq \alpha < 1)$ and $\beta(0 \leq \beta < 1)$. If $f(z) \in \mathcal{A}$ satisfies

$$\left|\arg\left(1+\frac{zf''(z)}{f'(z)}-\alpha\right)\right| < \frac{\pi}{2}\beta \quad (z \in \mathcal{U})$$

for some $\alpha(0 \leq \alpha < 1)$ and $\beta(0 < \beta \leq 1)$, then we say that $f(z)$ is strongly convex of order β and type α in $\mathcal{U}$, and we denote by $\overline{C}(\alpha;\beta)$ the class of all such functions.

The *linear multiplier differential operator* $D^m_{\lambda,\mu}f$ was defined by the authors in (see [2])

$$\begin{aligned}
D^0_{\lambda,\mu}f(z) &= f(z) \\
D^1_{\lambda,\mu}f(z) &= D_{\lambda,\mu}f(z) = \lambda\mu z^2(f(z))'' + (\lambda-\mu)z(f(z))' + (1-\lambda+\mu)f(z) \\
D^2_{\lambda,\mu}f(z) &= D_{\lambda,\mu}\left(D^1_{\lambda,\mu}f(z)\right) \\
&\vdots \\
D^m_{\lambda,\mu}f(z) &= D_{\lambda,\mu}\left(D^{m-1}_{\lambda,\mu}f(z)\right)
\end{aligned}$$

where $\lambda \geqslant \mu \geqslant 0$ and $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

If f is given by (3) then from the definition of the operator $D^m_{\lambda,\mu}f(z)$ it is easy to see that

$$D^m_{\lambda,\mu}f(z) = z + \sum_{n=2}^{\infty}[1+(\lambda\mu n+\lambda-\mu)(n-1)]^m a_n z^n. \tag{5}$$

It should be remarked that the $D^m_{\lambda,\mu}$ is a generalization of many other linear operators considered earlier. In particular, for $f \in \mathcal{A}$ we have the following:

- $D^m_{1,0}f(z) \equiv D^m f(z)$ the operator investigated by Sălăgean (see [4]).
- $D^m_{\lambda,0}f(z) \equiv D^m_{\lambda} f(z)$ the operator studied by Al-Oboudi (see [3]).
- $D^m_{\lambda,\mu}f(z)$ the operator firstly considered for $0 \leqslant \mu \leqslant \lambda \leqslant 1$, by Răducanu and Orhan (see [1]).

With the help of the differential operator $D^m_{\lambda,\mu}$, we say that a function belonging to $\mathcal{A}$ is said to be in the class $P^m_{\lambda,\mu}(\alpha;\beta,\eta)$ if it satisfies

$$\left|\arg\left(\frac{(1-\eta)z(D^m_{\lambda,\mu}f(z))' + \eta z(D^{m+1}_{\lambda,\mu}f(z))'}{(1-\eta)D^m_{\lambda,\mu}f(z) + \eta D^{m+1}_{\lambda,\mu}f(z)} - \alpha\right)\right| < \frac{\pi}{2}\beta \tag{6}$$

for some $\alpha(0 \leq \alpha < 1)$, $\eta(0 \leq \eta \leq 1)$, $\beta(0 < \beta \leq 1)$ and for all $z \in \mathcal{U}$. Note that $P^0_{\lambda,\mu}(\alpha;\beta,0) = \overline{S}^*(\alpha;\beta)$ and $P^1_{1,0}(\alpha;\beta,0) = P^0_{1,0}(\alpha;\beta,1) = \overline{C}(\alpha;\beta)$.

For the class $\mathcal{S}$ of analytic univalent functions, Fekete-Szegö [5] obtained the maximum value of $\left|a_3 - \xi a_2^2\right|$ when ξ is real. For various functions of $\mathcal{S}$, the upper bound for $\left|a_3 - \xi a_2^2\right|$ is investigated by many authors including ([12], [13], [14], [15], [16]).

In this paper we obtained sharp upper bound for $\left|a_3 - \xi a_2^2\right|$ when f belonging to the class of functions defined as follows:

Definition 1. *Let $\alpha(0 \leq \alpha < 1)$, $\eta(0 \leq \eta \leq 1)$, $\beta(0 < \beta \leq 1)$ and $f \in \mathcal{A}$. Then $f \in H^m_{\lambda,\mu}(\alpha;\beta,\eta)$ if and only if, there exists $g \in P^m_{\lambda,\mu}(\alpha;\beta,\eta)$ such that*

$$\Re\left(\frac{(1-\eta)z(D^m_{\lambda,\mu}f(z))' + \eta z(D^{m+1}_{\lambda,\mu}f(z))'}{(1-\eta)D^m_{\lambda,\mu}g(z) + \eta D^{m+1}_{\lambda,\mu}g(z)}\right) > 0 \tag{7}$$

where $g(z) = z + b_2z^2 + b_3z^3 + \dots$.

Note that $H^0_{\lambda,\mu}(0;\beta,0) = K(\beta)$ the class of close-to-convex functions defined in [6], $H^0_{\lambda,\mu}(0;1,0) = K(1)$ is the class of normalized close-to-convex functions defined by Kaplan [7], and $H^0_{\lambda,\mu}(\alpha;\beta,0) = M(\alpha;\beta)$ is the class introduced and studied by Frasin and Darus [8]. $H^m_{1,0}(\alpha;\beta,\eta) = H(\alpha;\beta,\eta,m)$ is the class defined and studied by Orhan *et al.* [9].

2 Main results

In order to derive our main results, we have to recall here the following lemma (see [10]).

Lemma 1. *Let $h \in \mathcal{P}$ i.e. h be analytic in $\mathcal{U}$ and be given by $h(z) = 1 + c_1z + c_2z^2 + \dots$, and $\Re(h(z)) > 0$ for $z \in \mathcal{U}$, then*

$$\left|c_2 - \frac{c_1^2}{2}\right| \leqslant 2 - \frac{|c_1|^2}{2}. \tag{8}$$

Theorem 1. *Let $f(z) \in H^m_{\lambda,\mu}(\alpha;\beta,\eta)$ and be given by (3). Then for $0 \leq \alpha < 1$, $0 \leq \eta \leq 1$, $0 < \beta \leq 1$ and $A_2\xi \geq A_1^2$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \frac{\beta^2\left[6\left(A_2\xi - A_1^2\right) + \alpha\left(8A_1^2 - 2\alpha A_1^2 - 3A_2\xi\right)\right]}{3A_1^2A_2(1-\alpha)^2(2-\alpha)} + \frac{(3A_2\xi - 2A_1^2)(2\beta+1-\alpha)}{3A_1^2A_2(1-\alpha)} \tag{9}$$

where

$$\begin{aligned} A_1 &= [1 + (2\lambda\mu + \lambda - \mu)]^m\,[1 + \eta(2\lambda\mu + \lambda - \mu)]\,, \\ A_2 &= [1 + 2(3\lambda\mu + \lambda - \mu)]^m\,[1 + 2\eta(3\lambda\mu + \lambda - \mu)]\,. \end{aligned}$$

Proof. Let $f(z) \in H^m_{\lambda,\mu}(\alpha;\beta,\eta)$. It follows from (7) that

$$(1-\eta)z(D^m_{\lambda,\mu}f(z))' + \eta z(D^{m+1}_{\lambda,\mu}f(z))' = \left[(1-\eta)D^m_{\lambda,\mu}g(z) + \eta D^{m+1}_{\lambda,\mu}g(z)\right]q(z) \tag{10}$$

for $z \in \mathcal{U}$ with $q \in \mathcal{P}$ given by $q(z) = 1 + q_1z + q_2z^2 + \dots$. Equating coefficients, we obtain

$$2A_1a_2 = q_1 + A_1b_2 \tag{11}$$

and

$$3A_2a_3 = q_2 + A_1b_2q_1 + A_2b_3. \tag{12}$$

Also, it follows from (6) that

$$
\begin{aligned}
&(1-\eta)z(D^{m}_{\lambda,\mu}g(z))' + \eta z(D^{m+1}_{\lambda,\mu}g(z))' - \alpha\left[(1-\eta)D^{m}_{\lambda,\mu}g(z) + \eta D^{m+1}_{\lambda,\mu}g(z)\right] \quad (13)\\
= &\left[(1-\eta)D^{m}_{\lambda,\mu}g(z) + \eta D^{m+1}_{\lambda,\mu}g(z)\right](p(z))^{\beta}
\end{aligned}
$$

where for $z \in \mathcal{U}$, $p \in \mathcal{P}$ and $p(z) = 1 + p_1 z + p_2 z^2 + ...$ Thus equating coefficients, we obtain

$$A_1(1-\alpha)b_2 = \beta p_1 \tag{14}$$

$$A_2(2-\alpha)b_3 = \beta\left(p_2 + \frac{\beta(3-\alpha)+\alpha-1}{2(1-\alpha)}p_1^2\right). \tag{15}$$

From (11), (12), (14) and (15) we have

$$
\begin{aligned}
a_3 - \xi a_2^2 \;=\; & \frac{1}{3A_2}\left(q_2 - \frac{1}{2}q_1^2\right) + \frac{2A_1^2 - 3A_2\xi}{12A_1^2A_2}q_1^2 + \frac{\beta}{3A_2(2-\alpha)}\left(p_2 - \frac{1}{2}p_1^2\right) \quad (16)\\
& + \frac{\left[2A_1^2 - 3A_2\xi\right]\beta}{6A_1^2A_2(1-\alpha)}p_1q_1 + \frac{\left[6A_1^2 + 2\alpha^2A_1^2 + 3\alpha A_2\xi - \left(6A_2\xi + 8\alpha A_1^2\right)\right]\beta^2}{12A_1^2A_2(1-\alpha)^2(2-\alpha)}p_1^2.
\end{aligned}
$$

Assume that $a_3 - \xi a_2^2$ is positive. Hence we now estimate $\Re\left(a_3 - \xi a_2^2\right)$, so from (16) and by using the lemma 1 and letting $p_1 = 2re^{i\theta}$, $q_1 = 2Re^{i\phi}$, $0 \le r \le 1$, $0 \le R \le 1$, $0 \le \theta \le 2\pi$ and $0 \le \phi \le 2\pi$, we obtain

$$
\begin{aligned}
3A_2\Re\left(a_3 - \xi a_2^2\right) \;=\; & \Re\left(q_2 - \frac{1}{2}q_1^2\right) + \frac{2A_1^2 - 3A_2\xi}{4A_1^2}\Re(q_1^2) + \frac{\beta}{(2-\alpha)}\Re\left(p_2 - \frac{1}{2}p_1^2\right)\\
& + \frac{\left[2A_1^2 - 3A_2\xi\right]\beta}{2A_1^2(1-\alpha)}\Re(p_1q_1)\\
& + \frac{\left[6A_1^2 + 2\alpha^2A_1^2 + 3\alpha A_2\xi - \left(6A_2\xi + 8\alpha A_1^2\right)\right]\beta^2}{4A_1^2(1-\alpha)^2(2-\alpha)}\Re(p_1^2)\\
\le\; & 2\left(1 - R^2\right) + \frac{2A_1^2 - 3A_2\xi}{A_1^2}R^2\cos 2\phi + \frac{2\beta}{(2-\alpha)}\left(1 - r^2\right)\\
& + \frac{2\left[2A_1^2 - 3A_2\xi\right]\beta}{A_1^2(1-\alpha)}rR\cos(\theta+\phi)\\
& + \frac{\left[6A_1^2 + 2\alpha^2A_1^2 + 3\alpha A_2\xi - \left(6A_2\xi + 8\alpha A_1^2\right)\right]\beta^2}{A_1^2(1-\alpha)^2(2-\alpha)}r^2\cos 2\theta\\
\le\; & \frac{3A_2\xi - 4A_1^2}{A_1^2}R^2 + \frac{2\left[3A_2\xi - 2A_1^2\right]\beta}{A_1^2(1-\alpha)}rR\\
& + \left(\frac{\left[6\left(A_2\xi - A_1^2\right) + \alpha\left(8A_1^2 - 2\alpha A_1^2 - 3A_2\xi\right)\right]\beta^2}{A_1^2(1-\alpha)^2(2-\alpha)} - \frac{2\beta}{2-\alpha}\right)r^2\\
& + \frac{2\beta}{2-\alpha} + 2\\
=\; & \Psi(r,R).
\end{aligned} \tag{17}
$$

Letting α, β, η and ξ fixed and differentiating $\Psi(r,R)$ partially when $0 \le \alpha < 1$, $0 \le \eta \le 1$, $0 < \beta \le 1$ and $A_2\xi \ge A_1^2$ we observe that

$$\begin{aligned}\Psi_{rr}\Psi_{RR} - (\Psi_{rR})^2 &= 16\beta A_1^4\left(2\alpha^2\beta + 2\alpha^2 + 4\beta + 2 - 4\alpha - 7\alpha\beta\right)\\ &\quad -12\beta\xi A_1^2 A_2\left(2\alpha^2\beta + 2\alpha^2 + 6\beta + 2 - 4\alpha - 8\alpha\beta\right) < 0.\end{aligned}$$

Therefore, the maximum of $\Psi(r,R)$ occurs on the boundaries. Thus the desired inequality follows by observing that

$$\begin{aligned}\Psi(r,R) &\le \Psi(1,1)\\ &= \frac{\beta^2\left[6\left(A_2\xi - A_1^2\right) + \alpha\left(8A_1^2 - 2\alpha A_1^2 - 3A_2\xi\right)\right]}{A_1^2(1-\alpha)^2(2-\alpha)}\\ &\quad + \frac{\left[3A_2\xi - 2A_1^2\right](2\beta+1-\alpha)}{A_1^2(1-\alpha)}. \qquad (18)\end{aligned}$$

□

The equality for (9) is attained when $p_1 = q_1 = 2i$ and $p_2 = q_2 = -2$.

Letting $\lambda = 1$, $\mu = 0$ in the Theorem 1 we get the result given by Orhan *et al.*[9]:

Corollary 1. *Let $f \in H_{1,0}^m(\alpha;\beta,\eta)$ and be given by (3). Then for $0 \le \alpha < 1$, $0 \le \beta < 1$ and $3^m(2\eta+1)\xi \ge 4^m(\eta+1)^2$ we have the sharp inequality*

$$\begin{aligned}&\left|a_3 - \xi a_2^2\right|\\ \leqslant\ & \frac{6\beta^2\left[3^m(2\eta+1)\xi - 4^m(\eta+1)^2\right]}{12^m 3(2\eta+1)(\eta+1)^2(1-\alpha)^2(2-\alpha)}\\ &+\frac{\alpha\beta^2\left[2^{2m+3}(\eta+1)^2 - 2^{2m+1}\alpha(\eta+1)^2 - 3^{m+1}(2\eta+1)\xi\right]}{12^m 3(2\eta+1)(\eta+1)^2(1-\alpha)^2(2-\alpha)}\\ &+\frac{\left[3^{m+1}(2\eta+1)\xi - 2^{2m+1}(\eta+1)^2\right](2\beta+1-\alpha)}{12^m 3(2\eta+1)(\eta+1)^2(1-\alpha)}.\end{aligned}$$

Letting $m = \alpha = \eta = 0$ in the Theorem 1 we get the result given by Jahangiri [11]:

Corollary 2. *Let $f \in K(\beta)$ and be given by (3). Then for $0 \le \beta < 1$ and $\xi \ge 1$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \beta^2(\xi - 1) + \frac{(2\beta+1)(3\xi-2)}{3}.$$

Letting $m = \eta = 0$ in the Theorem 1 we get the result given by Frasin and Darus [8]:

Corollary 3. *Let $f \in M(\alpha;\beta)$ and be given by (3). Then for $0 \le \alpha < 1$, $0 \le \beta < 1$ and $\xi \ge 1$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \frac{\beta^2\left[6(\xi-1) + \alpha\left(8 - 2\alpha - 3\xi\right)\right]}{3(1-\alpha)^2(2-\alpha)} + \frac{(2\beta+1-\alpha)(3\xi-2)}{3(1-\alpha)}.$$

Letting $\lambda = 1$, $\mu = 0$, $m = 0$, $\eta = 1$ in the Theorem 1 we have

Corollary 4. *Let $f \in H^0_{1,0}(\alpha;\beta,1)$ and be given by (3). Then for $0 \le \alpha < 1$, $0 \le \beta < 1$ and $\xi \ge 4/3$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \frac{\beta^2\left[6(3\xi-4)+\alpha\left(32-8\alpha-9\xi\right)\right]}{36(1-\alpha)^2(2-\alpha)} + \frac{(2\beta+1-\alpha)(9\xi-8)}{36(1-\alpha)}.$$

Letting $\lambda = 1$, $\mu = 0$, $m = 0$, $\eta = 1/2$ in the Theorem 1 we have

Corollary 5. *Let $f(z) \in H^0_{1,0}(\alpha;\beta,1/2)$ and be given by (3). Then for $0 \le \alpha < 1$, $0 \le \beta < 1$ and $\xi \ge 9/8$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \frac{\beta^2\left[6(8\xi-9)+\alpha\left(72-18\alpha-24\xi\right)\right]}{54(1-\alpha)^2(2-\alpha)} + \frac{(2\beta+1-\alpha)(4\xi-3)}{9(1-\alpha)}.$$

Letting $\lambda = 1$, $\mu = 0$, $m = 1$, $\eta = 1$ in the Theorem 1 we have

Corollary 6. *Let $f \in H^1_{1,0}(\alpha;\beta,1)$ and be given by (3). Then for $0 \le \alpha < 1$, $0 \le \beta < 1$ and $\xi \ge 16/9$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \frac{\beta^2\left[6(9\xi-16)+\alpha\left(128-32\alpha-27\xi\right)\right]}{432(1-\alpha)^2(2-\alpha)} + \frac{(2\beta+1-\alpha)(27\xi-32)}{432(1-\alpha)}.$$

Letting $\lambda = 1$, $\mu = 0$, $m = 1$, $\eta = 1/2$ in the Theorem1 we have

Corollary 7. *Let $f \in H^1_{1,0}(\alpha;\beta,1/2)$ and be given by (3). Then for $0 \le \alpha < 1$, $0 \le \beta < 1$ and $\xi \ge 3/2$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \frac{\beta^2(2\xi-3)+\alpha\left(4-\alpha-\xi\right)}{9(1-\alpha)^2(2-\alpha)} + \frac{(2\beta+1-\alpha)(\xi-1)}{9(1-\alpha)}.$$

Letting $\lambda = 1$, $\mu = 1$, $m = 1$, $\eta = 1$ in the Theorem1 we have

Corollary 8. *Let $f \in H^1_{1,1}(\alpha;\beta,1)$ and be given by (3). Then for $0 \le \alpha < 1$, $0 \le \beta < 1$ and $\xi \ge 81/49$ we have the sharp inequality*

$$\left|a_3 - \xi a_2^2\right| \leqslant \frac{\beta^2\left[6(49\xi-81)+\alpha\left(648-162\alpha-147\xi\right)\right]}{11907(1-\alpha)^2(2-\alpha)} + \frac{(2\beta+1-\alpha)(147\xi-162)}{11907(1-\alpha)}.$$

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