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PHYSICAL FINSLER COORDINATES IN SPACETIME

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Abstract

In Finsler geometry a Finsler coordinate is a coordinate in the tangent space manifold of a given base manifold. As such it has been given various definitions in the relativity and field theory literature and often even remains undefined physically. Physically meaningful coordinates of a point in the tangent bundle of spacetime are the spacetime and four-velocity coordinates of the measuring device. It is here emphasized that the four-velocity of the measuring device need not be the same as the four-velocity of the measured object, either classically or quantum mechanically. The four-velocity of a measured particle excitation of a Finslerian quantum field in the tangent space manifold of spacetime is not a suitable physical Finsler coordinate. The role of the Finsler coordinate is elaborated in a detailed example involving a Finslerian quantum field and associated microcausality.

Key words: Finsler geometry, Finslerian fields, quantum field theory, microcausality, maximal proper acceleration, spacetime tangent bundle, relativity, light cone, causal domain

1 Introduction

For the last thirty years I have been exploring possible physical implications of a possible physical upper bound on the curvature of worldlines in spacetime. Equivalently, it can be argued that there is a physical upper bound on the proper acceleration a_0 of any physical object relative to the vacuum and that it is of the order of one Planck length per squared Planck time [1], [2]. If, as one normally expects, the universal gravitational constant has the same value at submicroscopic distances as at macroscopic distances, then the maximal proper acceleration a_0 is of the order of 10^{52} m/s^2. (If this is not the case, as in currently popular theories of a running gravitational coupling constant or extra dimensions, then the maximal proper acceleration a_0 would be much less because what enters in its evaluation is the gravitational constant near the Planck scale [2].) In a long series of papers, it was argued that the universal upper limit on attainable proper acceleration relative to the vacuum imposes restrictions on the differential geometric structure of the

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tangent bundle of spacetime.[2]-[7] One is led naturally to a Finslerian structure for spacetime in which the spacetime metric depends not only on the spacetime coordinates, but also on the four-velocity coordinates of the tangent space manifold. Various features of the differential geometry of the tangent bundle of spacetime were obtained, including the bundle metric, connection, curvature, and geodesics [2]-[7]. In a personal communication, Anadi Das pointed out to me in 1991 that the differential geometric structure which I had obtained had a form very similar to that appearing in the mathematical work of Kentaro Yano and Evan Tom Davies on the tangent bundles of Finsler and Riemannian manifolds [6], [27], [28]. Exploiting this work of Yano and Davies, I undertook a series of investigations concerning possible differential geometric structures of a Finsler spacetime The Levi-Civita bundle connection coefficients and the Riemann curvature scalar were determined [8]. An almost complex structure was constructed on the bundle, and conditions were given that the tangent bundle be Kaehler and/or complex [9], [10]. The inclusion of bundle torsion was addressed [11], [12]. Possible physical implications were investigated for the differential geometric structure of spacetime and gravitation [4]-[6], [13]-[15]. Much of this work was summarized in 1995 at the Joint Summer Research Conference on Finsler Geometry organized by David Bao, Shing-Shen Chern, and Zhongmin Shen [13]. On that occasion, Chern requested that all of the speakers include in their papers a list of open problems. One of the problems posed by me (Problem 5) was to construct classical and quantum field theories defined covariantly on a Finslerian spacetime tangent bundle, and this problem motivated most of my subsequent work on Finslerian fields [14]-[26].

2 Physical Finsler coordinates

Throughout all of my earlier work, the question arose as to the physical interpretation of the tangent space Finlser coordinate, namely the four-velocity. It is the four velocity of what? The four-velocity appears implicitly in all possible Finslerian fields [5]. For example, the metric of the tangent bundle of spacetime adapted to the affine connection is

$$G^{MN}(x,v) = \begin{bmatrix} g_{\mu\nu}(x,v) & 0 \\ 0 & g_{\mu\nu}(x,v) \end{bmatrix}, \tag{1}$$

in which the spacetime and four-velocity coordinates are designated by:

$$\left\{x^M\right\} \equiv \left\{x^\mu, \rho_0 v^\mu\right\}, \quad \{M = 0, 2, ..., 7;\ \mu = 0, 1, 2, 3\}, \tag{2}$$

where $v^\mu = dx^\mu/ds$, $\rho_0 = c^2/a_0$ is a constant of the order of the Planck length, and c is the canonical speed of light in vacuum [5]. Evidently the metric field $g_{\mu\nu}(x,v)$ depends on the spacetime point x^μ at which the field is measured or else acts on some object, and x^μ would also be the spacetime coordinate of the measuring device. It follows that v^μ would be the four-velocity of the measuring device. Analogously, for example, the bundle connection also depends, through the metric on both x^μ and v^μ, and the geodesic equation yields the spacetime and four-velocity coordinates of an object such as the measuring device acted on by the gravitational field [5]. Also, for example, the Laplace Beltrami operator for the bundle depends on the spacetime coordinate of the measuring device and its four-velocity [14]. The four-velocity of the measuring device will be referred to in the following as the physical Finlser coordinate, it being the tangent space coordinate in the Finslerian tangent bundle of spacetime. As an explicit example of the possible role of physical Finsler coordinates, in the remainder of this paper I review its role in an analysis of microcausality in quantum field theory.

3 Example

In the following example, for simplicity, the role of the four-velocity Finsler coordinate is considered in a scalar quantum field theory in the spacetime tangent bundle restricted by the limiting proper acceleration [14]-[26]. For simplicity, a Minkowski spacetime in the base manifold is assumed. Of course, Minkowski spacetime is a very special case of a more general Finslerian spacetime, but understanding this simple case may facilitate future analyses involving a more general Finslerian spacetime The quantum field is Finslerian in the sense that it depends not only on the spacetime coordinates of the device measuring particle excitations of the quantum field, but also on the four-velocity of the measuring device.

Canonical quantum field theory in Minkowski spacetime suffers from the divergences occurring at very small distances and/or very high energies. This long standing issue is also manifested in the singular delta function appearing in the microcausality relation involving the commutator of the quantum field at two points separated in spacetime. It has been argued in earlier work that an implication of a physical upper bound on allowed proper acceleration relative to the vacuum is that the canonical microcausality relation is modified to include dependence of the field on the four-velocity of the device measuring the field, so that the delta function is replaced by a function concentrated near the Planck scale of spatial separation between the two devices measuring the field, or within a much larger separation when the relative speed of the two measuring devices is near the canonical speed of light [18], [19]. A consequence is that the causal boundary, canonically defined by the light cone, is warped at these scales so that the timelike region extends into the canonical spacelike region. The speed of the associated causal connectivity can exceed the canonical measured speed of light. The condition for this warp-speed causal connectivity to occur optimally with instantaneous transmission is when the spatial component of relative four-velocity of the two measuring devices is orthogonal to their spatial separation, and for spatial separations near the Planck scale. When the relative speed of the measuring devices is very large, the range for warp-speed causal connectivity may extend well beyond the Planck scale; however if the wavelength in the frame of the moving measuring device is much less than the range, the field is extremely reduced, and any warp-speed causal connectivity is exponentially suppressed.

The limiting proper acceleration a_0 determines the structure of the metric on the tangent bundle of spacetime [5]. Among the many differential geometric invariants determined by the bundle metric is the Laplace-Beltrami operator [14], [18]:

$$\mathcal{L} = G^{-1/2}\frac{\partial}{\partial x^M}\left(G^{1/2}G^{MN}\frac{\partial}{\partial x^N}\right). \tag{3}$$

This is the invariant generalization of the wave operator, or d'Alembertian, of standard field theory. A simple invariant field equation for a scalar field $\phi(x,v)$ is then given by [14]

$$\mathcal{L}\phi(x,v) = 0. \tag{4}$$

Again, as for any Finslerian field, x^μ denotes the location in spacetime where the field is measured, or equivalently, the location of a particle excitation of the quantum field or the location of the device measuring this excitation, and v^μ, the Finsler coordinates, denote the location in four-velocity space of the measuring device. It is important to stress that v^μ is not the four velocity of the particle excitation, and also that x^μ and v^μ are classical commuting variables since they are the coordinates of a measuring device, which is classical. For a flat Minkowski spacetime, the wave equation, Eq. (4), reduces to [16]:

$$\left(\frac{\partial^2}{\partial x^\mu \partial x_\mu} + \rho_0^{-2}\frac{\partial^2}{\partial v^\mu \partial v_\mu}\right)\phi(x,v) = 0, \tag{5}$$

for the Lorentz-invariant field $\phi(x,v)$, where ρ_0 is of the order of the Planck length. For this case, it was argued in earlier work that a scalar quantum field satisfying Eq.(4) is given by [14], [25]

$$\begin{aligned}\phi(x,v) = 2\int \tfrac{d^3\mathbf{p}}{(2\pi\hbar)^{3/2}(2p^0N)^{1/2}} & \left[e^{-ipx/\hbar}e^{-\rho_0 pv/\hbar}\theta(\rho_0 pv/\hbar)a(\mathbf{p})\right. \\ + & \left. e^{ipx/\hbar}e^{\rho_0 pv/\hbar}\theta(-\rho_0 pv/\hbar)a^\dagger(\mathbf{p})\right],\end{aligned} \tag{6}$$

where $\hbar$ is Planck's constant divided by 2π, p denotes the four-momentum $p^\mu = \{p^0, p^1, p^2, p^3\}$ of a particle excitation of the field, $a^\dagger(\mathbf{p})$ and $a(\mathbf{p})$ are particle creation and annihilation operators satisfying the commutation relations,

$$[a(\mathbf{p}), a^\dagger(\mathbf{p}')] = \delta^3(\mathbf{p}-\mathbf{p}'), \quad [a(\mathbf{p}), a(\mathbf{p}')] = 0, \quad [a^\dagger(\mathbf{p}), a^\dagger(\mathbf{p}')] = 0, \tag{7}$$

$\delta^3(\mathbf{p})$ is the three-dimensional Dirac delta function, and $\theta(z)$ is the Heaviside function,

$$\theta(z) = \begin{cases} 1, & z > 0 \\ \frac{1}{2}, & z = 0 \\ 0, & z < 0 \end{cases}. \tag{8}$$

Also in Eq.(6), N is a normalization factor. For vanishing ρ_0, or equivalently, infinite a_0, Eq. (6) reduces to a standard relativistic free scalar quantum field.

Next, it can be shown that the Pauli-Jordan function, expressing microcausality through the field commutator at two points (x,v) and (x',v') in the tangent bundle, generalized for the upper bound on proper acceleration, is given by [18], [19], [26]

$$\begin{aligned}
[\phi(x,v),\phi(x',v')] &= \frac{mc\hbar}{\pi^2 N}\left[\theta\left(\frac{\rho_0 mcv^0}{\hbar}\right)\theta\left(-\frac{\rho_0 mcv^{0\prime}}{\hbar}\right)-\theta\left(\frac{\rho_0 mcv^{0\prime}}{\hbar}\right)\theta\left(-\frac{\rho_0 mcv^{0\prime}}{\hbar}\right)\right]\\
&\times\frac{K_1\left((mc/\hbar)\left[-\left(x-x'-i\rho_0(v-v')\right)^2\right]^{1/2}\right)}{\left[-\left(x-x'-i\rho_0(v-v')\right)^2\right]^{1/2}},
\end{aligned} \tag{9}$$

where m is the mass of a particle excitation of the quantum field, $\theta(z)$ is the Heaviside function defined by Eq. (8), and $K_1(z)$ is the modified Bessel function of the third kind of order one. Equation (9) is divergent for

$$\left[x-x'-i\rho_0(v-v')\right]^2=0. \tag{10}$$

Equation (10) determines the causal boundary separating the future from the past and the spacelike region and describes a warping of the standard light cone near the origin in a region of the order of the Planck length, and at much larger distances for large relative four-velocities. The warped region is timelike, whereas without the warping, that region would be spacelike (outside the standard light cone). For vanishing ρ_0, and also for equal four-velocities, $v=v'$, Eq. (10) reduces to the standard light cone. Also, it can be argued that other bosonic and fermionic fields may also be expected to satisfy the same wave equation, and the same causal boundary, Eq. (10), will apply. Particle excitations of the field can be expected to propagate along the causal boundary. It is to be noted that the relative Finsler coordinates $(v-v')$ warp the causal boundary corresponding to the standard light cone.

Taking the real and imaginary parts of Eq. (10), one obtains the following two equations defining the causal boundary:

$$(x-x')^2=(\rho_0(v-v'))^2 \tag{11}$$

and

$$\rho_0(v-v')\cdot(x-x')=0. \tag{12}$$

Rewriting Eqs. (11) and (12) in explicit component form, they become:

$$(\Delta x^0)^2=\left|\overrightarrow{\Delta x}\right|^2+\rho_0^2(\Delta v^0)^2-\rho_0^2(\overrightarrow{\Delta v})^2, \tag{13}$$

and

$$\rho_0\Delta v^0\Delta x^0=\rho_0\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta x}, \tag{14}$$

where $\Delta x^0\equiv x^{0\prime}-x^0$, $\overrightarrow{\Delta x}\equiv\overrightarrow{x}'-\overrightarrow{x}$, $\Delta v^0\equiv v^{0\prime}-v$, and $\overrightarrow{\Delta v}=\overrightarrow{v}'-\overrightarrow{v}$. Next multiplying Eq. (13) by $(\Delta x^0)^2$ and substituting Eq. (14), one obtains

$$(\Delta x^0)^4-\left(\left|\overrightarrow{\Delta x}\right|^2-\rho_0^2(\overrightarrow{\Delta v})^2\right)(\Delta x^0)^2-\rho_0\left|\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta x}\right|^2=0. \tag{15}$$

Equation (15) has the solution:

$$\begin{aligned}\Delta x^0 &= \pm\left(\left|\overrightarrow{\Delta x}\right|^2 - \rho_0^2(\overrightarrow{\Delta v})^2\right)^{1/2} \\ &\times\left[\frac{1}{2}\pm\frac{1}{2}\left(1+\left(\frac{2\rho_0\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta x}}{\left|\overrightarrow{\Delta x}\right|^2-\rho_0^2(\overrightarrow{\Delta v})^2}\right)^2\right)^{1/2}\right]^{1/2}.\end{aligned} \tag{16}$$

Next dividing both sides of Eq. (16) by $\rho_0\left|\overrightarrow{\Delta v}\right|$, and choosing the positive sign inside the bracket in order that Δx^0 be real, one obtains

$$T = \pm\left\{\frac{1}{2}\left(X^2-1\right)+\frac{1}{2}\left[X^4+2\left(\cos 2\theta\right)X^2+1\right]^{1/2}\right\}^{1/2}, \tag{17}$$

in which the normalized temporal separation T is defined by

$$T = \frac{\Delta x^0}{\rho_0\left|\overrightarrow{\Delta v}\right|}, \tag{18}$$

and the normalized spatial separation X is

$$X = \frac{\left|\overrightarrow{\Delta x}\right|}{\rho_0\left|\overrightarrow{\Delta v}\right|}. \tag{19}$$

Also in Eq. (17), the angle θ between the spatial separation $\overrightarrow{\Delta x}$ and the relative spatial component $\overrightarrow{\Delta v}$ of four-velocity is

$$\theta = \cos^{-1}\frac{\left|\overrightarrow{\Delta v}\cdot\overrightarrow{\Delta x}\right|}{\left|\overrightarrow{\Delta v}\right|\left|\overrightarrow{\Delta x}\right|}. \tag{20}$$

Also, Eq. (14) becomes

$$V = \frac{X}{T}\cos\theta, \tag{21}$$

where the normalized relative time component four-velocity is defined by

$$V = \frac{\rho_0\Delta v^0}{\rho_0\left|\overrightarrow{\Delta v}\right|}. \tag{22}$$

For T, X, and V, the scale is here set by the relative spatial component of four-velocity $\left|\overrightarrow{\Delta v}\right|$ together with the factor ρ_0 of the order of the Planck length. Substituting Eq. (17) in Eq. (21), one obtains

$$V = \pm X\cos\theta\left\{\frac{1}{2}\left(X^2-1\right)+\frac{1}{2}\left[X^4+2\left(\cos 2\theta\right)X^2+1\right]^{1/2}\right\}^{-1/2}. \tag{23}$$

According to Eq. (17), near $\theta = \pi/2$, for $X^2 < 1$, or equivalently within the sphere $\left|\overrightarrow{\Delta x}\right|^2 \leq \rho_0^2(\overrightarrow{\Delta v})^2$, the temporal interval Δx^0 is near vanishing, and near instantaneous causal connectivity occurs between spacelike-separated points. This is consistent with the possible existence of extended excitations such as strings. The standard light cone, $X = T$, is effectively warped in this region. The biggest effect is infinitesimally near $\theta = \pi/2$ and for $\left|\overrightarrow{\Delta x}\right| \leq \rho_0 \overrightarrow{\Delta v}$, for which $\overrightarrow{\Delta x}/\Delta t$ is infinite. For vanishing θ, the standard light cone is not warped. Thus the warped light cone and associated causal boundary are anisotropic. Also, for $X \gg 1$, or equivalently for $\left|\overrightarrow{\Delta x}\right| \gg \rho_0 \overrightarrow{\Delta v}$, the warped light cone effectively becomes the standard light cone and becomes asymptotically isotropic and not warped.

When the measuring device detects a field excitation, the speed of the device is at the causal boundary, Eqs. (11) and (12), determined by the ϕ-field excitations. The measuring device at the origin is here taken to be at rest, and $d\overrightarrow{x}'/dt$ is defined to be the velocity of the moving device relative to the one at rest. The velocities of the two devices can be interchanged because only the magnitude of the relative velocity enters. We proceed to derive the velocity of the moving measuring device. First, according to Eq. (22), one has

$$\Delta v^0 = \left|\overrightarrow{\Delta v}\right| V. \tag{24}$$

It is important to note that, in accord with special relativity, the moving measuring device has time component of four-velocity $v^{0\prime} = \gamma' \equiv \left(1 - \left|\frac{d\overrightarrow{x}'}{cdt}\right|^2\right)^{-1/2}$ and spatial component of four-velocity $\overrightarrow{v}' = \gamma' \frac{d\overrightarrow{x}'}{cdt}$. The device at rest has time component $v^0 = 1$ and spatial component $\overrightarrow{v} = 0$. Therefore $\left|\overrightarrow{\Delta v}\right| = \gamma' \left|\frac{d\overrightarrow{x}'}{cdt}\right|$, and Eq. (24) becomes

$$\gamma' - 1 = \gamma' \left|\frac{d\overrightarrow{x}'}{cdt}\right| V, \tag{25}$$

or solving for γ', then,

$$\gamma' = \frac{1}{1 - \left|\frac{d\overrightarrow{x}'}{cdt}\right| V}, \tag{26}$$

or equivalently,

$$\left(1 - \left|\frac{d\overrightarrow{x}'}{cdt}\right|^2\right)^{-1/2} = \left(1 - \left|\frac{d\overrightarrow{x}'}{cdt}\right| V\right)^{-1}. \tag{27}$$

Solving Eq. (27), one obtains for the speed $\left|\frac{d\overrightarrow{x}'}{cdt}\right|$ of the measuring device in units of c :

$$\left|\frac{d\overrightarrow{x}'}{cdt}\right| = \frac{2V}{1 + V^2}. \tag{28}$$

We proceed to obtain an expression for the actual range $R = \left|\overrightarrow{\Delta x}\right| / \rho_0$ between the two measuring devices, expressed in units of ρ_0 (of the order of the Planck length). According to Eq (19), one has

$$\left|\overrightarrow{\Delta x}\right| = \rho_0 \left|\overrightarrow{\Delta v}\right| X, \tag{29}$$

and it then follows that the range is given by

$$R = \frac{\left|\overrightarrow{\Delta x}\right|}{\rho_0} = \gamma \left|\frac{d\overrightarrow{x}'}{cdt}\right| X = \frac{\left|\frac{d\overrightarrow{x}'}{cdt}\right|}{\left(1 - \left|\frac{d\overrightarrow{x}'}{cdt}\right|^2\right)^{1/2}} X. \tag{30}$$

It is to be noted that the range R approaches infinity as the speed of the moving detector $\left|\frac{d\overrightarrow{x}'}{dt}\right|$ approaches c, the canonical speed of light. Of course, extremely high energies are required to accelerate a detector to such high speed. For $\theta = .4999\pi$ and very small X, one calculates, for example, $R \sim 10^4$.

The speed in units of c, namely, $\left|\overrightarrow{dx'}/cdt\right|$, of the moving measuring device is determined by Eqs. (28) and (23). This gives the speed at which the moving measuring device must move for it to be at the causal boundary and detect a particle excitation. It can be shown that near standard light speed for the device is required in the region, $\theta = \pi/2$, $0 < X < 1$, in which warp-speed connectivity occurs.

The speed of the causal connectivity between the two measurements is here defined by $W = \left|\overrightarrow{\Delta x}\right| / \Delta x^0$ and is called the warp speed. It then follows from Eq. (17) that

$$W = \frac{\left|\overrightarrow{\Delta x}\right|}{\Delta x^0} = \frac{X}{T} = \frac{X}{\left\{\frac{1}{2}\left(X^2 - 1\right) + \frac{1}{2}\left[X^4 + 2\left(\cos 2\theta\right) X^2 + 1\right]^{1/2}\right\}^{1/2}}. \tag{31}$$

The warp speed W is the speed of causal connectivity expressed in units of the standard speed of light. For θ infinitesimally near $\pi/2$, and $X \leq 1$, the warp speed approaches infinity. For $\theta = .4999\pi$ and very small X, one calculates, for example, $W = 2000$. Thus warp-speed causal connectivity occurs near the Planck scale of spatial separation between the devices measuring the field, or at much larger separations when the relative speed of the two measuring devices is near the standard speed of light. However, it is argued below that the field is exponentially attenuated for wave lengths of the field excitation less than the spatial separation of the two points where the field is measured. For larger wave lengths, such connectivity is no surprise, since the location of the particle excitation is only definable up to a wavelength.

For particle excitations of negligible rest mass, according to Eq. (6) and [17], the field strength ϕ as a function wavelength λ of the excited particle is proportional to

$$\phi \sim e^{-\rho_0 |pv|/\hbar} = \exp\left\{-\frac{\rho_0}{\lambda}\gamma'(1 - \left|\frac{d\overrightarrow{x}'}{cdt}\right| \cos\theta')\right\}, \tag{32}$$

in which θ' is the angle between the wave vector of the excited field and the velocity of the device measuring the field. (It is significant to note in passing that the field has an automatic spectral cutoff beyond the Planck mass [20], [21].) Proceeding to evaluate Eq. (32), according to Eq. (19), one first has

$$\left|\overrightarrow{\Delta x}\right| = \rho_0 \left|\overrightarrow{\Delta v}\right| X, \tag{33}$$

or equivalently,

$$\frac{\left|\overrightarrow{\Delta x}\right|}{\rho_0\, X} = \gamma' \left|\frac{d\,\overrightarrow{x}'}{cdt}\right|, \tag{34}$$

and therefore solving for γ', one obtains

$$\gamma' = \left(1 + \left(\frac{\left|\overrightarrow{\Delta x}\right|}{\rho_0\, X}\right)^2\right)^{1/2}. \tag{35}$$

Thus for $\theta' = \pi/2$, Eq. (32) becomes

$$\phi \sim \exp\left(-\frac{\rho_0}{\lambda}\gamma\right) = \exp\left[-\frac{\rho_0}{\lambda}\left(1 + \left(\frac{\left|\overrightarrow{\Delta x}\right|}{\rho_0\, X}\right)^2\right)^{1/2}\right]. \tag{36}$$

For $\left|\overrightarrow{\Delta x}\right| \gg \rho_0$ and $X < 1$, Eq. (36) reduces to

$$\phi \sim \exp\left[-\left(\frac{\left|\overrightarrow{\Delta x}\right|}{\lambda}\right)\frac{1}{X}\right]. \tag{37}$$

One notes that for particle wavelength $\lambda \ll \left|\overrightarrow{\Delta x}\right|$, the field strength is greatly attenuated. As an example of the field attenuation, for $X = 0.001$ and $\theta = 0.4999\pi$, one obtains $\frac{\left|\overrightarrow{\Delta x}\right|}{\rho_0} = 7.1 \times 10^7 \approx \gamma$. Also, the corresponding speed of the measuring device is near the canonical speed of light, $\left|\frac{d\,\overrightarrow{x}'}{dt}\right| \sim c$, and the field is

$$\phi \sim \exp\left[-\left(\frac{\left|\overrightarrow{\Delta x}\right|}{\lambda}\right)\frac{1}{X}\right] = \exp\left[-\frac{7.1 \times 10^7 \rho_0}{\lambda}\right]. \tag{38}$$

The corresponding warp speed $W = 3,183$.

4 Conclusion

It has been argued that the appropriate Finsler coordinates for Finslerian fields such as the spacetime metric and any field defined over the spacetime tangent bundle are given by the four-velocity tangent space coordinates in the tangent space manifold of the tangent bundle of spacetime. The four-velocity here is that of a device measuring the field or any object acted on by the field. An example has been given of the role of physical Finsler coordinates in the analysis of microcausality in quantum field theory. The Finlser coordinate is the four-velocity of the measuring device measuring particle excitations of the quantum field. An implication of a physical upper bound on allowed proper acceleration relative to the vacuum is that near the Planck scale of spatial separation between the two devices measuring the field, or at much larger separations when the relative speed of the two measuring devices is near the canonical speed of light, the standard causal boundary, canonically defined by the light cone, is warped, so that the timelike region extends into the canonical spacelike region. The speed of the associated causal connectivity can exceed the canonical measured speed of light by many orders of magnitude. The condition for this warp-speed causal connectivity to occur optimally with instantaneous transmission is when the spatial component of the relative four-velocity of the two measuring devices is orthogonal to their spatial separation, and for spatial separations near the Planck scale. The range for warp-speed causal connectivity may extend well beyond the Planck scale when the relative speed of the measuring devices is very large, however for practical cases in which the wavelength is much less than the range, the field is extremely attenuated. Analogous behavior may also be expected not only for a scalar field but also for other bosonic and fermionic fields. It is also significant to note that the modified quantum field is Lorentz invariant, and causal connectivity backward in time remains impossible. A proper understanding of the appropriate physical Finsler coordinates is an essential ingredient in all of this analysis.

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