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A DISTINGUISHED RIEMANNIAN GEOMETRIZATION FOR QUADRATIC HAMILTONIANS OF POLYMOMENTA

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Abstract

In this paper we construct a distinguished Riemannian geometrization on the dual 1-jet space $J^{1*}(\mathcal{T}, M)$ for the multi-time quadratic Hamiltonian function

$$H = h_{ab}(t)g^{ij}(t,x)p_i^a p_j^b + U_{(a)}^{(i)}(t,x)p_i^a + \mathcal{F}(t,x).$$

Our geometrization includes a nonlinear connection N, a generalized Cartan canonical N-linear connection $C\Gamma(N)$ (together with its local d-torsions and d-curvatures), naturally provided by the given quadratic Hamiltonian function depending on polymomenta.

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1 Short introduction

In the last decades, numerous scientists have been preoccupied by the geometrization of Hamiltonians depending on polymomenta. In such a perspective, we point out that the Hamiltonian geometrizations are achieved in three distinct ways:

- the *multisymplectic Hamiltonian geometry* − developed by Gotay, Isenberg, Marsden, Montgomery and their peers (see [11], [10]);
- the *polysymplectic Hamiltonian geometry* − elaborated by Giachetta, Mangiarotti and Sardanashvily (see [8], [9]);

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- ♦ the *De Donder-Weyl Hamiltonian geometry* – studied by Kanatchikov (see the papers [12], [13], [14]).

In such a geometrical context, the recent studies of Atanasiu and Neagu ([4], [5], [6]) initiate the new way of distinguished Riemannian geometrization for Hamiltonians depending on polymomenta, which is in fact a natural "multi-time" extension of the already classical Hamiltonian geometry on cotangent bundles synthesized in the Miron et al.'s book [17]. Note that our distinguished Riemannian geometrization for Hamiltonians depending on polymomenta is different one by all three Hamiltonian geometrizations from above (multisymplectic, polysymplectic and De Donder-Weyl).

2 Metrical multi-time Hamilton spaces

Let us consider that $h = (h_{ab}(t))$ is a semi-Riemannian metric on the "multi-time" (*temporal*) manifold $\mathcal{T}^m$, where $m = \dim \mathcal{T}$. Let $g = \left(g^{ij}(t^c, x^k, p^c_k)\right)$ be a symmetric d-tensor on the dual 1-jet space $E^* = J^{1*}(\mathcal{T}, M^n)$, which has the rank $n = \dim M$ and a constant signature. At the same time, let us consider a smooth multi-time Hamiltonian function

$$E^* \ni (t^a, x^i, p^a_i) \to H(t^a, x^i, p^a_i) \in \mathbb{R},$$

which yields the *fundamental vertical metrical d-tensor*

$$G^{(i)(j)}_{(a)(b)} = \frac{1}{2}\frac{\partial^2 H}{\partial p^a_i \partial p^b_j},$$

where $a, b = 1, .., m$ and $i, j = 1, ..., n$.

Definition 1. *A multi-time Hamiltonian function $H : E^* \to \mathbb{R}$, having the fundamental vertical metrical d-tensor of the form*

$$G^{(i)(j)}_{(a)(b)}(t^c, x^k, p^c_k) = \frac{1}{2}\frac{\partial^2 H}{\partial p^a_i \partial p^b_j} = h_{ab}(t^c) g^{ij}(t^c, x^k, p^c_k),$$

is called a ***Kronecker h-regular multi-time Hamiltonian function****.*

In such a context, we can introduce the following important geometrical concept:

Definition 2. *A pair $MH^n_m = (E^* = J^{1*}(\mathcal{T}, M), H)$, where $m = \dim \mathcal{T}$ and $n = \dim M$, consisting of the dual 1-jet space and a Kronecker h-regular multi-time Hamiltonian function $H : E^* \to \mathbb{R}$, is called a* ***multi-time Hamilton space****.*

Remark 1. *In the particular case $(\mathcal{T}, h) = (\mathbb{R}, \delta)$, a* ***"single-time" Hamilton space*** *will be also called a* ***relativistic rheonomic Hamilton space*** *and it will be denoted by $RRH^n = (J^{1*}(\mathbb{R}, M), H)$.*

Example 1. *Let us consider the Kronecker h-regular multi-time Hamiltonian function* $H_1 : E^* \to \mathbb{R}$ *given by*

$$H_1 = \frac{1}{4mc} h_{ab}(t)\varphi^{ij}(x)p_i^a p_j^b, \tag{1}$$

where $h_{ab}(t)$ *(*$\varphi_{ij}(x)$*, respectively) is a semi-Riemannian metric on the temporal (spatial, respectively) manifold* $\mathcal{T}$ *(*M*, respectively) having the physical meaning of* ***gravitational potentials****, and* m *and* c *are the known constants from Theoretical Physics representing the* ***mass of the test body*** *and the* ***speed of light****. Then, the multi-time Hamilton space* $\mathcal{G}MH_m^n = (E^*, H_1)$ *is called the* ***multi-time Hamilton space of the gravitational field****.*

Example 2. *If we consider on* E^* *a symmetric d-tensor field* $g^{ij}(t,x)$*, having the rank* n *and a constant signature, we can define the Kronecker h-regular multi-time Hamiltonian function* $H_2 : E^* \to \mathbb{R}$*, by setting*

$$H_2 = h_{ab}(t)g^{ij}(t,x)p_i^a p_j^b + U_{(a)}^{(i)}(t,x)p_i^a + \mathcal{F}(t,x), \tag{2}$$

where $U_{(a)}^{(i)}(t,x)$ *is a d-tensor field on* E^**, and* $\mathcal{F}(t,x)$ *is a function on* E^**. Then, the multi-time Hamilton space* $\mathcal{NEDM}H_m^n = (E^*, H_2)$ *is called the* ***non-autonomous multi-time Hamilton space of electrodynamics****. The dynamical character of the gravitational potentials* $g_{ij}(t,x)$ *(i.e., the dependence on the temporal coordinates* t^c*) motivated us to use the word* ***"non-autonomous".***

An important role for the subsequent development of our distinguished Riemannian geometrical theory for multi-time Hamilton spaces is represented by the following result (proved in paper [4]):

Theorem 1. *If we have* $m = \dim \mathcal{T} \geq 2$*, then the following statements are equivalent:*

(i) H *is a Kronecker h-regular multi-time Hamiltonian function on* E^**.*

(ii) *The multi-time Hamiltonian function* H *reduces to a multi-time Hamiltonian function of non-autonomous electrodynamic type. In other words we have*

$$H = h_{ab}(t)g^{ij}(t,x)p_i^a p_j^b + U_{(a)}^{(i)}(t,x)p_i^a + \mathcal{F}(t,x). \tag{3}$$

Corollary 1. *The fundamental vertical metrical d-tensor of a Kronecker h-regular multi-time Hamiltonian function* H *has the form*

$$G_{(a)(b)}^{(i)(j)} = \frac{1}{2}\frac{\partial^2 H}{\partial p_i^a \partial p_j^b} = \begin{cases} h_{11}(t)g^{ij}(t,x^k,p_k^1), & m = \dim \mathcal{T} = 1 \\ h_{ab}(t^c)g^{ij}(t^c,x^k), & m = \dim \mathcal{T} \geq 2. \end{cases} \tag{4}$$

We recall that the transformations of coordinates on the dual 1-jet space $J^{1*}(\mathcal{T}, M)$ are given by

$$\widetilde{t}^a = \widetilde{t}^a\left(t^b\right), \quad \widetilde{x}^i = \widetilde{x}^i\left(x^j\right), \quad \widetilde{p}_i^a = \frac{\partial x^j}{\partial \widetilde{x}^i}\frac{\partial \widetilde{t}^a}{\partial t^b}p_j^b,$$

where $\det\left(\partial\widetilde{t}^a/\partial t^b\right) \neq 0$ and $\det\left(\partial\widetilde{x}^i/\partial x^j\right) \neq 0$. In this context, let us introduce the following important geometrical concept:

Definition 3. *A pair of local functions on $E^* = J^{1*}(\mathcal{T}, M)$, denoted by*

$$N = \left(\underset{1}{N}{}^{(a)}_{(i)b}, \ \underset{2}{N}{}^{(a)}_{(i)j} \right),$$

whose local components obey the transformation rules

$$\underset{1}{\widetilde{N}}{}^{(b)}_{(j)c} \frac{\partial \widetilde{t}^c}{\partial t^a} = \underset{1}{N}{}^{(c)}_{(k)a} \frac{\partial \widetilde{t}^b}{\partial t^c} \frac{\partial x^k}{\partial \widetilde{x}^j} - \frac{\partial \widetilde{p}^b_j}{\partial t^a},$$

$$\underset{2}{\widetilde{N}}{}^{(b)}_{(j)k} \frac{\partial \widetilde{x}^k}{\partial x^i} = \underset{2}{N}{}^{(c)}_{(k)i} \frac{\partial \widetilde{t}^b}{\partial t^c} \frac{\partial x^k}{\partial \widetilde{x}^j} - \frac{\partial \widetilde{p}^b_j}{\partial x^i},$$

is called a ***nonlinear connection*** *on E^*. The components $\underset{1}{N}{}^{(a)}_{(i)b}$ (resp. $\underset{2}{N}{}^{(a)}_{(i)j}$) are called the* ***temporal*** *(resp.* ***spatial****)* ***components*** *of N.*

Following now the geometrical ideas of Miron from [15], paper [4] proves that any Kronecker h-regular multi-time Hamiltonian function H produces a natural nonlinear connection on the dual 1-jet space E^*, which depends only on the given Hamiltonian function H:

Theorem 2. *The pair of local functions $N = \left(\underset{1}{N}{}^{(a)}_{(i)b}, \underset{2}{N}{}^{(a)}_{(i)j} \right)$ on E^*, where (χ^a_{bc} are the Christoffel symbols of the semi-Riemannian temporal metric h_{ab})*

$$\underset{1}{N}{}^{(a)}_{(i)b} = \chi^a_{bc} p^c_i,$$

$$\underset{2}{N}{}^{(a)}_{(i)j} = \frac{h^{ab}}{4} \left[\frac{\partial g_{ij}}{\partial x^k} \frac{\partial H}{\partial p^b_k} - \frac{\partial g_{ij}}{\partial p^b_k} \frac{\partial H}{\partial x^k} + g_{ik} \frac{\partial^2 H}{\partial x^j \partial p^b_k} + g_{jk} \frac{\partial^2 H}{\partial x^i \partial p^b_k} \right],$$

represents a nonlinear connection on E^*, which is called the ***canonical nonlinear connection of the multi-time Hamilton space*** *$MH^n_m = (E^*, H)$.*

Taking into account Theorem 1 and using the *generalized spatial Christoffel symbols* of the d-tensor g_{ij} which are given by

$$\Gamma^k_{ij} = \frac{g^{kl}}{2} \left(\frac{\partial g_{li}}{\partial x^j} + \frac{\partial g_{lj}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^l} \right),$$

we immediately obtain the following geometrical result:

Corollary 2. *For $m = \dim \mathcal{T} \geq 2$, the canonical nonlinear connection N of a multi-time Hamilton space $MH^n_m = (E^*, H)$, whose Hamiltonian function is given by (3), has the components*

$$\underset{1}{N}{}^{(a)}_{(i)b} = \chi^a_{bc} p^c_i, \qquad \underset{2}{N}{}^{(a)}_{(i)j} = -\Gamma^k_{ij} p^a_k + T^{(a)}_{(i)j},$$

where

$$T^{(a)}_{(i)j} = \frac{h^{ab}}{4} \left(U_{ib \bullet j} + U_{jb \bullet i} \right), \tag{5}$$

and

$$U_{ib} = g_{ik} U^{(k)}_{(b)}, \qquad U_{kb \bullet r} = \frac{\partial U_{kb}}{\partial x^r} - U_{sb} \Gamma^s_{kr}.$$

3 The Cartan canonical connection $C\Gamma(N)$ of a multi-time Hamilton space

Let us consider that $MH_m^n = (J^{1*}(\mathcal{T}, M), H)$ is a multi-time Hamilton space, whose fundamental vertical metrical d-tensor is given by (4). Let

$$N = \left(\underset{1}{N}{}^{(a)}_{(i)b},\ \underset{2}{N}{}^{(a)}_{(i)j} \right)$$

be the canonical nonlinear connection of the multi-time Hamilton space MH_m^n.

Theorem 3 (the generalized Cartan canonical N-linear connection). *On the multi-time Hamilton space $MH_m^n = (J^{1*}(\mathcal{T}, M), H)$, endowed with the canonical nonlinear connection N, there exists a unique h-normal N-linear connection*

$$C\Gamma(N) = \left(\chi^a_{bc},\ A^i_{jc},\ H^i_{jk},\ C^{i(k)}_{j(c)} \right),$$

having the metrical properties:

(i) $g_{ij|k} = 0, \quad g^{ij}|^{(k)}_{(c)} = 0,$

(ii) $A^i_{jc} = \dfrac{g^{il}}{2}\dfrac{\delta g_{lj}}{\delta t^c}, \quad H^i_{jk} = H^i_{kj}, \quad C^{i(k)}_{j(c)} = C^{k(i)}_{j(c)},$

where "$_{/a}$", "$_{|k}$" and "$|^{(k)}_{(c)}$" represent the local covariant derivatives of the h-normal N-linear connection $C\Gamma(N)$.

Proof. Let $C\Gamma(N) = \left(\chi^a_{bc},\ A^i_{jc},\ H^i_{jk},\ C^{i(k)}_{j(c)} \right)$ be an h-normal N-linear connection, whose local coefficients are defined by the relations

$$\begin{aligned} A^a_{bc} &= \chi^a_{bc}, \qquad A^i_{jc} = \frac{g^{il}}{2}\frac{\delta g_{lj}}{\delta t^c},\\ H^i_{jk} &= \frac{g^{ir}}{2}\left(\frac{\delta g_{jr}}{\delta x^k} + \frac{\delta g_{kr}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^r} \right),\\ C^{j(k)}_{i(c)} &= -\frac{g_{ir}}{2}\left(\frac{\partial g^{jr}}{\partial p^c_k} + \frac{\partial g^{kr}}{\partial p^c_j} - \frac{\partial g^{jk}}{\partial p^c_r} \right). \end{aligned}$$

Taking into account the local expressions of the local covariant derivatives induced by the h-normal N-linear connection $C\Gamma(N)$, by local calculations, we deduce that $C\Gamma(N)$ satisfies conditions (i) and (ii).

Conversely, let us consider an h-normal N-linear connection

$$\tilde{C}\Gamma(N) = \left(\tilde{A}^a_{bc},\ \tilde{A}^i_{jc},\ \tilde{H}^i_{jk},\ \tilde{C}^{i(k)}_{j(c)} \right)$$

which satisfies conditions (i) and (ii). It follows that we have

$$\tilde{A}^a_{bc} = \chi^a_{bc}, \quad \tilde{A}^i_{jc} = \frac{g^{il}}{2}\frac{\delta g_{lj}}{\delta t^c}.$$

Moreover, the metrical condition $g_{ij|k} = 0$ is equivalent with

$$\frac{\delta g_{ij}}{\delta x^k} = g_{rj}\tilde{H}^r_{ik} + g_{ir}\tilde{H}^r_{jk}.$$

Applying now a Christoffel process to indices $\{i, j, k\}$, we find

$$\tilde{H}^i_{jk} = \frac{g^{ir}}{2}\left(\frac{\delta g_{jr}}{\delta x^k} + \frac{\delta g_{kr}}{\delta x^j} - \frac{\delta g_{jk}}{\delta x^r}\right).$$

By analogy, using the relations $C^{i(k)}_{j(c)} = C^{k(i)}_{j(c)}$ and $g^{ij}|^{(k)}_{(c)} = 0$, together with a Christoffel process applied to indices $\{i, j, k\}$, we obtain

$$\tilde{C}^{j(k)}_{i(c)} = -\frac{g_{ir}}{2}\left(\frac{\partial g^{jr}}{\partial p^c_k} + \frac{\partial g^{kr}}{\partial p^c_j} - \frac{\partial g^{jk}}{\partial p^c_r}\right).$$

In conclusion, the uniqueness of the *generalized Cartan canonical connection* $C\Gamma(N)$ on the dual 1-jet space $E^* = J^{1*}(\mathcal{T}, M)$ is clear. □

Remark 2. (i) *Replacing the canonical nonlinear connection N of the multi-time Hamilton space MH^n_m with an arbitrary nonlinear connection $\hat{N}$, the preceding Theorem holds good.*

(ii) *The generalized Cartan canonical connection $C\Gamma(N)$ of the multi-time Hamilton space MH^n_m verifies also the metrical properties*

$$h_{ab/c} = h_{ab|k} = h_{ab}|^{(k)}_{(c)} = 0, \quad g_{ij/c} = 0.$$

(iii) *In the case $m = \dim \mathcal{T} \geq 2$, the coefficients of the generalized Cartan canonical connection $C\Gamma(N)$ of the multi-time Hamilton space MH^n_m reduce to*

$$A^a_{bc} = \chi^a_{bc}, \quad A^i_{jc} = \frac{g^{il}}{2}\frac{\partial g_{lj}}{\partial t^c}, \quad H^i_{jk} = \Gamma^i_{jk}, \quad C^{i(k)}_{j(c)} = 0. \tag{6}$$

4 Local d-torsions and d-curvatures of the Cartan canonical connection $C\Gamma(N)$

Applying the formulas that determine the local d-torsions and d-curvatures of an h-normal N-linear connection $D\Gamma(N)$ (see these formulas in [23]) to the generalized Cartan canonical connection $C\Gamma(N)$, we obtain the following important geometrical results:

Theorem 4. *The torsion tensor $\mathbb{T}$ of the generalized Cartan canonical connection $C\Gamma(N)$*

of the multi-time Hamilton space MH^n_m is determined by the local d-components

	$h_{\mathcal{T}}$	h_M		v	
	$m \geq 1$	$m = 1$	$m \geq 2$	$m = 1$	$m \geq 2$
$h_{\mathcal{T}} h_{\mathcal{T}}$	0	0	0	0	$R^{(f)}_{(r)ab}$
$h_M h_{\mathcal{T}}$	0	T^r_{1j}	T^r_{aj}	$R^{(1)}_{(r)1j}$	$R^{(f)}_{(r)aj}$
$v h_{\mathcal{T}}$	0	0	0	$P^{(1)\ (j)}_{(r)1(1)}$	$P^{(f)\ (j)}_{(r)a(b)}$
$h_M h_M$	0	0	0	$R^{(1)}_{(r)ij}$	$R^{(f)}_{(r)ij}$
$v h_M$	0	$P^{r(j)}_{i(1)}$	0	$P^{(1)\ (j)}_{(r)i(1)}$	0
vv	0	0	0	0	0

where

(i) *for $m = \dim \mathcal{T} = 1$, we have*

$$T^r_{1j} = -A^r_{j1}, \quad P^{r(j)}_{i(1)} = C^{r(j)}_{i(1)}, \quad P^{(1)\ (j)}_{(r)1(1)} = \frac{\partial \underset{1}{N}{}^{(1)}_{(r)1}}{\partial p^1_j} + A^j_{r1} - \delta^j_r \chi^1_{11},$$

$$P^{(1)\ (j)}_{(r)i(1)} = \frac{\partial \underset{2}{N}{}^{(1)}_{(r)i}}{\partial p^1_j} + H^j_{ri}, \quad R^{(1)}_{(r)1j} = \frac{\delta \underset{1}{N}{}^{(1)}_{(r)1}}{\delta x^j} - \frac{\delta \underset{2}{N}{}^{(1)}_{(r)j}}{\delta t},$$

$$R^{(1)}_{(r)ij} = \frac{\delta \underset{2}{N}{}^{(1)}_{(r)i}}{\delta x^j} - \frac{\delta \underset{2}{N}{}^{(1)}_{(r)j}}{\delta x^i};$$

(ii) *for $m = \dim \mathcal{T} \geq 2$, using the equality (5) and the notations*

$$\chi^c_{fab} = \frac{\partial \chi^c_{fa}}{\partial t^b} - \frac{\partial \chi^c_{fb}}{\partial t^a} + \chi^d_{fa}\chi^c_{db} - \chi^d_{fb}\chi^c_{da},$$

$$\mathfrak{R}^r_{kij} = \frac{\partial \Gamma^r_{ki}}{\partial x^j} - \frac{\partial \Gamma^r_{kj}}{\partial x^i} + \Gamma^p_{ki}\Gamma^r_{pj} - \Gamma^p_{kj}\Gamma^r_{pi},$$

we have

$$T^r_{aj} = -A^r_{ja}, \quad P^{(f)\ (j)}_{(r)a(b)} = \delta^f_b A^j_{ra}, \quad R^{(f)}_{(r)ab} = \chi^f_{gab} p^g_r,$$

$$R^{(f)}_{(r)aj} = -\frac{\partial \underset{2}{N}{}^{(f)}_{(r)j}}{\partial t^a} - \chi^f_{ca} T^{(c)}_{(r)j},$$

$$R^{(f)}_{(r)ij} = -\mathfrak{R}^k_{rij} p^f_k + \left[T^{(f)}_{(r)i|j} - T^{(f)}_{(r)j|i}\right].$$

Theorem 5. *The curvature tensor $\mathbb{R}$ of the generalized Cartan canonical connection $C\Gamma(N)$ of the multi-time Hamilton space MH^n_m is determined by the following adapted* *local curvature d-tensors:*

	$h_{\mathcal{T}}$	h_M		v	
	$m \geq 1$	$m = 1$	$m \geq 2$	$m = 1$	$m \geq 2$
$h_{\mathcal{T}} h_{\mathcal{T}}$	χ^d_{abc}	0	R^l_{ibc}	0	$-R^{(d)(i)}_{(l)(a)bc}$
$h_M h_{\mathcal{T}}$	0	R^l_{i1k}	R^l_{ibk}	$-R^{(1)(l)}_{(i)(1)1k} = -R^l_{i1k}$	$-R^{(d)(i)}_{(l)(a)bk}$
$v h_{\mathcal{T}}$	0	$P^{l\ \ (k)}_{i1(1)}$	0	$-P^{(1)(l)\ (k)}_{(i)(1)1(1)} = -P^{l\ \ (k)}_{i1(1)}$	0
$h_M h_M$	0	R^l_{ijk}	$\mathfrak{R}^l_{ijk}$	$-R^{(1)(l)}_{(i)(1)jk} = -R^l_{ijk}$	$-R^{(d)(i)}_{(l)(a)jk}$
$v h_M$	0	$P^{l\ (k)}_{ij(1)}$	0	$-P^{(1)(l)\ (k)}_{(i)(1)j(1)} = -P^{l\ (k)}_{ij(1)}$	0
vv	0	$S^{l(j)(k)}_{i(1)(1)}$	0	$-S^{(1)(l)(j)(k)}_{(i)(1)(1)(1)} = -S^{l(j)(k)}_{i(1)(1)}$	0

where, for $m \geq 2$, *we have the relations*

$$-R^{(d)(i)}_{(l)(a)bc} = \delta^i_l \chi^d_{abc} - \delta^d_a R^i_{lbc}, \quad -R^{(d)(i)}_{(l)(a)bk} = -\delta^d_a R^i_{lbk}, \quad -R^{(d)(l)}_{(i)(a)jk} = -\delta^d_a \mathfrak{R}^l_{ijk},$$

and, generally, the following formulas are true:

(i) *for* $m = \dim \mathcal{T} = 1$, *we have* $\chi^1_{111} = 0$ *and*

$$R^l_{i1k} = \frac{\delta A^l_{i1}}{\delta x^k} - \frac{\delta H^l_{ik}}{\delta t} + A^r_{i1} H^l_{rk} - H^r_{ik} A^l_{r1} + C^{l(r)}_{i(1)} R^{(1)}_{(r)1k},$$

$$R^l_{ijk} = \frac{\delta H^l_{ij}}{\delta x^k} - \frac{\delta H^l_{ik}}{\delta x^j} + H^r_{ij} H^l_{rk} - H^r_{ik} H^l_{rj} + C^{l(r)}_{i(1)} R^{(1)}_{(r)jk},$$

$$P^{l\ \ (k)}_{i1(1)} = \frac{\partial A^l_{i1}}{\partial p^1_k} - C^{l(k)}_{i(1)/1} + C^{l(r)}_{i(1)} P^{(1)\ \ (k)}_{(r)1(1)},$$

$$P^{l\ (k)}_{ij(1)} = \frac{\partial H^l_{ij}}{\partial p^1_k} - C^{l(k)}_{i(1)|j} + C^{l(r)}_{i(1)} P^{(1)\ (k)}_{(r)j(1)},$$

$$S^{l(j)(k)}_{i(1)(1)} = \frac{\partial C^{l(j)}_{i(1)}}{\partial p^1_k} - \frac{\partial C^{l(k)}_{i(1)}}{\partial p^1_j} + C^{r(j)}_{i(1)} C^{l(k)}_{r(1)} - C^{r(k)}_{i(1)} C^{l(j)}_{r(1)};$$

(ii) *for* $m = \dim \mathcal{T} \geq 2$, *we have*

$$\chi^d_{abc} = \frac{\partial \chi^d_{ab}}{\partial t^c} - \frac{\partial \chi^d_{ac}}{\partial t^b} + \chi^f_{ab} \chi^d_{fc} - \chi^f_{ac} \chi^d_{fb},$$

$$R^l_{ibc} = \frac{\partial A^l_{ib}}{\partial t^c} - \frac{\partial A^l_{ic}}{\partial t^b} + A^r_{ib} A^l_{rc} - A^r_{ic} A^l_{rb},$$

$$R^l_{ibk} = \frac{\partial A^l_{ib}}{\partial x^k} - \frac{\partial \Gamma^l_{ik}}{\partial t^b} + A^r_{ib} \Gamma^l_{rk} - \Gamma^r_{ik} A^l_{rb},$$

$$\mathfrak{R}^l_{ijk} = \frac{\partial \Gamma^l_{ij}}{\partial x^k} - \frac{\partial \Gamma^l_{ik}}{\partial x^j} + \Gamma^r_{ij} \Gamma^l_{rk} - \Gamma^r_{ik} \Gamma^l_{rj}.$$

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