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TRANSIENT MHD FREE CONVECTION FLOW WITH HALL CURRENTS PAST A SEMI-INFINITE ISOTHERMAL VERTICAL FLAT PLATE

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1 Introduction

Free convective flows of electrically conducting fluids under transverse magnetic filed have important applications in Nuclear engineering, Cooling system, Solar collectors, Furnaces, Cylindrical heaters, Plasma studies, Heat exchangers. Steady free convective flow of a viscous incompressible fluid past a semi-infinite vertical plate was first studied by Pohlhausen [1] and Ostrach [2]. Transient free convective flow past a semi-infinite vertical plate was studied by Siegel [3] by integral method and Gebhart [4] by approximate method. Unsteady free convective flow with mass transfer past a semi-infinite vertical plate was studied by Callahan and Marner [5]. Soundalgekar and Ganesan [6] restudied the same problem by using implicit finite difference technique.

In this work, we investigate the effect of Hall current on the transient free convective flow past a semi-infinite vertical flat plate of a viscous incompressible and electrically conducting fluid, resulting from buoyancy forces which arise due to temperature differences. This work is undertaken for the first time in the literature. Governing partial differential equations of the flow for unsteady state, have been solved by implicit finite-difference scheme, which is unconditionally stable.

2 Mathematical analysis

Consider an unsteady flow of an incompressible viscous, electrically conducting fluid past a semi-infinite vertical plate in the presence of a strong non uniform magnetic field normal to the plate. Initially, fluid is at the temperature T_∞. At time $t' \succ 0$, the temperature of the plate is suddenly raised to T_W from T_∞ and maintained at the same value.

For weakly ionized gases, the thermoelectric pressure and ion slip are considered negligible. It is also assumed that viscous and electrical dissipation are negligible.

Using rectangular Cartesian coordinate system, we take X-axis along the plate, Y-axis normal to the plate and Z-axis coincides with leading edge of the plate The effects of Hall

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current give rise to a force in the Z- direction, inducing a cross flow in that direction, making the flow three-dimensional.

To simplify the problem, we assume there is no variation of flow and heat transfer in the Z-direction. We also assume that viscous and electrical dissipation are negligible. The equation of conservation of electric charge $\nabla \cdot \overline{J} = 0$ gives y component of electric current density vector $\overline{J}$ as constant, which is in fact zero every where in the flow as $J_y = 0$ at the plate which is electrically non-conducting.

Under the Boussinesq's approximation, the flow can be shown to be governed by following boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) - \frac{\sigma B_y^2}{\rho\left(1 + m^2\right)}\left(u + mw\right), \tag{2.2}$$

$$\frac{\partial w}{\partial t'} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \nu\frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_y^2}{\rho(1 + m^2)}(w - mu), \tag{2.3}$$

$$\frac{\partial T}{\partial t'} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_P}\frac{\partial^2 T}{\partial y^2}. \tag{2.4}$$

The initial and boundary conditions are:

$$\left.\begin{array}{llllll} t' \leq 0, & w = 0, & u = 0, & v = 0, & T = T_\infty, & \text{for all } y \\ t' \succ 0, & w = 0, & u = 0, & v = 0, & T = T_\infty, & \text{at } x = 0 \\ & w = 0, & u = 0, & v = 0, & T = T_w, & \text{at } y = 0 \\ & w = 0, & u = 0, & & T \to T_\infty. & \text{as } y \to \infty \end{array}\right\} \tag{2.5}$$

Here u , v and w are the velocity components of the fluid along the x , y and z-directions respectively. T is the temperature of the fluid, T_∞ is the temperature of the fluid far away from the plate, ρ shows the density, m is the Hall parameter, $\alpha = \frac{k}{\rho c_P}$ is the thermal diffusivity, β denotes the coefficient of volume expansion, and g is the acceleration due to gravity.

Non-dimensional quantities are introduced by:

$$\left.\begin{array}{lll} X = \frac{xu_\infty}{\nu}, & Y = \frac{yu_\infty}{\nu}, & U = \frac{u}{u_\infty}, \\ W = \frac{w}{w_\infty}, & V = \frac{v}{v_\infty}, & B_y, \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, & t = \frac{t'(u_\infty)^2}{\nu}, & u_\infty = \frac{\nu X}{x}, \\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{(u_\infty)^3}, & M = \frac{\sigma B_0^2 \nu}{\rho u_\infty^2}. & \end{array}\right\} \tag{2.6}$$

Using (2.6) in equations (2.1)-(2.4), they are reduced to following equations:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{2.7}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+Gr\theta-\frac{M}{1+m^2}\cdot\frac{1}{X^2}\left(U+mW\right), \tag{2.8}$$

$$\frac{\partial W}{\partial t}+U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}=\frac{\partial^2 W}{\partial Y^2}-\frac{M}{1+m^2}\cdot\frac{1}{X^2}\left(W-mU\right), \tag{2.9}$$

$$\frac{\partial \theta}{\partial t}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\cdot\frac{\partial^2 \theta}{\partial Y^2}. \tag{2.10}$$

The initial and boundary conditions are:

$$\left.\begin{array}{lllll} t\le 0, & U=V=0, & W=0, & \theta=0, & \text{for all } Y\\ t\succ 0, & U=V=0, & W=0, & \theta=0, & \text{at } X=0\\ & U=V=0, & W=0, & \theta=1, & \text{at } Y=0\\ & U=0, & W=0, & \theta=0. & \text{as } Y\to\infty \end{array}\right\} \tag{2.11}$$

3 Method of solution

We aim to solve the unsteady non-linear coupled partial differential equations. We employ an implicit finite difference method of Crank-Nicolson type. We define semi-infinite plate as $L = 1.0$ with a rectangular region with X varying from 0 to 1.0 and Y varying from 0 to Y_{Max}. The value of $X = 1.0$ corresponds to the height of the plate and $Y_{Max} = 20$ is regarded as ∞. It is ensured that Y_{Max} lies well outside the momentum and thermal boundary layers.

The implicit finite-difference equations of Crank-Nicholson type corresponding to equation (2.10) is:

$$\left.\begin{array}{l} \frac{\theta_{i,j}^{n+1}-\theta_{i,j}^{n}}{\Delta t}+\frac{U_{i,j}^{n}}{2}\left[\frac{\theta_{i,j}^{n+1}-\theta_{i-1,j}^{n+1}+\theta_{i,j}^{n}-\theta_{i-1,j}^{n}}{\Delta X}\right]+\frac{V_{i,j}^{n}}{2}\left[\frac{\theta_{i,j+1}^{n+1}-\theta_{i,j-1}^{n+1}+\theta_{i,j+1}^{n}-\theta_{i,j-1}^{n}}{2\Delta Y}\right]\\ =\frac{1}{\Pr}\left[\frac{\theta_{i,j-1}^{n+1}-2\theta_{i,j}^{n+1}+\theta_{i,j+1}^{n+1}+\theta_{i,j-1}^{n}-2\theta_{i,j}^{n}+\theta_{i,j+1}^{n}}{2(\Delta Y)^2}\right]. \end{array}\right\} \tag{3.12}$$

which is simplified to:

$$A_1\theta_{i,j-1}^{n+1}+B_1\theta_{i,j}^{n+1}+D_1\theta_{i,j+1}^{n+1}=E_1, \tag{3.13}$$

where:

$$A_1 = -\frac{V_{i,j}^n}{4\Delta Y} - \frac{1}{2\,\mathrm{Pr}\,(\Delta Y)^2},$$
$$B_1 = \frac{1}{\Delta t} + \frac{U_{i,j}^n}{2\Delta X} + \frac{1}{\mathrm{Pr}\,(\Delta Y)^2},$$
$$D_1 = \frac{V_{i,j}^n}{4\Delta Y} - \frac{1}{2\,\mathrm{Pr}\,(\Delta Y)^2},$$
$$E_1 = \frac{\theta_{i,j}^n}{\Delta t} - \frac{U_{i,j}^n}{2}\left[\frac{\theta_{i,j}^n - \theta_{i-1,j}^n - \theta_{i-1,j}^{n+1}}{\Delta X}\right] - \frac{V_{i,j}^n}{4}\left[\frac{\theta_{i,j+1}^n - \theta_{i,j-1}^n}{\Delta Y}\right] + \frac{1}{\mathrm{Pr}}\left[\frac{\theta_{i,j-1}^n - 2\theta_{i,j}^n + \theta_{i,j+1}^n}{2\,(\Delta Y)^2}\right].$$

Similarly, we use the finite-difference for all equations (2.7) ,(2.8),(2.9).

Then, the initial and boundary conditions are:

$$\left.\begin{array}{llll} \theta_{i,j}^0 = 0 & U_{i,j}^0 = 0 & W_{i,j}^0 = 0 & V_{i,j}^0 = 0 \\ \theta_{i,0}^n = 1 & U_{i,0}^n = 0 & W_{i,0}^n = 0 & V_{i,0}^n = 0 \\ \theta_{0,j}^n = 0 & U_{0,j}^n = 0 & W_{0,j}^n = 0 & V_{0,j}^n = 0 \\ \theta_{i,N}^n = 0 & U_{i,N}^n = 0 & W_{i,N}^n = 0 & \end{array}\right\} \tag{3.14}$$

Here the subscript i designates the grid-point along X-coordinate $\sum_{k=1}^{i} \Delta X_K$, j designates the grid-point along Y-coordinate $\sum_{k=1}^{j} \Delta Y_K$and the subscript n designates a value of time $t = n\Delta t$.

During the computations, the coefficients $U_{i,j}^n$, $W_{i,j}^n$, and $V_{i,j}^n$ appearing in equations.

4 Results and discussion

We have computed the time required to reach the steady state for temperature θ, axial velocity U and transverse velocity W for different values of all the parameters. Initially the time step Δt was chosen as 0.01 and the time required to reach the steady state was calculated. Then to check the accuracy of the result, Δt was chosen as 0.005. The time required to reach the steady state remained unchanged for all the variables. Convergence criterion is $\varepsilon = 10^{-5}$. Whenever the difference in the values of the variables for two successive time steps is less than 10^{-5}, for all values of $X$and Y we presume that steady state is reached.

In ***FIG.1.***, the transient temperature profiles are shown for $X = 1.0$, $M = 0.5$, $Gr = 0.5$ and different values of Hall parameter m. It is observed that when $m = 0.5$, time required to reach the steady state is $t = 4.53$. This shows that as m increases from 0.5 to ∞, time required to reach the steady state increases. An increase in m, leads to decrease in temperature.

From ***FIG.2.***, we study the effects of magnetic field parameter M on temperature profiles for $m = 0.5$. For $M = 1.0$ time required to reach the steady state is $t = 5.99$. As

M decreases, there is a fall in the time required to reach steady state and also there is a fall in temperature.

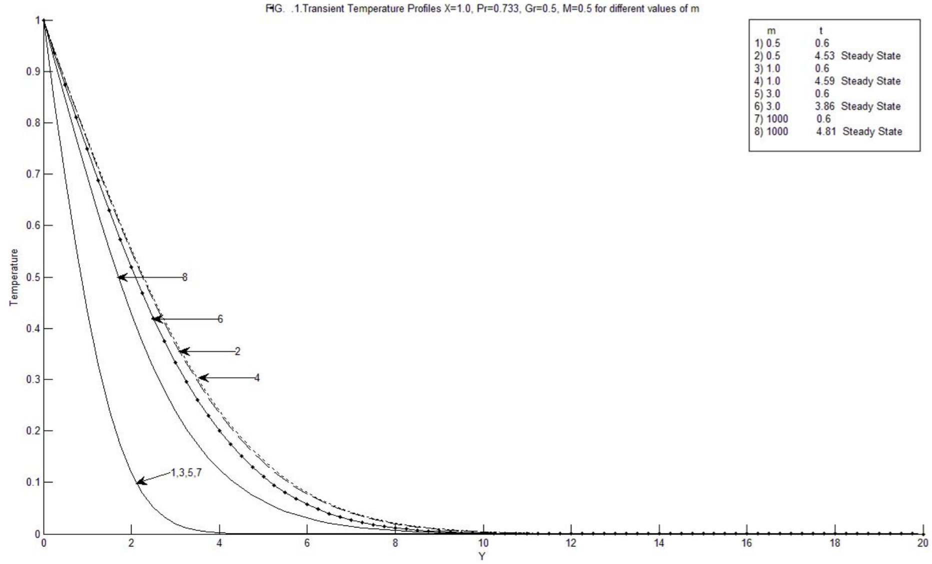
Transient axial velocity profiles are shown in ***FIG.3.*** for $M = 0.5$, $Gr = 0.5$ and different values of m. It is observed that steady state velocity increases as m increases.

FIG.4., shows the transient axial velocity profiles for $m = 0.5$ and different values of M. Steady state axial velocity increases as M decreases. We observe from this figure that as M increases, time required to reach the steady state increases and also axial velocity decreases.

FIG.5., shows Local skin-friction profiles for $\Pr = 0.733$,$Gr = 0.5$ and different values of m and M. It clearly shows that Local skin-friction increases as m decreases, for same values of M. Also, it increases as M decreases, for same values of m. We observe from this figure that Local skin-friction is at maximum without magnetic field and Hall parameter $(m = M = 0.0)$ and it is at minimum when $(m = M = 1.0)$.

FIG.6., shows Local Nusselt number for $\Pr = 0.733$, $Gr = 0.5$ and different values of m and M. We observe from this figure that Local Nusselt number is at maximum for $m = M = 1.0$ and at minimum for $m = M = 0.0$. For same values of m, Local Nusselt number decreases as M decreases. Also, it increases as m increases for same values of M.

FIG. .1.Transient Temperature Profiles X=1.0, Pr=0.733, Gr=0.5, M=0.5 for different values of m



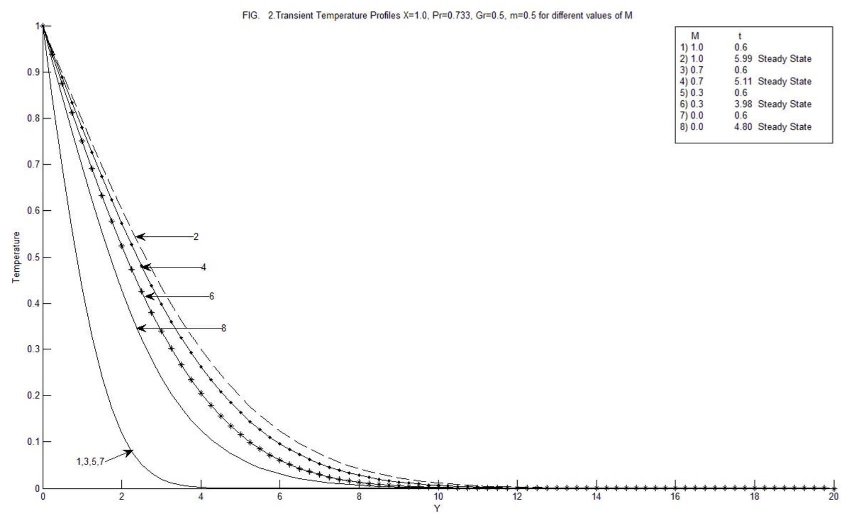


FIG. 2.Transient Temperature Profiles X=1.0, Pr=0.733, Gr=0.5, m=0.5 for different values of M
M t
1) 1.0 0.6
2) 1.0 5.99 Steady State
3) 0.7 0.6
4) 0.7 5.11 Steady State
5) 0.3 0.6
6) 0.3 3.98 Steady State
7) 0.0 0.6
8) 0.0 4.80 Steady State
Temperature
Y
1,3,5,7

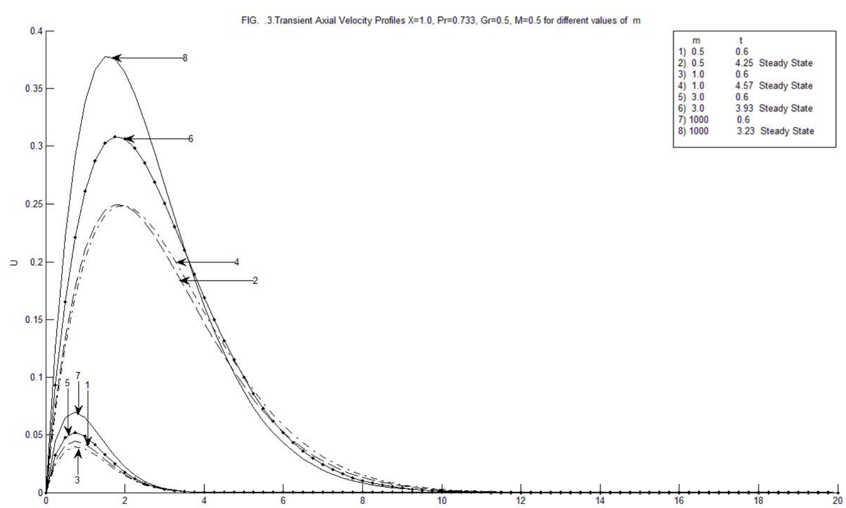


FIG. 3.Transient Axial Velocity Profiles X=1.0, Pr=0.733, Gr=0.5, M=0.5 for different values of m
m t
1) 0.5 0.6
2) 0.5 4.25 Steady State
3) 1.0 0.6
4) 1.0 4.57 Steady State
5) 3.0 0.6
6) 3.0 3.93 Steady State
7) 1000 0.6
8) 1000 3.23 Steady State
U
Y

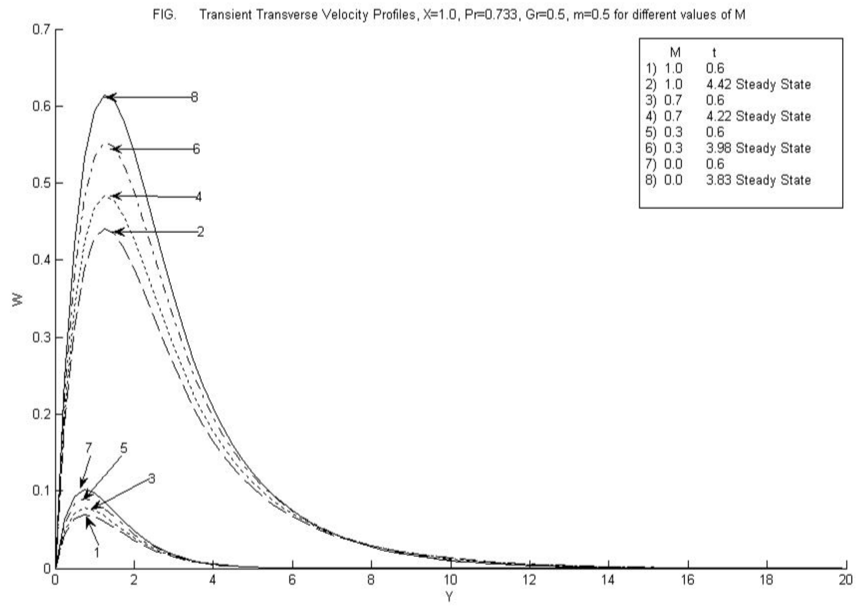


FIG. Transient Transverse Velocity Profiles, X=1.0, Pr=0.733, Gr=0.5, m=0.5 for different values of M
M t
1) 1.0 0.6
2) 1.0 4.42 Steady State
3) 0.7 0.6
4) 0.7 4.22 Steady State
5) 0.3 0.6
6) 0.3 3.98 Steady State
7) 0.0 0.6
8) 0.0 3.83 Steady State
W
Y

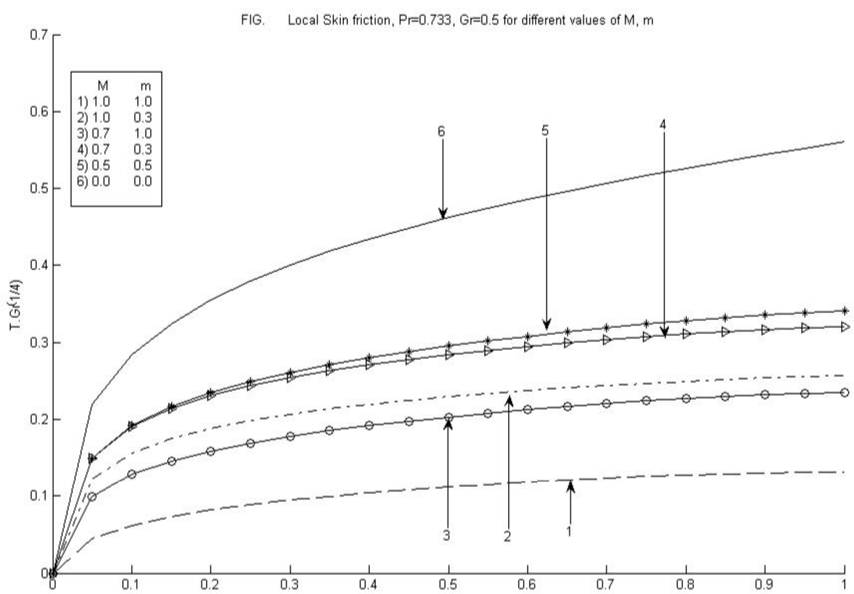
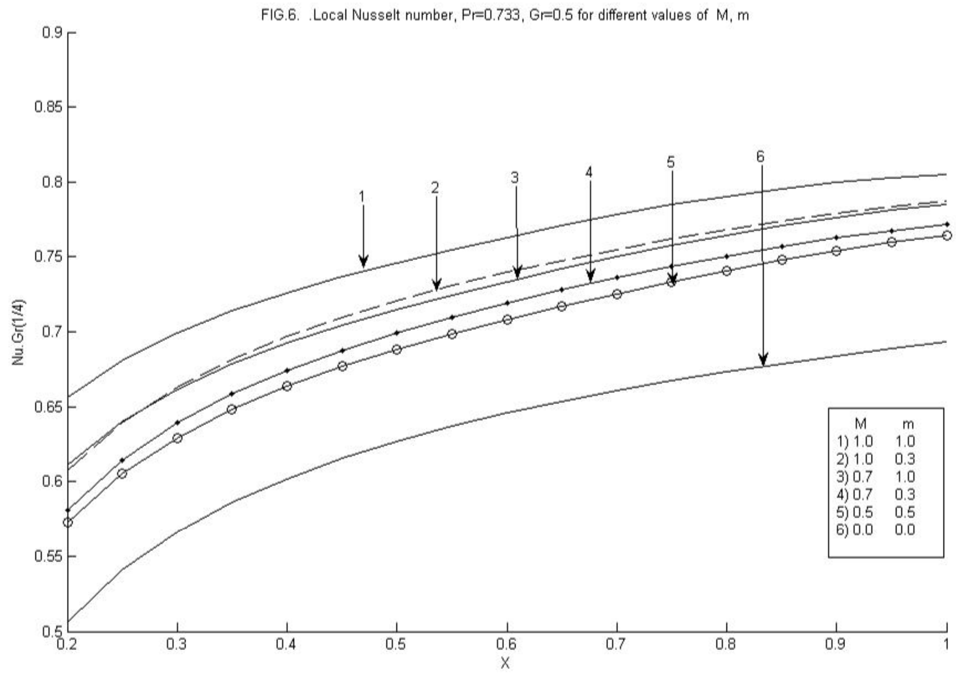


FIG. Local Skin friction, Pr=0.733, Gr=0.5 for different values of M, m
M m
1) 1.0 1.0
2) 1.0 0.3
3) 0.7 1.0
4) 0.7 0.3
5) 0.5 0.5
6) 0.0 0.0
T.G(1/4)
X



5 Conclusions

1. As m increases, time required to reach the steady state temperature increases.
2. An increase in m, leads to decrease in temperature.
3. As M decreases, there is a fall in temperature.
4. As m increases, transient transverse velocity increases.
5. Local skin-friction increases, as M and m decreases.
6. Local Nusselt number decreases, as M and m decreases.

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