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THE CONNECTIVE COVER OF THE STELLAR MODEL HAVING THE CHEMICAL COMPOSITION: $X = 0.7405$ AND $Z = 0.0135$

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Abstract

In this paper we present the way in which the integrated equations describe the structure of the convective cover. We give numerical results on the convective cover considering null and not-null conditions at the surface of the star.

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1 Formulation of the problem

We consider a model of the star having the radiative nucleus and a convective cover. In [12] the way we have obtained the numerical results for the radiative nucleus is showed.

For the whole convective cover of the star, we assume the validity of the equation of the hydrostatic equilibrium, the equation of the mass distribution and the adiabatic equation (see, e.g. [1] , [7] , [9]).

$$\begin{aligned}
\frac{dP(r)}{dr} &= -\frac{GM(r)\rho(r)}{r^2} \qquad (1)\\
\frac{dM(r)}{dr} &= 4\pi r^2 \rho(r)\\
P(r) &= K \cdot \rho^{\gamma}(r)\\
& \quad or\\
P(r) &= K_1 \cdot T(r)^{\frac{\gamma}{\gamma-1}}
\end{aligned}$$

with $\gamma = \frac{5}{3}$ and the law of the perfect gas is assumed to hold:

$$P(r) = \frac{k}{\mu \cdot H}\rho(r) \cdot T(r) \qquad (2)$$

Using Schwarzschild's equations [9]:

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$$\begin{aligned} P(r) &= p\frac{GM^2}{4\pi R^4} \\ T(r) &= t\frac{\mu H}{k}\frac{GM}{R} \\ M(r) &= q \cdot M, \\ r &= x \cdot R \end{aligned} \tag{3}$$

We apply these equations to system (1) ,where x, q, t, p are adimensional variables. System (1) becomes:

$$\begin{aligned} \frac{dp}{dx} &= -\frac{pq}{tx^2}, \\ \frac{dq}{dx} &= \frac{px^2}{t}, \\ p &= E \cdot t^{2.5} \end{aligned} \tag{4}$$

or

$$\frac{dt}{dx} = -\frac{1}{2.5E} \cdot \frac{pq}{t^{2.5}x^2}.$$

System (4) can be integrated using the null conditions for pressure and temperature at the surface of the star:

$$\begin{aligned} x &= 1 \\ t &= p = 0 \\ q &= 1 \end{aligned} \tag{5}$$

or using the not-null conditions at the surface of the star. Due to the fact that pressure is not completely known at the base of the atmosphere of the star, we have assumed $T(R) = T_{ef} = 5785^oK$ and we did some interpretations for (4) using different values for E.

At the surface of the star we have $r = R$ and from (3) we obtain:

$$t = \frac{k \cdot R}{\mu HGM} \cdot T(R) = \frac{k \cdot R \cdot T_{ef}}{\mu HGM}. \tag{6}$$

The not-null conditions at the surface of the star are:

$$\begin{gathered} x = 1 \\ q = 1 \\ t = \frac{1.3805.6.96 \cdot 5.785}{1.672 \cdot 6.67 \cdot 1.99 \cdot 10^4} \cdot \frac{1}{\mu} \end{gathered} \tag{7}$$

where t determined in (7) is obtained from (6) by replacing the following constants k, H, G, M, R and T_{ef} with the values presented in the paper. The values of M and R are the same with the ones of the actual Sun, and T_{ef} is close to $T_{ef\odot} = 5770K$.

The mean molecular weight is measured in proton masses, is given by [9]:

$$\frac{1}{\mu} = 2X = \frac{3}{4}Y + \frac{1}{2}Z. \tag{8}$$

Eliminating the proportion of the helium Y from the following relation:

$$X = Y = Z = 1 \tag{9}$$

we obtain for μ the next formulae:

$$\mu = \frac{4}{3 + 5X - Z}. \tag{10}$$

For the radiative nucleus [12], and also for the convective cover, we have assumed:

$$\begin{aligned} X &= 0.7405 \\ Z &= 0.0135 \end{aligned} \tag{11}$$

representing the proportion of the hydrogen, respectively the abundance of metals.

We introduce three new parameters:

$$\begin{aligned} U &= \frac{d \log M(r)}{d \log r} \\ V &= \frac{d \log P(r)}{d \log r} \\ (n+1) &= \frac{d \log P(r)}{d \log T(r)}. \end{aligned} \tag{12}$$

After making the calculations in (12) , we obtain:

$$\begin{aligned} U &= 4\pi r^3 \frac{\rho(r)}{M(r)} = \frac{px^3}{qt} \\ V &= \frac{\rho(r)}{P(r)} \cdot \frac{GM(r)}{r} = \frac{q}{tx} \end{aligned} \tag{13}$$

2 The numerical solution of the problem

Starting with the integration of the equations of the radiative nucleus [12] from center, it stops when $(n = 1)_{rad} = 2.5$. In the point where the integration of the equation of the nucleus has stopped and where we fit the solution of the radiative nucleus with the one of the convective cover, we obtain the values [12]:

$$\begin{aligned} U_0 &= 0.006 \\ V_0 &= 24.866. \end{aligned} \tag{14}$$

2.1 The integration of the equation of the convective cover using null conditions at the surface of the star

We use for the system (4) the limit conditions (5). System (4) has a nondetermination of the type $\frac{0}{0}$ for $x = 1$. Using the development in Taylor series around the point $x = 1$, we obtained:

$$\begin{aligned} p\left(x\right) &= \frac{E}{\left(2.5\right)^{2.5}} \cdot \left(1-x\right)^{2.5} + ... \\ q\left(x\right) &= 1 - \frac{E}{\left(2.5\right)^{2.5}} \cdot \left(1-x\right)^{2.5} + ... \\ 2t\left(x\right) &= \frac{1}{2.5}\left(1-x\right) + \frac{14E}{4+25E}\left(1-x\right)^{2} + ... \end{aligned} \tag{15}$$

From (15) we have obtained the values of three parameters p, q and t in a point around $x - 1$. Next, the integration of the system (4) is made using the Runge-Kutta method (see,e.g. [8],[12]). We make integration choosing different values for the constant E.

I each point of integration x_j, we calculate $U\left(x_j\right)$ and $V\left(x_j\right)$. We integrate system (4) until:

$$V\left(x_{ji}\right) < V_0 \tag{16}$$

In point x_j in which we apply (16), we verify if

$$\left|U\left(x_j\right) - U_0\right| < \varepsilon_1, \varepsilon_1 = 10^{-5}. \tag{17}$$

If condition (17) is fulfilled, we consider the integration of another value for E,value which we note E_1.

$$\begin{aligned} E_1 &= E + h_1 \\ \text{if } U_0 - U\left(x_j\right) &> \varepsilon_1 \\ E_1 &= E - h_1 \\ \text{if } U_0 - U\left(x_j\right) &< -\varepsilon_1. \end{aligned} \tag{18}$$

At the begining, we consider $h_1 = 0.2$, and if condition (17) is not satisfied, we start a new integration of $\frac{h_1}{2}$.The value for $h_1 = 0.2$ was chosen by the author. There is the possibility to choose any other value for h_1, and the following algorithm is the classical one of splitting in half the interval, which is in our case $[0; 0.2]$. Between the constant h_1 and the integration step h there is no connection. The integration step is

$$\begin{aligned} h &= 0.001032 \\ x_j &= 1 - j \cdot h. \end{aligned} \tag{19}$$

2.2 The integration of the equation of the convective cover using not-null conditions at the surface of the star

We integrate system (4) using the conditions (7) for the surface of the star. The integration is done by choosing different values for parameter E, and considering the conditions (16)−(18) . System (4) has no longer a singularity in the point $x = 1$,which means that the integration starts directly with the Runge-Kutta method.

3 Results and conclusions

If we consider the null conditions (5) for pressure and temperatuse at the surface of the star, we obtain:

$$E = 0.80$$

and Table 1 presents the results for pressure P, the reduced mass q, the temperature T and the density ρ, which correspond to the convective cover.

Table 1

x	P	q	T	ρ
1	0	1	0	0
0.999	$1.2031.10^{-11}$	1	0.0107	$8.2503.10^{-6}$
0.9952	$1.9001.10^{-10}$	1	0.0325	$4.3715.10^{-5}$
0.9848	$2.3140.10^{-9}$	1	0.0875	$1.9314.10^{-4}$
0.9748	$8.0421.10^{-9}$	1	0.1454	$4.0025.10^{-4}$
0.9646	$1.8325.10^{-8}$	1	0.1980	$6.5381.10^{-4}$
0.9546	$3.5042.10^{-8}$	1	0.2681	$9.7603.10^{-4}$
0.9444	$6.0031.10^{-8}$	1	0.3241	$1.3241.10^{-3}$
0.9344	$9.0245.10^{-8}$	0.9999	0.3846	$1.7238.10^{-3}$
0.9242	$1.3781.10^{-7}$	0.9999	0.4442	$2.2610.10^{-3}$
0.9142	$1.8725.10^{-7}$	0.9998	0.5103	$2.6532.10^{-3}$
0.904	$2.7881.10^{-7}$	0.9998	0.5901	$3.3721.10^{-3}$
0.899	$3.2215.10^{-7}$	0.9997	0.6102	$3.6204.10^{-3}$

If we consider the not-null conditions (7) for pressure and temperature from the surface of the star, we obtain:

$$E = 0.82$$

and Table 2 presents the numerical results for pressure, reduced mass, temperature and density.

Table 2

x	P	q	T	ρ
1	$2.5545.10^{-12}$	1	$5.785.10^{-3}$	$3.2560.10^{-6}$
0.999	$1.4272.10^{-11}$	1	0.0114	$9.0241.10^{-6}$
0.9898	$9.7392.10^{-10}$	1	0.0627	$1.1224.10^{-4}$
0.9796	$5.1076.10^{-9}$	1	0.1208	$3.1041.10^{-4}$
0.9694	$1.4501.10^{-8}$	1	0.1801	$5.8145.10^{-4}$
0.9592	$2.9147.10^{-8}$	1	0.2409	$8.8010.10^{-4}$
0.9490	$5.2014.10^{-8}$	1	0.3025	$1.2349.10^{-3}$
0.9388	$8.2301.10^{-8}$	0.9999	0.3660	$1.6704.10^{-3}$
0.9286	$1.2452.10^{-7}$	0.9999	0.4301	$2.1045.10^{-3}$
0.9184	$1.8241.10^{-7}$	0.9999	0.4965	$2.5819.10^{-3}$
0.9	$3.1045.10^{-7}$	0.9998	0.6011	$3.5002.10^{-3}$

In Table 1 and 2, pressure P is calculated in units of $10^{18}\frac{dyne}{cm^2}$, the temperature T in units of $10^6\ {}^\circ K$, the density ρ in $\frac{gr}{cm^3}$, and q is the reduced mass.

We will compare the results that we have obtained in this paper with the ones from [12] in the fitting point x_j. Table 4 exhibits the values that correspond to the point x_i, using the null conditions at the surface of the star, and Table 5 exhibits the corresponding results using the not-null conditions at the surface of the star.

Table 3

x_i	P	q	T	ρ
0.8952	$0.375E-6$	0.9997	0.7231	0.0037

Table 4

x_i	P	q	T	ρ
0.899	$3.2215E-6$	0.9997	0.6102	$3.6204E-3$

Table 5

x_i	P	q	T	ρ
0.9	$3.1045E-7$	0.9998	0.6011	$3.5002E-3$

On the one hand, by composing the results which have been presented in Table 3-5, we can conclude that the results that we have obtained in this paper are in accordance with the ones that have been obtained in [12] for the radiative nucleus. On the other hand, another conclusion that can be expressed is that the utilisation of the null conditions at the surface of the star is not restrictive and the corresponding numerical results are good.

The values of the constants that appear in this paper are:

$$\begin{aligned} G &= 6.672 \cdot 10^{-8} cm^3 g^{-1} s^{-1} \\ R &= 6.96 \cdot 10^{10} cm \\ H &= 1.6725 \cdot 10^{-24} gr \\ M &= 1.99 \cdot 10^{33} gr \\ k &= 1.3805 \cdot 10^{-16} \frac{erg}{K}. \end{aligned}$$

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