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FRACTAL MODEL FOR SIMULATION AND INFLATION CONTROL

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Abstract

The theory of chaos and fractals are completing each other. The fractal geometry can be seen as a language that describes models and analyzes complex forms from nature. The basics of fractal geometry are algorithms that can be visualized as structures and different forms using the computer.

The simplest example of a nonlinear iteration procedure in a complex number is given by the transformation $z \to z^2$. Using the transformation $z \to z^2$, we reach a dynamic dichotomy: the complex plane of initial values is divided into two subsets, one with points for which the iteration escapes, called *the escape set E*, and the other one with points for other initial values that remain in a bounded region forever, called *the prisoner set P*. The bounded between E and P is called *the Julia set of the iteration.*

The Julia set for the parameter c is built of the iteration $f_c(z) = z^2 + c$. In our case study, c is a complex number of the form $c = a + i \cdot b$, where a is the *Consumer Price Index* and b is the *Inflation Rate* between the years 1991-2013.

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1 Introduction

In this paper we propose a fractal model for the economy field. The model can help the managers to take a correct decision in real time about Inflation Rate and can show how Consumer Price Index can influence this indicator. The advantage is that the results are presented through suggestive images.

The paper is organized as follows: in Section 2 we present a mathematical background about fractals and Julia sets, in Section 3 we give the formulas for Consumer Price Index and Inflation Rate and the evolution of these two indicators between the years 1991-2013. The data sets will be used in our case study in Section 4, that presents the results obtained with a Java software program that generates the Julia sets. The Julia software follows the algorithm presented in section 2. Finally, in Section 5 we present our conclusion.

2 Fractal theory and Julia sets

The chaos theory has known a very good development in the mathematics field. This is explained by the fact that systems apparently simple can model the reality that is very complex (chaotic). From this point of view, the numerical analysis of time series represents the start for different applications in many fields, like: economic sciences, medicine, financial field, etc. Real systems are defined by the following characteristics:

- they are dynamical, it means that they can be changed;
- they are complex, it means that they depend on many parameters;
- they are iterative, it means that the laws that lead the behavior can be described by feedback.

The interactions of such a dynamic system cannot be described completely by using mathematical formulas, but the researches are focused on the regularities that form the system and, particularly, on finding simple mathematical models that can simulate the system behavior. Generally, the notion *chaotic* has the meaning of unpredictability. A chaotic behavior is based on the fact that small changes in the quantity can produce unexpected chaotic effects. The research area in this field is interdisciplinary.

For understanding the iterations of a polynomials like $x^2 + c$ or $x^3 + c$, etc., where $c = a + b \cdot i \in \mathbb{C}$, the Julia sets are crucial. Julia sets are defined in the complex plane ([9]).

The complex plane of the initial points is divided into two subsets: a subset with points for which the iteration escapes (the escape set E), and a subset with points for other initial values that remain in a bounded region forever (the prisoner set P). The P set represents the disc around 0, with center in origin and radius 1. The E set contains the points that do not belong to the disc. The border between the sets E and P forms the circle of radius 1. In this context we obtain the Julia set for the iteration. The set P can be considered an attractor and the Julia set is the border between the points that escape and those that are prisoners. The prisoner points are colored using a different color and in this way we can visualize the Julia set as the border around the prisoner set, for each value $c \in \mathbb{C}$.

We have the following notations for $c \in \mathbb{C}$:
$E_c = \{z_0 | |z_n| \to \infty\}$ - the escape set;
$P_c = \{z_0 | z_0 \notin E_c\}$ - the prisoner set;
$z_{n+1} = {z_n}^2 + c$ - is the recurrence relationship.

For every point in plan we will draw an orbit. The points z_k that belong to P_c will be colored in black and the points z_k for which the orbit will converge to ∞ will belong to the E_c set. The problem is to determine the number of iterations for which the point will converge to ∞.

Let us consider a number $r(c) = max\{|c|, 2\}$. We have the following situations:

- If $|z_k| > r(c)$ then the point belongs to E_c;
- If $|z_k| \leq r(c)$ then the point belongs to P_c.

In practice, we will stop at a certain number of iterations, and if the point does not belong to the outside of the circle, then we will consider it from the set P_c.

From [8] we have the following theorem:

Theorem 1. *Let $z \in \mathbb{C}$ be a complex number not less than c and greater than 2 in absolute value. Then z is an escaping point for the iteration $z \to z^2 + c$.*

We have implemented a Java program in order to delimit the points P_c. The main steps are the following:

- We chose a parameter $c \in \mathbb{C}$;
- We calculate $r(c)$;
- The orbit must escape to infinity if any one point in an orbit for $z \to z^2 + c$ is larger than $r(c)$;
- We define a region for the complex plane that will be placed on a surface with pixels. The pixels correspond to the initial points for the testing orbits. We verify if the initial points $|z_0| > r(c)$ exist. They are the escape points.
- The first approximation of the set P_c is the disc with radius $r(c)$ centered in origin. For this approximation we introduce the notation ${Q_c}^0 = \{z_0 | |z_0| \leq r(c)\}$. We will make one iteration for every remained pixel and we will obtain one z_1;
- We apply again the rule for the orbits that escape;
- We obtain the set ${Q_c}^{-1} = \{z_0 | |z_1| \leq r(c)\}$;
- We repeat a number of iterations: ${Q_c}^{-k} = \{z_0 | |z_k| \leq r(c)\}$. For $k \to \infty$ we obtain the prisoner P_c set.

3 Consumer Price Index (CPI) and Inflation Rate (IR)

Inflation is an obstacle in the way of implementing economic policies of economic growth, because forecasts can no longer be carried out correctly. That means a waste of resources and no confidence in the Government's macroeconomic policies.

Inflation can be measured through several indicators, like Consumer Price Index (CPI), Production Price Index (PPI), and the General Price Index (GPI). Consumer price index (CPI) measures the evolution of prices of a basket of goods to expenditure incurred by a representative household. The components of this basket and their share in total expenses are determined by the National Statistics Institute on the basis of studies conducted by the survey of households in Romania. According to the methodology applied in Romania, the CPI is calculated based on the volume and structure of public expenditure in the sample family budget. Most often, the CPI is calculated as a Laspeyres index (with weights in the base period) ([1]): $CPI = \frac{\sum p_{i1} q_{i0}}{\sum p_{i0} q_{i0}}$, where p_i prices are the prices of goods and services purchased in the base period (p_{i0}) and in the current period (p_{i1}); q_{i0} represent the quantities purchased in the base period.

Consumer price indices measure the overall evolution of prices of goods bought and tariffs of services used by people in a certain period (current period) from a previous period (base or reference) ([2]):

- with the mobile base: a series showing the comparison is made between the sizes of sequentially adjacent periods;

- fixed base indices: the index that is part of a series that is constantly comparing the size of the base period, which remains unchanged.

The Consumer Price Index used to measure the monthly inflation rate, the real indicators of private consumption calculation, the calculation of real income, real wages, real pension, the indexation of wages, pensions, etc. Not every price increase is inflationary, but only a cumulative, long-time increase. The inflation is characterized by simultaneous increase in general prices and a decrease in the purchasing power of money. The rate inflation formula is: $IR = CPI - 100$.

The economic consequences of inflation affect society as a whole. These consequences can be both negative and positive. The negative consequences of inflation are felt by buyers, who have to pay higher prices for the goods purchased. Most affected are low and/or fixed income buyers. The positive consequences of inflation are felt by borrowers because the reimbursement of amounts borrowed in terms of purchasing power of money. Economic agents who convert cash availability to more stable currencies of other countries and turn them after some time in local currency, will gain from the difference between the domestic inflation rate ([3]).

The final objective of the fight against inflation first pursues the sluggish growth in prices and then removes the causes that triggered this process. For this purpose policies against inflation are adopted. Starting from the different dimensions of inflation, its reduction and policies against inflation are different from country to country and can be approached from many angles([11]).

Inflation Rate in Romania is reported by the National Institute of Statistics. Inflation averaged 53.48 Percent from 1991 until 2014, reaching an all time high of 316.90 Percent in November of 1993 and a record low of 1.04 percent in March of 2014. We design a model for managers to take a correct decision in real time about inflation rate and how CPI can influence this indicator, who can be a real tool for all economic agents because we can compare very fast the report between CPI and IR. Another advantage is that the results are presented through images. The study period is between 1991 and 2013 as in the Figure 1.

Year	CPI (%)	IR (%)
1991	270.20	170.20
1992	310.40	210.40
1993	356.10	256.10
1994	236.70	136.70
1995	132.30	32.30
1996	138.80	38.80
1997	254.80	154.80
1998	159.10	59.10
1999	145.80	45.80
2000	145.70	45.70
2001	134.50	34.50
2002	122.50	22.50
2003	115.30	15.30
2004	111.90	11.90
2005	109.00	9.00
2006	106.56	6.56
2007	104.84	4.84
2008	107.85	7.85
2009	105.59	5.59
2010	106.09	6.09
2011	105.79	5.79
2012	103.33	3.33
2013	103.98	3.98

Figure 1: CPI and IR between 1991 and 2013.

CPI evolution between the period 1991-2013 is presented in Figure 2.

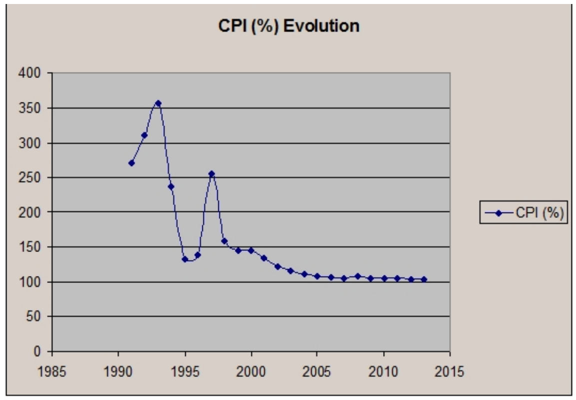


Figure 2: CPI evolution between 1991 and 2013.

4 Case study

The implementation of the software that generated the Julia set has been made using Java programming language ([5],[10]). Java is an object-oriented programming language initially developed by James Gosling at Sun Microsystems at begining of 1990. It was been launched in 1995. The newest technological evolutions allow the use of Java also on mobile ([7]). A Java software program can be used on any platform on which a Java virtual machine is installed. The Julia software follows the algorithm presented in section 2.

Our case study focuses on the data sets provided by the National Institute of Statistics ([12]) regarding the IR and CPI evolution between the years 1991 and 2013. We designed a fractal model for managers in order to take a correct decision in real time about inflation rate and how CPI can influence this indicator. This model is studied from the fractal theory point of view. In our graphical representations, the values have been scaled with 1000.

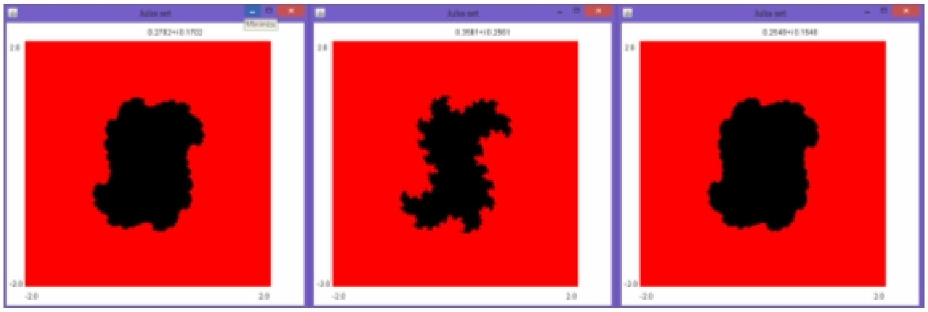
We have considered the point $c = a + b \cdot i \in \mathbb{C}$ where a is the *Consumer Price Index* and b is the *Inflation Rate* between the years 1991-2013. For an arbitrary choice of c, we obtain only a visualization of the escape set E_c and prisoner set P_c and the common boundary between E_c and P_c is the Julia set. In Figure 3 we can observe the Julia sets:

For $a = 0.1002$ and $b = 0.0002$ we can see a different Julia set, like in Figure 4.

In Figure 4 we observe that the prison set is included in a circle, so the Inflation Rate is under control.

5 Conclusion

The fractal geometry can be used to model different forms from many research fields, like medicine, economy, finance, etc. In this paper we propose a fractal model for the study of two economic indicators: CPI and IR. We presented a



a=0.2702 b=0.1702 | a=0.3561 b=0.2561 | a=0.2548 b=0.1548

Figure 3: Julia sets for $c = 0.2702 + i \cdot 0.1702$, $c = 0.3561 + i \cdot 0.2561$ and $c = 0.2548 + i \cdot 0.1548$.

a=0.1002
b=0.0002

Figure 4: Julia set for $c = 0.1002 + i \cdot 0.0002$.

case study using the IR and CPI data sets between the years 1991 and 2013. For certain values of IR and CPI we represented the Julia sets. We observed that for $CPI = 0.1002$ and $IR = 0.0002$ the shape of Julia set is like a circle, so the conclusion is that the Inflation Rate is under control.

Our approach can be used by managers in taking decisions in real time about inflation rate and how CPI can influence this indicator.

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