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REDUCTION OF THE CRITICAL PATH FINDING PROBLEM TO AN ORDINARY TRANSPORTATION TASK IN EXCEL

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Abstract

The problem of determining the critical path for the scheduling tasks has been considered. An original technique for practical calculations using the available Solver add-on in Excel has been described. The proposed approach is based on reducing the task of solving the ordinary transportation problem, in particular the problem of finding the longest path. Examples of solutions with the test input data and corresponding screenshots are given. Consistently we describe practical steps of the users action in the process of direct solution in Excel. The analysis of the results of the proposed method has been performed and presented. It was established that the presented method for calculating the critical path requires minimal efforts from users, regardless of the dimension of the tasks.

Key words: transportation problem, definition of the critical path, add-on for finding a solution in Excel.

1 Introduction

As it is well known, there are many practical problems, including transport character and project management which are formulated and solved with the use of network models.

Almost in each manual on economic-mathematical methods and models mathematical bases of network planning and management [1–6] are stated.

The necessity of developing of effective ways of planning complex processes has led to the creation of essentially new methods of network planning and management.

More often for the construction of network models five basic algorithms are used: findings of minimal tree; findings of the shortest way; definitions of the maximal

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stream; minimization of cost of a stream in a network with the limited throughput; findings of a critical path (way) [6].

Thus the algorithm of a critical path is the best known method in planning, drawing up of time schedules and managements of projects.

2 Description of the research

The main, basic problem in calendar network planning of manufacture is definition of "a critical path". It represents a sequence of the operations which do not have a reserve, a stock of time.

The operation is considered as critical if the delay of its beginning leads to an increase in a termination date of all process (part of which is considered operation) as a whole.

In case of graphic representation of the network schedule, its arrows (focused arches) represent the certain operations. The figure near each arrow means duration of corresponding operation. Initial and final points of any operation correspond to meeting events (initial and final). The operations which start with some event cannot begin while the operations entering this event will not be completed yet.

"The critical path (way)" on the network schedule represents the continuous chain of operations connecting the initial event of a network with the finishing.

The purpose of our work is the reduction of a problem of search of a critical path to usual transport task, namely a task of search of the longest way.

By analogy to a transport task we shall consider units of the network schedule (except for initial and final), as transit points. Certainly, thus for a critical way the requirement that it is possible to arrive to each transit point only from the previous point and to go only to one subsequent point is carried out.

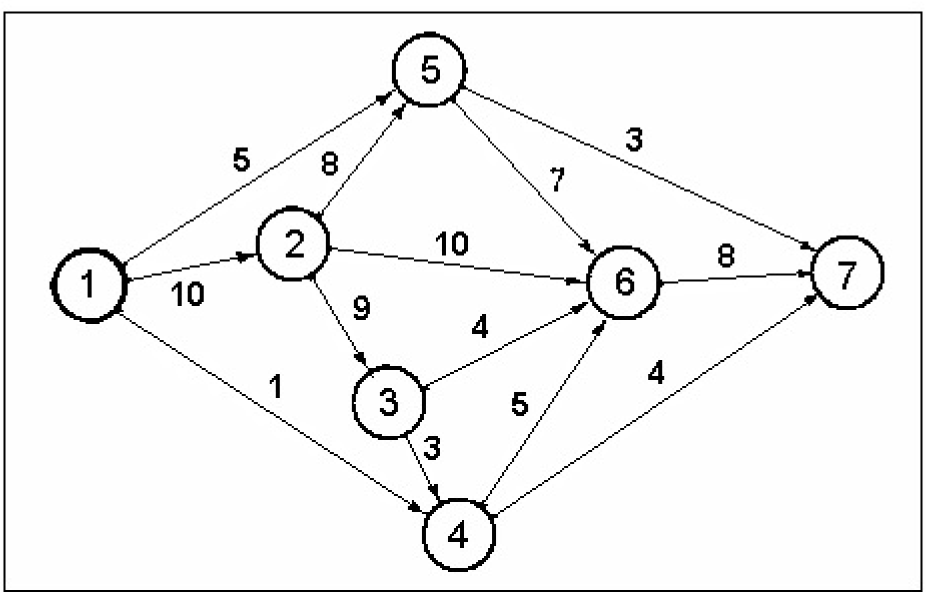

Figure 1. The network schedule for a test example

Earlier in the work of authors [5] the task of finding the shortest way by "Solver" tool in spreadsheet Excel has been examined.

The offered approach can be applied to calculation of a critical path. For an illustration of the offered approach we shall consider an example of the network schedule (fig. 1) from the work of Hemdi A. Taha [4].

It is necessary to emphasize, that, as a whole, the idea of the use of methods of linear programming for the definition of a critical path and the solving by computer is important. In fact practical network models can be much more difficult than simple graphs, represented in fig. 1.

Therefore it is necessary to organize calculations so that it is feasible for the usual user. Usually in real network model there are about ten events (points). Accordingly, it is necessary to fill tables of initial data of the big size with dimension in about ten elements. There is a problem to reduce this work up to a possible minimum and to receive thus the optimum decision.

Let's consider an offered technique and practical actions for an example of graph, shown in fig. 1.

First we shall enter in Excel corresponding with the network schedule (fig. 1) data for durations of works t_{ij} between each pair of points T_iT_j (fig. 2).

	I	J	K	L	M	N	O
4	**Duration of works between each pair of points**						
5	t_{ij}	T_2	T_3	T_4	T_5	T_6	T_7
6	T_1	10	–100	1	5	–100	–100
7	T_2	0	9	–100	8	10	–100
8	T_3	–100	0	3	–100	4	–100
9	T_4	–100	–100	0	–100	5	4
10	T_5	–100	–100	–100	0	7	3
11	T_6	–100	–100	–100	–100	0	8

Figure 2. Input data for the test example

All points, except for the last point T_7 , we will consider as points of departure. They are listed in the left column of the table t_{ij}. All points, except for the start point T_1, we will count as points of destinations. They are listed in the top line of the table t_{ij}. Transit points $T_2 - T_6$ are considered both as points of departure and as points of destinations.

The durations of works between identical transit points $T_k - T_k$ are equal to zero. Between some points there are no communications. Therefore we set corresponding duration of works equal to a very big negative number ($t_ij = -100$), that by critical search (the longest way) these forbidden transitions automatically were rejected.

Let's describe practical steps for filling the given table. To exclude from consideration fictitious durations on the forbidden transitions, it is expedient to represent (by means of conditional formatting) them by their grey color on a grey background. For this purpose we bring in any free cell number -100 (duration for the forbidden transitions) and copy it in the buffer of an exchange.

Then, keeping pressed key Ctrl, we allocate by the mouse the table t_{ij} (without headings) and insert contents from the buffer of an exchange (at once into all cells of the table). Further (not removing allocation) in menu Format we use item Conditional formatting. On the panel of conditional formatting we fill a field Condition 1: Cell Value Is and less than by -99. We press the button Format and on panel Format Cells (item Font) we set color of numbers, and on item Patterns - color of a background. After that we enter real duration of works for all possible transitions between points.

Similarly zero duration of works between pairs of identical transit points can also be entered by one operation. For this purpose it is necessary to copy a cell with value 0 on the buffer of an exchange. Further it is necessary to allocate by mouse (with pressed key Ctrl) the cells on diagonal T_kT_k and to insert 0 from the buffer of an exchange at once into all allocated cells. By the described actions work on data input is shown up to a necessary minimum.

In the following table (the same size) for x_ij we shall define transitions between points (fig. 3). If between points T_iT_j there is no transition then we accepted $x_{ij} = 0$ and if transition exists then $x_{ij} = 1$. First we fill all cells of table x_{ij} with 1 (all $x_{ij} = 1$). Naturally, they are written into all cells of the table by one operation (by copying from buffer of an exchange at once in all of the allocated cells).

	I	J	K	L	M	N	O	P
13			Transitions between points					
14	x_{ij}	T_2	T_3	T_4	T_5	T_6	T_7	Sum 1
15	T_1	1	1	1	1	1	1	6
16	T_2	1	1	1	1	1	1	6
17	T_3	1	1	1	1	1	1	6
18	T_4	1	1	1	1	1	1	6
19	T_5	1	1	1	1	1	1	6
20	T_6	1	1	1	1	1	1	6
21	Sum 2	6	6	6	6	6	6	36

Figure 3. Transitions between points start (initial) position

However, as follows from a condition of a problem, the critical path passes through transit points only once. Therefore in each line and in each column of the table x_{ij} for a critical path there should be only one 1 (similarly a task about destinations). Therefore in table x_{ij} it is added the right column and the last line in which formulas of summation (by function SUM) are entered.

For this purpose it is necessary to allocate table x_{ij} without headings, but with additional right column and the last line, and to press on tools panel the auto summa button Σ. Then in all cells of an additional column and an additional line down of formulas of summation will be automatically written. Certainly, for a critical path of all these sums should be equal to 1.

On an empty place of spreadsheet Excel for calculation of critical path duration we write formula SUMPRODUCT (Range t_{ij} ; Range t_{ij}). At first, before calculation's start, this duration is equal -1723 (fig. 4).

	K	L	M
22	Duration of a critical path		
23		Max	
24	Function of the purpose	–1723	

Figure 4. Length of a critical path

Now we shall go directly to search of a critical path. We put the table cursor on a target cell and through the menu Tools call "Solver" Add-In (fig. 5).

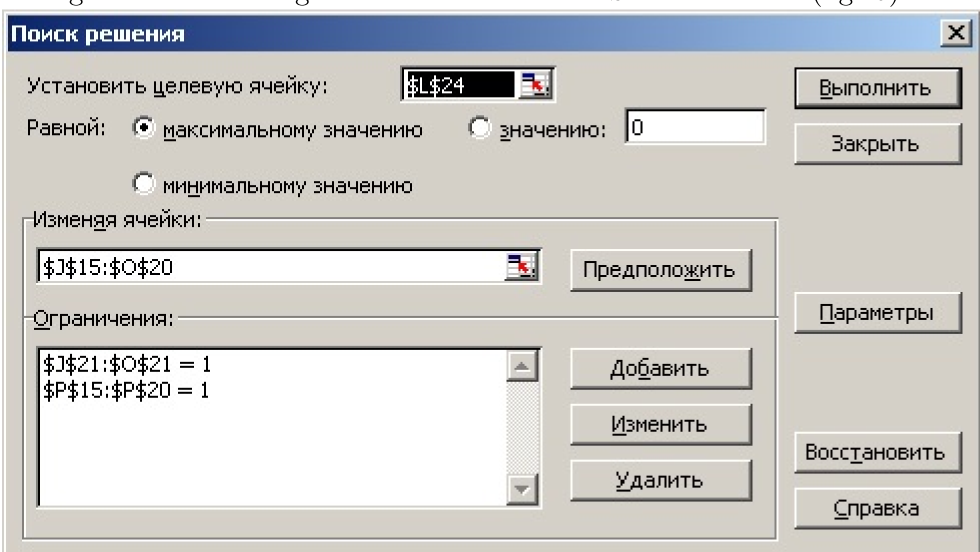

Figure 5. Window with the parameters of decision search in Solver Add-In.

On the Solver panel window we set Target Cell equal Maximal value. In a field Changing cells we specify a range x_{ij}.

We set two restrictions: Sum1 on table lines and Sum2 on table columns should be equal 1.

Then we press the button Parameters and put flags Linear model and Non-negative values.

Finally we press the button To execute and have received the optimum decision.

Now, in the transformed table of transitions (fig. 6) on each line and on each column there is only one 1 unit, all other numbers are zero.

	I	J	K	L	M	N	O	P
13	Transitions between points							
14	x_{ij}	T_2	T_3	T_4	T_5	T_6	T_7	Sum 1
15	T_1	1	0	0	0	0	0	1
16	T_2	0	1	0	0	0	0	1
17	T_3	0	0	1	0	0	0	1
18	T_4	0	0	0	0	1	0	1
19	T_5	0	0	0	1	0	0	1
20	T_6	0	0	0	0	0	1	1
21	Sum 2	1	1	1	1	1	1	6

Figure 6. The received decision

For simplification of results visualization is expediently to allocate all zeros in the table so that they do not prevent us from seeing a critical path.

For example, by means of conditional formatting it is possible to show zero (numbers, smaller than 0,01) as a grey color on a grey background. Numbers on diagonal $T_k T_k$ also do not have any helpful information. Therefore it is possible to set a grey background for these cells too for what it is necessary to click mouse (at pressed key Ctrl) on diagonal cells and to set a demanded background at once for all of them.
Now on fig. 6 only the critical path is allocated. From initial point T_1 there is a transition to point T_2. From point T_2 there is a transition to T_3. Further from point T_3 there is a transition to T_4 , and from point T_4 there is a transition to T_6. At last, from point T_6 there exists transition at once to finish point T_7 . The critical path does not take place through point T_5 , therefore it is specified in the optimum decision fictitious transition from T_5 to T_5.

In fig. 7 critical path $T_1 \to T_2 \to T_3 \to T_4 \to T_6 \to T_7$ is represented.

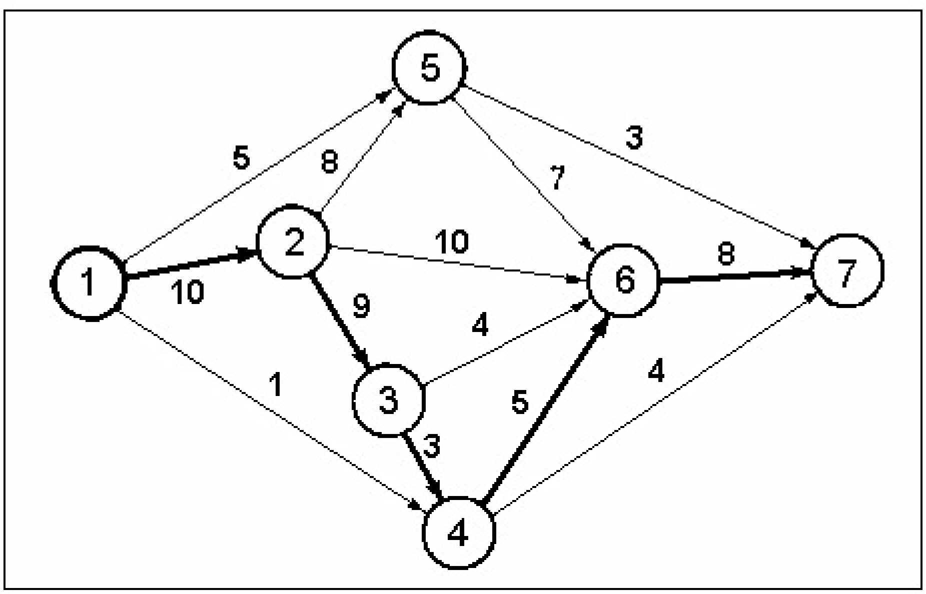

Figure 7. Founding of critical path.

Duration of a critical path is equal to $10 + 9 + 3 + 5 + 8 = 35$. As clear from fig. 3, there is a reserve of time in 21 units for performance of work 1-4; 4 units for work 3-6; 7 units for 2-6; in the sum of 2 units for works 2-5 and 5-6; in the sum of 27 units for works 1-5 and 5-7.

Let's shortly examine one more example of calculation of a critical path for the network schedule represented in fig. 8. The filled corresponding table for work's durations is shown in fig. 9.

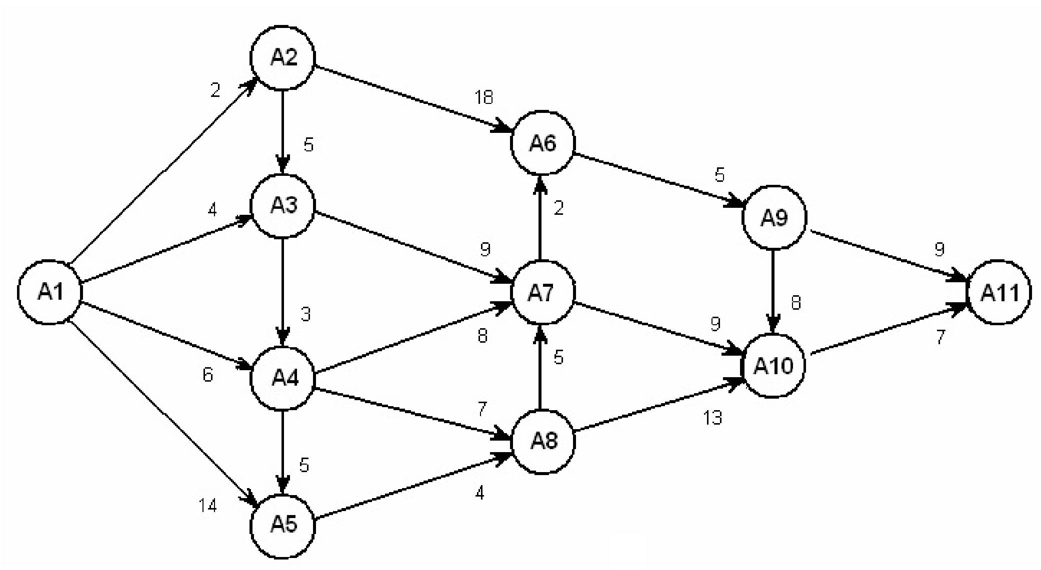

Figure 8. The network schedule for a test example of greater dimension

t_{ij}	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
A_1	2	4	6	14	–100	–100	–100	–100	–100	–100
A_2	0	5	–100	–100	18	–100	–100	–100	–100	–100
A_3	–100	0	3	–100	–100	9	–100	–100	–100	–100
A_4	–100	–100	0	5	–100	8	7	–100	–100	–100
A_5	–100	–100	–100	0	–100	–100	4	–100	–100	–100
A_6	–100	–100	–100	–100	0	–100	–100	5	–100	–100
A_7	–100	–100	–100	–100	2	0	–100	–100	9	–100
A_8	–100	–100	–100	–100	–100	5	0	–100	13	–100
A_9	–100	–100	–100	–100	–100	–100	–100	0	8	9
A_{10}	–100	–100	–100	–100	–100	–100	–100	–100	0	7

Figure 9. Duration of works for a test example of greater dimension

The size of the second task is much greater than the previous. Data which is necessary for typing manually, make a small part of the table (for this example nearly 20%). The optimum decision was received in the form (see fig. 10) where owing to conditional formatting the critical path is allocated. We write out it from the lines (fig. 10): $A_1 \to A_2 \to A_3 \to A_4 \to A_5 \to A_8 \to A_7 \to A_6 \to A_9 \to A_{10} \to A_{11}$. The summary (total) duration of works on a critical path is equal to $2+5+3+5+4+5+2+5+8+7 = 46$ time units.

x_{ij}	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	Sum 1
A_1	1	0	0	0	0	0	0	0	0	0	1
A_2	0	1	0	0	0	0	0	0	0	0	1
A_3	0	0	1	0	0	0	0	0	0	0	1
A_4	0	0	0	1	0	0	0	0	0	0	1
A_5	0	0	0	0	0	0	1	0	0	0	1
A_6	0	0	0	0	0	−100	−100	1	0	0	1
A_7	0	0	0	0	1	0	0	0	0	0	1
A_8	0	0	0	0	0	1	0	0	0	0	1
A_9	0	0	0	0	0	0	0	0	1	0	1
A_{10}	0	0	0	0	0	0	0	−100	0	1	1
Sum 2	1	1	1	1	1	1	1	1	1	1	10

Figure 10. The final decision for a test example of greater dimension

In fig. 11 the found critical way on the network schedule is represented. Numbers in brackets near arrows show reserves of time for performance of noncritical operations.

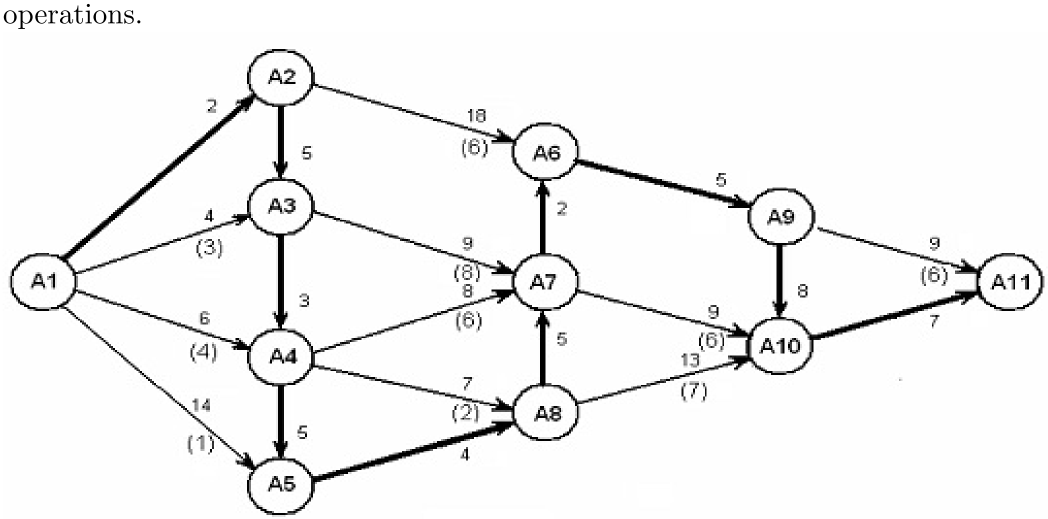

Figure 11. The critical path on the network schedule for the test example

Finally, it is necessary to deal with two important moments confirming an opportunity of practical use of the offered technique.

The first concern is the maximal dimension of a task. As it is well known, standard tool Solver (built in standard Excel) has the general limitation on the quantity of cells with initial data (up to about 200 cells). For overcoming this restriction in practical tasks with big dimensions we recommend to use more powerful tool Premium Solver (accessible free of charge on a site of the developer) which practically presupposes usage of matrices of any dimensions.

The second concern is the usage of newer versions of spreadsheet Excel. Though the material stated in this paper has been received in Excel 2003, check of a technique in Excel 2010 shows its working capacity. Certainly, the sequence of commands, their arrangement on panels and names can be different.

3 Conclusions

The described design procedure for finding the critical path demands the minimal labor expenditures from the user irrespective of the tasks sizes. In spite of the fact that the special methods considering their structure are developed for network models, many network tasks can be solved as tasks of linear programming (in particular, in transportation).

In this paper the expediency of solving the examined tasks by their reduction to the problem of the longest path search has been shown. The demanded decision is easy to be obtained by using Solver tool from spreadsheet Excel.

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