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ABOUT MATHEMATICAL METHODS FOR MULTI-CRITERIA DECISION

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Abstract

The Multi Criteria Decision Making (MCDM) delivers strong decision making in areas where selection of the best alternative is highly complex. This paper reviews and explains the main condensations of MCDM models and practices in detail. The purpose is explain and identify various applications and approaches and to suggest different MCDM models for different decision making issues and to select the best alternatives. The MCDM methods have helped to choose the best alternatives where many criteria are present. The best can be selected and analyzed the different scopes of the criteria, using weights for the criteria and the choice of the ideal ones using any multi criteria decision making methods.

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1 Introduction

In our daily life, so many decisions are being made by taking into account various criteria, so decisions can be made by providing weights to different criteria and all the weights are obtained from expert groups (Keivan, 2015). It is essential to determine the structure of the problem and explicitly evaluate multi criteria.

Multi-Criteria Decision Making (MCDM) method has been used over the last decades. MCDM refers to making decisions in the presence multiple but usually conflicting criteria (Naderi, 2011). MCDM approach is used for the problems that are more complicated and usually of large scale (Albayrak, 2005). For instance, many organizations in the US and Europe are conducting self-assessment using many criteria and sub-criteria sets in Quality Management System (QSM) business excellence model (Zaeri, 2012). In large organizations the purchasing departments often need to evaluate their suppliers by using the range of criteria in a different area, such as after sale service, quality management and financial stability.

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Although MCDM problems are extensive all the time, MCDM as a discipline only has a relatively short history of about 30 years (Zarghami, 2011). MCDM is an advantageous tool in many economies, material selection, manufacturing, construction problems, etc. It particularly plays a significant role in investment decision, project evaluation, staff appraisal and so on (Zhang, 2011). There have been many techniques offered to solve multiple attribute decision making problems. Multi-Attribute Decision Making (MCDM) is a study of classifying and selecting alternatives based on the values and preference of the decision maker (Peng, 2012). In MCDM making a decision implies that there are alternative choices to be considered and in such case we won't only identify as many of these alternatives as possible, but take up the one that best fits the ultimate goal, objective, desires and value (Peng, 2011).

The development and progress of the MCDM disciple is closely connected to the progression of computer technology. Moreover, the fast development of computer technology in recent years has made it possible to demeanor systematic analysis of complex MCDM process in problems (Mazumdar, 2010). On the other hand, the extensive use of computers and information technology has generated a huge amount of information that makes MCDM increasingly important and useful in supporting business decision making (Shimming, 2011).

Alternatives often represent different choices of action that are available to the decision maker (Schinnas, 2007). For instance, the general goal depends on two or 3 criteria or sub criteria that is under criteria (Zaeri, 2011). Since different criteria represent different dimensions of alternatives, they may conflict with each other. For example, costs may conflict with profit. Essentially, MCDM problems can be interpreted by solving with choosing best alternatives from a set of available alternatives, choosing a small set of good alternatives.

2 Review and analysis of MCDM methods

MCDM methods have been applied to different applications and the best explanation to select the best alternative have been found.

There are many methods for solving MCDM problems as studied by Hwang and Yoon (2008), nonetheless some of the methods were criticized as being to certain degree unjustified on theoretical and empirical grounds (Stewart, 2009). According to Zimmermann (2001) MCDM is separated into Multi-Attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM), where MODM studies decision problems in which the decision space is continuous. A typical example is mathematical Programming problems with multiple objective functions, the first reference is known as "Vector-Maximum" problem (Tucker, 1991). The main focus was directed on problems with a predeterminate set of decision alternatives. Even though MCDM methods may be extensively diverse, many of them have certain features in common (Chen, 2006). These notions of alternatives and attributes, are also called decision criteria.

2.1 Analytic Hierarchical Process (AHP)

The basic knowledge of AHP is to capture the expert's ideas of the phenomena under study. The use of the concepts of fuzzy set theory and the hierarchical systematic approach is the way followed for alternatives selection and justification problem (Miranda, 2013). AHP is based on decomposing a complex MCDM problem into a system of hierarchies (more on these hierarchies can be found in (Saaty, 2005).

Decision makers often find that it is more confident to give interval judgments than fixed value judgments (Martin, 2013). When a user preference is not defined explicitly due to fuzzy nature this method can be applied. AHP consists of the opinions of experts and multi-criteria evaluation. It is not capable of reflecting human's imprecise thoughts (Dalalah, 2011). The classical AHP considers the definite judgments of decision makers, thus the fuzzy set theory makes the comparison process more flexible and capable of explaining experts' preferences. The Analytic Hierarchy Process (AHP) decomposes a difficult MCDM problem into a systematic hierarchy procedure (Haghighi, 2011).

The final step in the AHP deals with the structure of an $M \times N$ matrix (where M is the number of alternatives and N is the number of criteria). This matrix is made by using the comparative importance of alternatives in terms of each criterion (Roy, 2011). The vector $(a_{i1}, a_{i2}, ..., a_{in})$ for each i is the principle eigenvector of an $N \times N$ reciprocal matrix which is determined by alternatives on i-the criterion.

Thomas Satty (1980) first developed the Analytic Hierarchical Process to allow decision making in situations characterized by multiple attributes and alternatives. AHP is one of the multi criteria decision making techniques. AHP has been applied effectively in many areas of decision making. In short, it is a method to derive ratio scales from paired comparison.

Satty examined the method used in AHP to process the a_{ij} values after they been determined. The entry age, in the $M \times N$ matrix, represents the relative value of the alternative A_i when it is considered in terms of criterion C_j.

According to Satty the best alternative in AHP (in Maximization case) is shown by the following relationship:

$$AAHP^* = \max \sum_{j=1}^{N} a_{ij} w_j, \text{ for } i = 1, 2, 3, ..., M. \tag{1}$$

The AHP uses relative values instead of actual ones. Hence, it can be used in single or multi-dimensional decision making problems (Belton, 2009). It uses a series of pairwise comparisons to determine the relative performance of each alternative in terms of each one of the decision criteria. AHP uses relative data instead of absolute data.

It is known that fuzzy AHP uses fuzzy set theory to express the uncertain comparison judgments as fuzzy numbers.

2.2 ELECTRE

ELECTRE, along with its many iterations, is an outranking method based on concordance analysis. Its major advantage is that it takes into account uncertainty and vagueness. One disadvantage is that its process and outcomes can be hard to explain in layman's terms. Further, due to the way preferences are incorporated, the lowest performances under certain criteria are not displayed. The outranking method causes the strengths and weaknesses of the alternatives not to be directly identified, no results and impacts to be verified (Konidari and Mavrakis, 2007). ELECTRE has been used in energy, economics, environmental, water management and transportation problems. Like other methods, it also takes uncertainty and vagueness into account, which many of the mentioned applications appear to need.

2.3 TOPSIS

TOPSIS (The Technique for order Preference by Similarity to Ideal Solution) was developed by Hwang Yoon (1981) as an alternative approach to the ELECTRE method. The basic concept of the TOPSIS method is that the selected alternative should have the shortest distance from idea solution and the farthest distance from negative-ideal solution in a geometrical sense (Erkoyuncu, 2015). TOPSIS undertakes that each attribute has a tendency of monotonically increasing or decreasing utility. Therefore, it is easy to find the ideal and negative ideal solution. The Euclidean distance approach is used to appraise the relative familiarity of alternative to ideal solution (Sabaei, 2015). Hence, the preference order of alternative is yielded through comparing these relative distances.

The TOPSIS approach evaluates the following decision matrix that refers to M as alternative which is evaluated in terms of N as criteria.

A disadvantage is that its use of Euclidean Distance does not consider the correlation of attributes. It is difficult to weight attributes and keep consistency of judgement, particularly with additional attributes (Valesquez, 2013).

2.4 PROMETHEE

PROMETHEE is similar to ELECTRE in that it also has several repetitions and it also an outranking method (Okeola, 2012). The PROMETHEE family of outranking methods, including PROMETHEE I for partial ranking of alternatives and the PROMETHEE II for complete ranking of alternatives, were developed and presented for the first time in 1982.

Few ears later, several versions of the PROMETHEE method such as PROMETHEE III for ranking based on interval and PROMETHEE IV for entire or partial ranking of alternatives when the set of feasible answer is continuous, the PROMETHEE V for the problems with division and subdivision. Constrains, the PROMETHEE VI for the human brain representation (Behzadian, 2010).

PROMETHEE method has benefits such that it is easy to be implemented. But the limitation of PROMETHEE method according to Aghdasi (2010) is that it does not provide clear method by which to assign weights and it requires the assignment of values but does not provide a clear process by which to assign the value. The area PROMETHEE has been used are mostly environmental management hydrology and water management, financial management, logistic and transportation management (Kazemzadeh, 2010)

2.5 Grey Theory

The Grey Theory is the method that is used to study uncertainty and being superior in the mathematical analysis of systems with uncertain and undefined information. The Grey Theory method is used to solve uncertain issues with separate data and incomplete information (Erkan, 2015). The Grey Theory was first introduced and applied by Julong Deng (1982) in order to manage with situations categorized by partially known and partially unknown information.

The Grey theory method includes several major parts: Grey Prediction, Grey Rational Analysis, Grey Decision and Grey Control (Temiz, 2011).

3 Conclusion

Many MCMD methods have been introduced and applied in various fields in the last several decades. This paper assessed the common methods of MCDM in order to help experts, consultants or practitioners to choose a method for solving a specific problem.

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