Bulletin of the *Transilvania* University of Braşov • Vol 9(58), No. 2 - 2016
Series III: Mathematics, Informatics, Physics, 71-78

ON THE EXISTENCE OF POSITIVE WEAK SOLUTIONS FOR A CLASS OF CHEMICALLY REACTING SYSTEMS WITH SIGN-CHANGING WEIGHTS

S. H. RASOULI*[1] and B. SALEHI[2]

Abstract

We study the existence of positive weak solutions for a class of nonlinear systems

$$\begin{cases} -\Delta_p u = \lambda a(x)\Big(f(v) - \frac{1}{u^\alpha}\Big), & x \in \Omega, \\ -\Delta_q v = \lambda b(x)\Big(g(u) - \frac{1}{v^\beta}\Big), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

where $\Delta_s z = div(|z|^{s-2}\nabla z)$, $s > 1$, λ is a positive parameter and Ω is a bounded domain with smooth boundary, $\alpha\ \beta \in (0,1)$. Here $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe negative near the boundary and f, g are C^1 nondecreasing functions such that $f, g : (0,\infty) \to (0,\infty)$; $f(s) > 0$, $g(s) > 0$ for $s > 0$ and $\lim_{s\to\infty} \frac{f(Mg(s)^{\frac{1}{q-1}})}{s^{p-1}} = 0$. We discuss the existence of positive weak solutions when f, g , $a(x)$ and $b(x)$ satisfy certain additional conditions. We use the method of sub-supersolution to establish our results.

2010 *Mathematics Subject Classification:* 35J55, 35J65.
Key words: positive solutions, chemically reacting systems, sub-supersolutions.

1 Introduction

In this paper, we consider the existence of positive solutions for the nonlinear system

$$\begin{cases} -\Delta_p u = \lambda a(x)\Big(f(v) - \frac{1}{u^\alpha}\Big), & x \in \Omega, \\ -\Delta_q v = \lambda b(x)\Big(g(u) - \frac{1}{v^\beta}\Big), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

[1]**Corresponding author*, Department of Mathematics, Faculty of Basic Sciences, Babol Noshirvani University of Technology, Babol, Iran, e-mail: s.h.rasouli@nit.ac.ir

[2]Department of Mathematics, Faculty of Basic Sciences, Babol Noshirvani University of Technology, Babol, Iran

where $\Delta_s z = div(|z|^{s-2}\nabla z)$, $s > 1$, λ is a positive parameter and Ω is a bounded domain with smooth boundary, $\alpha\ \beta \in (0,1)$. Here $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe negative near the boundary and f, g are C^1 non-decreasing functions such that $f, g : (0,\infty) \to (0,\infty)$; $f(s) > 0$,$g(s) > 0$ for $s > 0$.

Systems of singular equations like (1) are the stationary counterpart of general evolutionary problems of the form

$$\begin{cases} u_t = \eta\Delta_p u + \lambda\Big(f(v) - \frac{1}{u^\alpha}\Big), & x \in \Omega, \\ v_t = \delta\Delta_q v + \lambda\Big(g(u) - \frac{1}{v^\beta}\Big), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

where η and δ are positive parameters. This system is motivated by an interesting application in chemically reacting systems, where u represents the density of an activator chemical substance and v is an inhibitor. The slow diffusion of u and the fast diffusion of v is translated into the fact that η is small and δ is large (see [1]).

Also, systems of the form (1) arise in several context in biology and engineering. It provides a simple model to describe, for instance, the interaction of two diffusing biological species. u, v represent the densities of two species. See [6] for more results on the physical models involving more general elliptic problems.

Recently, such infinite problems have been studied in [3, 4, 5]. Also in [5], the authors have studied the existence results for system (1) in the case $a \equiv 1$, $b \equiv 1$. Here we focus on further extending the study in [4] to system (1). In fact, we study the existence of positive solution to the system (1) with sign-changing weight functions $a(x), b(x)$. Due to these weight functions, the extensions are challenging and nontrivial. Our approach is based on the method of sub-super solutions (see [7]).

To precisely state our existence result we consider the eigenvalue problem

$$\begin{cases} -\Delta_r\phi = \lambda\,|\phi|^{r-2}\,\phi, & x \in \Omega, \\ \phi = 0, & x \in \partial\Omega. \end{cases} \tag{2}$$

Let $\phi_{1,r}$ be the eigenfunction corresponding to the first eigenvalue $\lambda_{1,r}$ of (2) such that $\phi_{1,r}(x) > 0$ in Ω, and $\|\phi_{1,r}\|_\infty = 1$ for $r = p, q$ (see [2].) Let $m, \mu, \delta > 0$ be such that

$$\mu \le \phi_{1,r} \le 1, \quad x \in \Omega - \overline{\Omega_\delta}, \tag{3}$$

$$(1-\frac{sr}{r-1+s})|\nabla\phi_{1,r}|^r \geq m, \quad x\in\overline{\Omega_\delta}, \tag{4}$$

for $r=p,q$, and $s=\alpha,\beta$, where $\overline{\Omega_\delta} := \{x\in\Omega \,|\, d(x,\partial\Omega)\leq\delta\}$. This is possible since $|\nabla\phi_{1,r}|\neq 0$ on $\partial\Omega$ while $\phi_{1,r}=0$ on $\partial\Omega$ for $r=p,q$. We will also consider the unique solution $e_r\in W_0^{1,r}(\Omega)$ (for $r=p,q$) of the boundary value problem

$$\begin{cases} -\Delta_r e_r = 1, & x\in\Omega, \\ e_r = 0, & x\in\partial\Omega, \end{cases}$$

to discuss our existence result, it is known that $e_r>0$ in Ω and $\frac{\partial e_r}{\partial n}<0$ on $\partial\Omega$.

Here we assume that the weight functions $a(x)$ and $b(x)$ take negative values in $\overline{\Omega_\delta}$, but require $a(x)$ and $b(x)$ be strictly positive in $\Omega-\overline{\Omega_\delta}$. To be precise we assume that there exist positive constants a_0 , a_1 , b_0 and b_1 such that $a(x)\geq -a_0$, $b(x)\geq -b_0$ on $\overline{\Omega_\delta}$ and $a(x)\geq a_1$, $b(x)\geq b_1$ on $\Omega-\overline{\Omega_\delta}$.

2 Existence result

In this section, we shall establish our existence result by constructing a positive weak subsolution $(\psi_1,\psi_2)\in W^{1,p}(\Omega)\bigcap C(\overline{\Omega})\times W^{1,q}(\Omega)\bigcap C(\overline{\Omega})$ and a supersolution $(z_1,z_2)\in W^{1,p}(\Omega)\bigcap C(\overline{\Omega})\times W^{1,q}(\Omega)\bigcap C(\overline{\Omega})$ of (1) such that $\psi_i\leq z_i$ for $i=1,2$. That is, ψ_i, z_i satisfies $(\psi_1,\psi_2)=(0,0)=(z_1,z_2)$ on $\partial\Omega$, and

$$\begin{aligned}
&\int_\Omega |\nabla\psi_1|^{p-2}\nabla\psi_1\nabla\xi dx \leq \lambda\int_\Omega a(x)[f(\psi_2)-\frac{1}{\psi_1^\alpha}]\xi dx,\\
&\int_\Omega |\nabla\psi_2^{q-2}|\nabla\psi_2\nabla\xi dx \leq \lambda\int_\Omega b(x)[g(\psi_1)-\frac{1}{\psi_2^\beta}]\xi dx,\\
&\int_\Omega |\nabla z_1|^{p-2}\nabla z_1\nabla\xi dx \geq \lambda\int_\Omega a(x)[f(z_2)-\frac{1}{z_1^\alpha}]\xi dx,\\
&\int_\Omega |\nabla z_2|^{q-2}\nabla z_2\nabla\xi dx \geq \lambda\int_\Omega b(x)[g(z_1)-\frac{1}{z_2^\beta}]\xi dx,
\end{aligned}$$

for all $\xi\in W := \{\zeta\in C_0^\infty(\Omega) : \zeta\geq 0 \ \ in\ \Omega\}$. Then the following result holds :

Lemma 2.1 (see [7]) Suppose there exist sub and super-solutions (ψ_1,ψ_2) and (z_1,z_2) respectively of (1) such that $(\psi_1,\psi_2)\leq(z_1,z_2)$. Then (1) has solution (u,v) such that $(u,v)\in[(\psi_1,\psi_2),(z_1,z_2)]$.

To state our results precisely we introduce the following hypotheses:

(H1) $f,g : [0,\infty)\to[0,\infty)$ are C^1 nondecreasing functions such that $f(s)$, $g(s)>0$ for $s>0$, and $\lim_{s\to\infty} g(s)=\infty$.

(H2) $\lim_{s\to\infty} \frac{f(Mg(s)^{\frac{1}{q-1}})}{s^{p-1}} = 0$, for all $M > 0$.

(H3) Suppose that there exists $\epsilon > 0$ such that :

(i) $f\Big(\frac{\mu(q-1+\beta)}{q}\epsilon^{\frac{1}{q-1}}\Big) > \Big(\frac{p}{(p-1+\alpha)\mu\epsilon^{\frac{1}{p-1}}}\Big)^{\alpha}$,

(ii) $g\Big(\frac{\mu(p-1+\alpha)}{p}\epsilon^{\frac{1}{p-1}}\Big) > \Big(\frac{q}{(q-1+\beta)\mu\epsilon^{\frac{1}{q-1}}}\Big)^{\beta}$,

(iii) $\frac{1}{m}f(\epsilon^{\frac{1}{q-1}}) \le \min\left\{\frac{p^{\alpha}}{\epsilon^{\frac{\alpha}{p-1}}(p-1+\alpha)^{\alpha}\lambda_{1,p}}, \frac{Na_1}{a_0\lambda_{1,p}}, \frac{b_0 q^{\beta}}{a_0\epsilon^{\frac{\beta}{q-1}}(q-1+\beta)^{\beta}\lambda_{1,q}}, \frac{Mb_1}{a_0\lambda_{1,q}}\right\}$,

(iv) $\frac{1}{m}g(\epsilon^{\frac{1}{p-1}}) \le \min\left\{\frac{q^{\beta}}{\epsilon^{\frac{\beta}{q-1}}(q-1+\beta)^{\beta}\lambda_{1,q}}, \frac{Na_1}{b_0\lambda_{1,p}}, \frac{a_0 p^{\alpha}}{b_0\epsilon^{\frac{\alpha}{p-1}}(p-1+\alpha)^{\alpha}\lambda_{1,q}}, \frac{Mb_1}{b_0\lambda_{1,q}}\right\}$,

where

$$N = f\Big(\frac{\mu(q-1+\beta)}{q}\epsilon^{\frac{1}{q-1}}\Big) - \Big(\frac{p}{(p-1+\alpha)\mu\epsilon^{\frac{1}{p-1}}}\Big)^{\alpha},$$

and

$$M = g\Big(\frac{\mu(p-1+\alpha)}{p}\epsilon^{\frac{1}{p-1}}\Big) - \Big(\frac{q}{(q-1+\beta)\mu\epsilon^{\frac{1}{q-1}}}\Big)^{\beta}.$$

We are now ready to give our existence result .

Theorem 2.2. Let (H1)-(H3) hold. Then there exists a positive solution of (1) for every $\lambda \in [\lambda_*(\epsilon), \lambda^*(\epsilon)]$, where

$$\lambda^* = \min\left\{\frac{m\epsilon}{a_0 f(\epsilon^{\frac{1}{q-1}})}, \frac{m\epsilon}{b_0 g(\epsilon^{\frac{1}{p-1}})}\right\},$$

and

$$\lambda_* = \max\left\{\frac{\lambda_{1,p}(\frac{p-1+\alpha}{p})^{\alpha}\epsilon^{\frac{p-1+\alpha}{p-1}}}{a_0}, \frac{\lambda_{1,p}\epsilon}{Na_1}, \frac{\lambda_{1,q}(\frac{q-1+\beta}{q})^{\beta}\epsilon^{\frac{q-1+\beta}{q-1}}}{b_0}, \frac{\lambda_{1,q}\epsilon}{Mb_1}\right\}.$$

Remark 2.3. Note that $(H3)$ implies $\lambda_* < \lambda^*$.

Example 2.4. Let $f(s) = e^{\frac{s}{s+1}}$, $g(s) = e^{s}$. Here $f(s)$, $g(s) > 0$ for $s > 0$, f , g are non-decreasing functions and

$$\lim_{s\to\infty} \frac{f(Mg(s)^{\frac{1}{q-1}})}{s^{p-1}} = 0,$$

for all $M > 0$, and $\lim_{s\to\infty} g(s) = \infty$. We can choose $\epsilon > 0$ so small that f, g satisfy (H3).

Proof. of Theorem 2.2 We shall verify that

$$(\psi_1, \psi_2) = (\frac{p-1+\alpha}{p}\ \epsilon^{\frac{1}{p-1}}\ \phi_{1,p}^{\frac{p}{p-1+\alpha}}, \frac{q-1+\beta}{q}\ \epsilon^{\frac{1}{q-1}} \phi_{1,q}^{\frac{q}{q-1+\beta}}),$$

is a sub-solution of (1). Let $w \in W$. Then a calculation shows that

$$\nabla\psi_1 = \epsilon^{\frac{1}{p-1}} \nabla\phi_{1,p}\, \phi_{1,p}^{\frac{1-\alpha}{p-1+\alpha}},$$

and we have

$$\int_\Omega |\nabla\psi_1|^{p-2}\, \nabla\psi_1 \cdot\, \nabla w\, dx = \epsilon \int_\Omega \phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}} |\nabla\phi_{1,p}|^{p-2} \nabla\phi_{1,p} \nabla w dx$$

$$\begin{aligned}
&= \epsilon \int_\Omega |\nabla\phi_{1,p}|^{p-2}\nabla\phi_{1,p} \left\{ \nabla(\phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}} w) - w\nabla(\phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}}) \right\} dx \\
&= \epsilon \left\{ \int_\Omega \left[\lambda_{1,p}\, \phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}} - |\nabla\phi_{1,p}|^{p-2}\nabla\phi_{1,p}\nabla(\phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}})\right] w dx \right\} \\
&= \epsilon \left\{ \int_\Omega \left[\lambda_{1,p}\, \phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}} - |\nabla\phi_{1,p}|^{p}\, (1 - \frac{\alpha p}{p-1+\alpha}) \phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}\right] w dx \right\} \\
&= \epsilon\, \phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}} \left\{ \int_\Omega \left[\lambda_{1,p}\, \phi_{1,p}^{p} - |\nabla\phi_{1,p}|^{p}\, (1 - \frac{\alpha p}{p-1+\alpha})\right] w dx \right\}.
\end{aligned}$$

Similarly

$$\int_\Omega |\nabla\psi_2|^{q-2}\, \nabla\psi_2 \cdot \nabla w\, dx = \epsilon\, \phi_{1,q}^{-\frac{\beta q}{q-1+\beta}} \left\{ \int_\Omega [\lambda_{1,q}\, \phi_{1,q}^{q} - |\nabla\phi_{1,q}|^{q}\, (1 - \frac{\beta q}{q-1+\beta})] w dx \right\}.$$

First we consider the case when $x \in \overline{\Omega_\delta}$. We have

$$(1 - \frac{sr}{r-1+s})|\nabla\phi_{1,r}|^r \geq m.$$

Then

$$-\epsilon(1 - \frac{\alpha p}{p-1+\alpha})|\nabla\phi_{1,p}|^p \phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}} \leq -m\epsilon.$$

Since $\lambda \leq \lambda^*$ then

$$\lambda \leq \frac{m\epsilon}{a_0 f(\epsilon^{\frac{1}{q-1}})}.$$

Hence

$$\begin{aligned}
-\epsilon(1 - \frac{\alpha p}{p-1+\alpha})|\nabla\phi_{1,p}|^p \phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}} &\leq -\lambda a_0 f(\epsilon^{\frac{1}{q-1}}) \\
&\leq -\lambda a_0 f(\frac{q-1+\beta}{q} \epsilon^{\frac{1}{q-1}} \phi_{1,q}^{\frac{q}{q-1+\beta}}) \qquad (5) \\
&= -\lambda a_0 f(\psi_2).
\end{aligned}$$

Also since $\lambda_* \leq \lambda$, then

$$\frac{\lambda_{1,p}\,(\frac{p-1+\alpha}{p})^{\alpha}\,\epsilon^{\frac{p-1+\alpha}{p-1}}}{a_0} \leq \lambda.$$

Therefore

$$\begin{aligned} \lambda_{1,p}\,\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}}\,\epsilon &\leq \lambda_{1,p}\,\epsilon\,\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}} \\ &\leq \frac{\lambda a_0}{(\frac{p-1+\alpha}{p}\,\epsilon^{\frac{1}{p-1}}\,\phi_{1,p}^{\frac{p}{p-1+\alpha}})^{\alpha}} \\ &= \frac{\lambda\,a_0}{\psi_1^{\alpha}}. \end{aligned} \tag{6}$$

Combining (5) and (6) we see that

$$\begin{aligned} &\epsilon[\lambda_{1,p}\,\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}} - (1-\frac{\alpha p}{p-1+\alpha})|\nabla\phi_{1,p}|^p\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}] \\ &= \lambda_{1,p}\,\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}}\,\epsilon - \epsilon(1-\frac{\alpha p}{p-1+\alpha})|\nabla\phi_{1,p}|^p\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}} \\ &\leq \frac{\lambda\,a_0}{\psi_1^{\alpha}} - \lambda a_0 f(\psi_2) \\ &\leq \lambda a(x)[f(\psi_2) - \frac{1}{\psi_1^{\alpha}}]. \end{aligned} \tag{7}$$

On the other hand on $\Omega - \overline{\Omega_\delta}$ we have $\mu \leq \phi_{1,p}^{\frac{p}{p-1+\alpha}} \leq 1$, for $\mu > 0$, and therefore for $\lambda \geq \lambda_*$, we have $\frac{\lambda_{1,p}\epsilon}{Na_1} \leq \lambda$. Hence

$$\begin{aligned} &\epsilon[\lambda_{1,p}\,\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}} - |\nabla\phi_{1,p}|^p\,(1-\frac{\alpha p}{p-1+\alpha})\,\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}] \\ &\leq \epsilon\lambda_{1,p}\,\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}} \\ &\leq \epsilon\lambda_{1,p} \\ &\leq \lambda N a_1 \\ &= \lambda a_1[f(\frac{\mu\,(q-1+\beta)}{q}\,\epsilon^{\frac{1}{q-1}}) - (\frac{p}{(p-1+\alpha)\,\mu\,\epsilon^{\frac{1}{p-1}}})^{\alpha}] \\ &\leq \lambda a_1[f(\psi_2) - \frac{1}{\psi_1^{\alpha}}] \\ &\leq \lambda a(x)[f(\psi_2) - \frac{1}{\psi_1^{\alpha}}]. \end{aligned} \tag{8}$$

Combining (7) and (8) on Ω, for $\lambda \in [\lambda_*(\epsilon), \lambda^*(\epsilon)]$, we see that

$$\epsilon[\lambda_{1,p}\,\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}} - |\nabla\phi_{1,p}|^p\,(1-\frac{\alpha p}{p-1+\alpha})\,\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}] \leq \lambda a(x)[f(\psi_2) - \frac{1}{\psi_1^{\alpha}}].$$

Similarly for $\lambda \in [\lambda_*, \lambda^*]$ we get

$$\epsilon[\lambda_{1,q}\, \phi_{1,q}^{q-\frac{\beta q}{q-1+\beta}} - |\nabla \phi_{1,q}|^q\, (1 - \frac{\beta q}{q-1+\beta})\, \phi_{1,q}^{-\frac{\beta q}{q-1+\beta}}] \leq \lambda b(x)[g(\psi_1) - \frac{1}{\psi_2^\beta}].$$

Hence

$$\int_\Omega |\nabla \psi_1|^{p-2} \nabla \psi_1 \nabla w dx \leq \lambda \int_\Omega a(x)[f(\psi_2) - \frac{1}{\psi_1^\alpha}] w dx$$

and

$$\int_\Omega |\nabla \psi_2|^{q-2} \nabla \psi_2 \nabla w dx \leq \lambda \int_\Omega b(x)[g(\psi_1) - \frac{1}{\psi_2^\beta}] w dx,$$

i.e., (ψ_1, ψ_2) is a sub-solution of (1) for $\lambda \in [\lambda_*, \lambda^*]$.

Now we construct a supersolution $(z_1, z_2) \geq (\psi_1, \psi_2)$. We will prove there exists $c \gg 1$ such that

$$(z_1, z_2) = (c\, e_p(x)\, ,\, [\lambda \|b\|_\infty g(c\|e_p\|_\infty)]^{\frac{1}{q-1}} e_q(x)),$$

is a supersolution of (1). A calculation shows that

$$\begin{aligned} \int_\Omega |\nabla z_1|^{p-2}\, \nabla z_1\, \nabla w\, dx &= c^{p-1} \int_\Omega |\nabla e_p|^{p-2}\, \nabla e_p\, \nabla w\, dx \\ &= c^{p-1} \int_\Omega w\, dx, \end{aligned}$$

by (H2) we know that, for $c \gg 1$,

$$\frac{1}{\lambda \|a\|_\infty} \geq \frac{f\Big([\lambda \|b\|_\infty g(c\|e_p\|_\infty)]^{\frac{1}{q-1}} \|e_q\|_\infty\Big)}{c^{p-1}}.$$

Hence

$$\begin{aligned} c^{p-1} &\geq \lambda ||a||_\infty f\Big([\lambda \|b\|_\infty g(c\|e_p\|_\infty)]^{\frac{1}{q-1}} \|e_q\|_\infty\Big) \\ &\geq \lambda ||a||_\infty f\Big([\lambda \|b\|_\infty g(c\|e_p\|_\infty)]^{\frac{1}{q-1}} e_q(x)\Big) \\ &= \lambda a(x) f(z_2) \\ &\geq \lambda a(x)[f(z_2) - \frac{1}{z_1^\alpha}]. \end{aligned}$$

Therefore

$$\int_\Omega |\nabla z_1|^{p-2} \nabla z_1 \nabla w dx \geq \lambda \int_\Omega a(x)[f(z_2) - \frac{1}{z_1^\alpha}] w dx.$$

Also

$$\begin{aligned}\int_\Omega |\nabla z_2|^{q-2}\nabla z_2 \nabla w dx &= \lambda \|b\|_\infty g(c\|e_p\|_\infty) \int_\Omega w dx \\ &\geq \lambda \int_\Omega b(x) g(c\, e_p(x)) w dx \\ &= \lambda \int_\Omega b(x) g(z_1) w dx \\ &\geq \lambda \int_\Omega b(x)[g(z_1) - \frac{1}{z_2^\beta}] w dx,\end{aligned}$$

i.e., (z_1, z_2) is a supersolution of (1) with $z_i \geq \psi_i$ for c large, $i = 1, 2$. (This is possible since $|\nabla e_r| \neq 0; \partial\Omega$ for $r = p, q$). Thus, there exists a positive solution (u, v) of (1) such that $(\psi_1, \psi_2) \leq (u, v) \leq (z_1, z_2)$ and Theorem 2.2 is proven. □

References

[1] Aris, R., *Introduction to the analysis of chemical reactors*, Prentice Hall, Englewood Cliffs, NJ, 1965.

[2] Drábek, P. and Milota, J. *Methods of nonlinear analysis, Applications to differential equations*, Birkhauser Advanced Texts, Basel - Boston - Berlin 2007.

[3] Lee, E.K., Shivaji, R. and Ye, J. *Classes of infinite semipositone systems*, Proc. Roy. Soc. Edinb. **139A** (2009), 853-865.

[4] Lee, E.K. and Shivaji, R., *Posiotive solutions for infinit semipositone problems with falling zeros*, Nonlinear Analysis **72** (2010), 4475-4479.

[5] Lee, E.K., Shivaji, R. and Ye, J. *Classes of infinite semipositone $n \times n$ systems*, Diff. Int. Eqs, **24**, no. 3-4, (2011), 361-370.

[6] Marin, M., *Some basic theorems in elastostatics of micropolar materials with voids*, J. Comp. Appl. Math. **70**, no. 1, (1996), 115-126.

[7] Rasouli, S.H., Halimi, Z. and Mashhadban, Z., *A remark on the existence of positive weak solution for a class of (p,q)-laplacian nonlinear system with sign-changing weight*, Nonl. Anal, **73** (2010) 385-389.