Bulletin of the *Transilvania* University of Braşov • Vol 10(59), No. 1 - 2017
Series III: Mathematics, Informatics, Physics, 155-164

# RIEMANNIAN MANIFOLDS ADMITTING A PROJECTIVE SEMI-SYMMETRIC CONNECTION

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**Abstract**

The object of the present paper is to study some curvature properties of a Riemannian manifold admitting projective semi-symmetric connection.

2010 *Mathematics Subject Classification:* 53C05, 53D15.
*Key words:* Riemannian Manifolds, Projective Semi-symmetric connection.

## 1 Introduction

The idea of semi-symmetric connection was introduced by A. Friedman and J. A. Schouten [4] in 1924. In 1932, H. A. Hayden [5] introduced the semi-symmetric linear connection on a Riemannian manifold and this was further developed by K. Yano [14], M. C. Chaki and A. Konar [1], M. Prvanović ([6],[7],[8],[9]), U. C. De [3], U. C. De and B. K. De [2], P. Zhao et al [15, 16] and many others.

A linear connection $\overline{\nabla}$ defined on $(M^n, g)$ is said to be semi-symmetric [4] if its torsion tensor $\overline{T}$ with respect to the connection $\overline{\nabla}$ is of the form

$$\overline{T}(X,Y) = \pi(Y)X - \pi(X)Y, \tag{1}$$

where $\pi$ is a 1-form defined by

$$\pi(X) = g(X,\rho), \tag{2}$$

where $\rho$ is associate vector field and for all vector fields X $\in \chi(M)$, $\chi(M)$ is the set of all differentiable vector field on $M^n$.

A linear connection $\overline{\nabla}$ defined on $(M^n, g)$ is said to be semi-symmetric metric connection [14] if its torsion tensor $\overline{T}$ with respect to the connection $\overline{\nabla}$ satisfies (1) and $\nabla g = 0$.

A Riemannian manifold $(M^n, g)$ is called locally symmetric if its curvature tensor $R$ is parallel, that is, $\nabla R = 0$, where $\nabla$ is the Levi-Civita connection. The

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notion of semi-symmetric manifold, a proper generalization of locally symmetric manifold, is defined by $R(X,Y)\cdot R = 0$, where $R(X,Y)$ is considered as a field of linear endomorphisms, acting on $R$. A complete intrinsic classification of these manifolds was given by Szabó in [11].

In a recent paper P. Zhao [17] introduced the projective semi-symmetric connection on a Riemannian manifold. The projective semi-symmetric connection has also been studied by P. Zhao and H. Song [15], S. K. Pal and et al [10] and many others. This paper is organized as follows:

After the introduction we give some preliminary results in section 2. In Section 3, we obtain some results on the projective semi-symmetric connection whose the torsion tensor is recurrent. Section 4, deals with Riemannian manifold admitting a projective semi-symmetric connection whose curvature tensor vanishes and torsion tensor is recurrent. Finally, we obtain some sufficient conditions for a compact orientable Riemannian manifold admitting a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor $\overline{R}$.

## 2 Preliminaries

Let $(M^n,\ g)$ $(n \geq 3)$ be a Riemannian manifold and $\nabla$ be the Levi-Civita connection associated with the metric $g$. In a Riemannian manifold, a linear connection $\overline{\nabla}$ is called a semi-symmetric connection if its torsion tensor $\overline{T}$ defined by

$$\overline{T}(X,Y) = \overline{\nabla}_X Y - \overline{\nabla}_Y X - [X,Y], \tag{3}$$

satisfies (1).

In this paper, we study a type of projective semi-symmetric connection $\overline{\nabla}$ in a Riemannian manifold introduced by P. Zhao [17]. The connection is given by

$$\overline{\nabla}_X Y = \nabla_X Y + \psi(Y)X + \psi(X)Y + \phi(Y)X - \phi(X)Y, \tag{4}$$

where the 1-forms $\phi$ and $\psi$ are given by

$$\phi(X) = \frac{1}{2}\pi(X) \text{ and } \psi(\mathrm{X}) = \frac{(\mathrm{n}-1)}{2(\mathrm{n}+1)}\pi(\mathrm{X}). \tag{5}$$

Making use of (3), the above equations gives

$$\overline{T}(X,Y) = \pi(Y)X - \pi(X)Y. \tag{6}$$

It follows that the connection $\overline{\nabla}$ defined by (4) and (5) satisfies the condition (1). Therefore the connection $\overline{\nabla}$ is semi-symmetric [4].

Let $\overline{R}$ and $R$ be the curvature tensors with respect to the projective semi-symmetric connection $\overline{\nabla}$ and the Levi-Civita connection $\nabla$ respectively. The curvature tensor $\overline{R}$ and $R$ are related by [17] that

$$\overline{R}(X,Y)Z = R(X,Y)Z + \beta(X,Y)Z + \alpha(X,Z)Y - \alpha(Y,Z)X, \tag{7}$$

where $\beta(X,Y)$ and $\alpha(X,Y)$ are given by the following relations

$$\beta(X,Y) = \Psi'(X,Y) - \Psi'(Y,X) + \Phi'(Y,X) - \Phi'(X,Y), \tag{8}$$

$$\alpha(X,Y) = \Psi'(X,Y) + \Phi'(Y,X) - \psi(X)\phi(Y) - \phi(X)\psi(Y), \tag{9}$$

$$\Psi'(X,Y) = (\nabla_X\psi)(Y) - \psi(X)\psi(Y), \tag{10}$$

and

$$\Phi'(X,Y) = (\nabla_X\phi)(Y) - \phi(X)\phi(Y). \tag{11}$$

Contracting $X$ in (7), we have [17]

$$\overline{S}(Y,Z) = S(Y,Z) + \beta(Y,Z) - (n-1)\alpha(Y,Z), \tag{12}$$

where $\overline{S}$ and $S$ are the Ricci tensors with respect to the connections $\overline{\nabla}$ and $\nabla$ respectively.

If $\overline{r}$ and $r$ are scalar curvatures of the manifold with respect to connections $\overline{\nabla}$ and $\nabla$ respectively, then we have

$$\overline{r} = r + b - (n-1)a, \tag{13}$$

where

$$b = \sum \beta(e_i, e_i) \text{ and } a = \sum \alpha(e_i, e_i). \tag{14}$$

The Weyl projective curvature tensor $\overline{P}$ on a Riemannian manifold with respect to the connection $\overline{\nabla}$ is defined by

$$\overline{P}(X,Y)Z = \overline{R}(X,Y)Z - \frac{1}{(n-1)}[\overline{S}(Y,Z)X - \overline{S}(X,Z)Y]. \tag{15}$$

## 3 Projective semi-symmetric connection with recurrent torsion tensor

In this section, we consider a projective semi-symmetric connection $\overline{\nabla}$ given by (4), whose torsion tensor $\overline{T}$ is recurrent, that is, the torsion tensor $\overline{T}$ satisfies the condition

$$(\overline{\nabla}_X\overline{T})(Y,Z) = \pi(X)\overline{T}(Y,Z), \tag{16}$$

where the 1-form $\pi$ is defined by (2). From (4), we get

$$(C_1^1\overline{T})(Y) = (n-1)\pi(Y), \tag{17}$$

where $C_1^1$ denotes the operation of contraction.
From (17), it follows that

$$(\overline{\nabla}_X C_1^1\overline{T})(Y) = (n-1)(\overline{\nabla}_X\pi)(Y). \tag{18}$$

Using (16), we get

$$(\overline{\nabla}_X C_1^1 \overline{T})(Y) = \pi(X)(C_1^1 \overline{T})(Y). \tag{19}$$

Making use of (17) and (19), we obtain

$$(\overline{\nabla}_X C_1^1 \overline{T})(Y) = (n-1)\pi(X)\pi(Y). \tag{20}$$

Equating the right hand side of (18) and (20) implies

$$(\overline{\nabla}_X \pi)(Y) = \pi(X)\pi(Y). \tag{21}$$

Interchanging $X$ by $Y$ in the above equation, we get

$$(\overline{\nabla}_Y \pi)(X) = \pi(X)\pi(Y). \tag{22}$$

Thus from (21) and (22), we have

$$(\overline{\nabla}_X \pi)(Y) = (\overline{\nabla}_Y \pi)(X). \tag{23}$$

Hence the 1-form $\pi$ is closed with respect to $\overline{\nabla}$.
Again

$$(\overline{\nabla}_X \pi)(Y) = \overline{\nabla}_X \pi(Y) - \pi(\overline{\nabla}_X Y). \tag{24}$$

Applying (4) in (24), we get

$$(\overline{\nabla}_X \pi)(Y) = (\nabla_X \pi)(Y) - \psi(Y)\pi(X) - \psi(X)\pi(Y) - \phi(Y)\pi(X) + \phi(X)\pi(Y). \tag{25}$$

With the help of (5), the above equation yields

$$(\overline{\nabla}_X \pi)(Y) = (\nabla_X \pi)(Y) - \frac{(n-1)}{(n+1)}\pi(X)\pi(Y). \tag{26}$$

From (26), it follows that

$$(\overline{\nabla}_X \pi)(Y) - (\overline{\nabla}_Y \pi)(X) = (\nabla_X \pi)Y - (\nabla_Y \pi)X. \tag{27}$$

Since $\pi$ is closed with respect to the connection $\overline{\nabla}$, it follows that the 1-form $\pi$ is closed with respect to the connection $\nabla$.

It is easy to verify that both the 1-forms $\phi$ and $\psi$ are closed with respect to $\overline{\nabla}$ and $\nabla$. Also that the tensors $\Phi'$ and $\Psi'$ are symmetric. Consequently, we have

$$\beta(X,Y) = 0. \tag{28}$$

and

$$\alpha(X,Y) = \alpha(Y,X). \tag{29}$$

In view of (23) and (28), the expressions (7), (12) and (13) reduce to

$$\overline{R}(X,Y)Z = R(X,Y)Z + \alpha(X,Z)Y - \alpha(Y,Z)X, \tag{30}$$

$$\overline{S}(Y,Z) = S(Y,Z) - (n-1)\alpha(Y,Z), \tag{31}$$

$$\overline{r} = r - (n-1)a, \tag{32}$$

respectively. We easily observe that the Ricci tensor $\overline{S}$ is symmetric. Again using (9), (10) and (11) in (30), we obtain

$$\begin{aligned}\overline{R}(X,Y)Z &= R(X,Y)Z + [\Psi'(X,Z) + \Phi'(Z,X) - \psi(X)\phi(Z) - \phi(X)\psi(Z)]Y \\ &\quad -[\Psi'(Y,Z) + \Phi'(Z,Y) - \psi(Y)\phi(Z) - \psi(Z)\phi(Y)]X \\ &= R(X,Y)Z + [(\nabla_X\psi)Z - \psi(X)\psi(Z) + (\nabla_Z\phi)X - \phi(X)\phi(Z) \\ &\quad -\psi(X)\psi(Z) - \phi(X)\psi(Z)]Y - [(\nabla_Y\psi)Z - \psi(Y)\psi(Z) \\ &\quad +(\nabla_Z\phi)Y - \phi(Z)\phi(Y) - \psi(Y)\phi(Z) - \phi(Y)\psi(Z)]X \\ &= R(X,Y)Z + \frac{n}{(n+1)}[(\nabla_X\pi)(Z)Y - (\nabla_Y\pi)(Z)X] \\ &\quad -\frac{n^2}{(n+1)^2}[\pi(X)\pi(Z)Y - \pi(Y)\pi(Z)X].\end{aligned} \tag{33}$$

Contracting $X$ in (33), we get

$$\overline{S}(Y,Z) = S(Y,Z) - \frac{n(n-1)}{(n+1)}(\nabla_Y\pi)(Z) + \frac{n^2(n-1)}{(n+1)^2}\pi(Y)\pi(Z). \tag{34}$$

Making use of (33), (34) and closed 1-form $\pi$ in (15), we have

$$\overline{P}(X,Y)Z = P(X,Y)Z. \tag{35}$$

By the above discussion we can state the following:

**Theorem 1.** *If ($M^n$, $g$) ($n \geq 3$) is a Riemannian manifold admitting a projective semi-symmetric connection $\overline{\nabla}$, whose torsion tensor $\overline{T}$ is recurrent with respect to $\overline{\nabla}$, then the Weyl projective curvature tensor is invariant.*

Now we define (0,4) type tensors $\overline{\widetilde{R}}$ and $\widetilde{R}$ with respect to $\overline{\nabla}$ and $\nabla$ respectively,where

$$\widetilde{R}(X,Y,Z,W) = g(R(X,Y)Z,W) \tag{36}$$

and

$$\overline{\widetilde{R}}(X,Y,Z,W) = g(\overline{R}(X,Y)Z,W). \tag{37}$$

Then from (33), we have

$$\overline{\widetilde{R}}(X,Y,Z,W) = -\overline{\widetilde{R}}(Y,X,Z,W), \tag{38}$$

$$\overline{\widetilde{R}}(X,Y,Z,W) + \overline{\widetilde{R}}(X,Y,W,Z) \neq 0 \tag{39}$$

and

$$\overline{\widetilde{R}}(X,Y,Z,W) + \overline{\widetilde{R}}(Z,W,X,Y) \neq 0. \tag{40}$$

Thus we have the following:

**Theorem 2.** *If $(M^n, g)$ $(n \geq 3)$ is a Riemannian manifold admitting a projective semi-symmetric connection $\overline{\overline{\nabla}}$, whose torsion tensor is recurrent with respect to $\overline{\overline{\nabla}}$. Then*

*(a)* $\widetilde{\overline{\overline{R}}}(X,Y,Z,W) + \widetilde{\overline{\overline{R}}}(Y,X,Z,W) = 0$*,*

*(b)* $\widetilde{\overline{\overline{R}}}(X,Y,Z,W) + \widetilde{\overline{\overline{R}}}(X,Y,W,Z) \neq 0$*, in general,*

*(c)* $\widetilde{\overline{\overline{R}}}(X,Y,Z,W) + \widetilde{\overline{\overline{R}}}(Z,W,X,Y) \neq 0$*, in general.*

## 4 Projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor $\overline{R}$

In this section we consider a projective semi-symmetric connection $\overline{\overline{\nabla}}$ whose curvature tensor $\overline{R}$ vanishes and torsion tensor $\overline{T}$ is recurrent with respect to $\overline{\overline{\nabla}}$. Then (33) becomes

$$\begin{aligned} R(X,Y)Z \;&=\; \frac{n}{(n+1)}[(\nabla_Y\pi)(Z)X - (\nabla_X\pi)(Z)Y] \\ &\quad + \frac{n^2}{(n+1)^2}[\pi(X)\pi(Z)Y - \pi(Y)\pi(Z)X]. \end{aligned} \tag{41}$$

Now

$$\begin{aligned} &(R(X,Y)\cdot R)(U,V)W \\ &= R(X,Y)\cdot R(U,V)W - R(R(X,Y)U,V)W \\ &-R(U,R(X,Y)V)W - R(U,V)R(X,Y)W \\ &= \frac{n}{(n+1)}[(\nabla_V\pi)(W)R(X,Y)U - (\nabla_U\pi)(W)R(X,Y)V \\ &-(\nabla_Y\pi)(U)R(X,V)W + (\nabla_X\pi)(U)R(Y,V)W - (\nabla_Y\pi)(V)R(U,X)W \\ &+(\nabla_X\pi)(V)R(U,Y)W - (\nabla_Y\pi)(W)R(U,V)X + (\nabla_X\pi)(W)R(U,V)Y] \\ &-\frac{n^2}{(n+1)^2}[\pi(V)\pi(W)R(X,Y)U - \pi(U)\pi(W)R(X,Y)V \\ &-\pi(Y)\pi(U)R(X,V)W + \pi(X)\pi(U)R(Y,V)W - \pi(Y)\pi(V)R(U,X)W \\ &+\pi(X)\pi(V)R(U,Y)W - \pi(Y)\pi(W)R(U,V)X + \pi(X)\pi(W)R(U,V)Y] \end{aligned} \tag{42}$$

Now using (41) and closed 1-form $\pi$ in (42), we get

$$(R(X,Y)\cdot R)(U,V)W = 0 \tag{43}$$

This leads to the following:

**Theorem 3.** *Let $(M^n, g)$ $(n \geq 3)$ be a Riemannian manifold admitting a projective semi-symmetric connection $\overline{\overline{\nabla}}$, whose torsion tensor is recurrent with respect to the connection $\overline{\overline{\nabla}}$ and vanishing curvature tensor $\overline{R}$. Then the manifold is semi-symmetric with respect to the Levi-Civita connection $\nabla$.*

Now suppose that vector field $\rho$ is a unit vector field defined by $g(X, \rho) = \pi(X)$. Contracting $X$ in (41), we get

$$S(Y, Z) = \frac{n(n-1)}{(n+1)}(\nabla_Y \pi)Z - \frac{n^2(n-1)}{(n+1)^2}\pi(Y)\pi(Z). \tag{44}$$

Using (21) in (26), we get

$$(\nabla_X \pi)Y = \frac{2}{(n+1)}\pi(X)\pi(Y). \tag{45}$$

Applying (45) in (44), we get

$$S(Y, Z) = \lambda\pi(Y)\pi(Z), \tag{46}$$

where $\lambda = \frac{n(n-1)(2-n)}{(n+1)^2} \neq 0$ for $n \geq 3$.
From (46), we get

$$S(X, X) = \lambda[g(X, \rho)]^2 \text{ for all } X. \tag{47}$$

$$\text{Therefore } S(\rho, \rho) = \lambda, \text{ since } \rho \text{ is a unit vector.} \tag{48}$$

Let $\theta$ be the angle between $\rho$ and an arbitrary vector X,
then $cos\theta = \frac{g(X,\rho)}{\sqrt{g(\rho,\rho)}\sqrt{g(X,X)}} = \frac{g(X,\rho)}{\sqrt{g(X,X)}}$ [by our hypothesis $g(\rho, \rho) = 1$].
Since $cos\theta \leq 1$, so $[g(X, \rho)]^2 \leq g(X, X) = |X|^2$.
Thus from (46), we have

$$S(X, X) \leq \lambda|X|^2. \tag{49}$$

Let $l^2$ be the square length of the Ricci tensor. Then

$$l = S(Le_i, e_i), \tag{50}$$

where $L$ is the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor $S$, that is $g(LX, Y) = S(X, Y)$ for all $X$, $Y$ and $\{ e_i\}$ i = 1, 2, 3,..., $n$ is an orthonormal basis of the tangent space at a point.
Making use of (47), the above equations gives

$$\begin{aligned} l^2 &= S(Le_i, e_i) \\ &= \lambda\pi(Le_i)\pi(e_i) \\ &= \lambda g(Le_i, \rho)g(e_i, \rho) \\ &= \lambda g(L\rho, \rho) \\ &= \lambda S(\rho, \rho) \\ &= \lambda.\lambda \\ &= \lambda^2 \end{aligned} \tag{51}$$

This leads to the following:

**Lemma 1.** *The length of the Ricci tensor of a Riemannian manifold ($M^n$, g) ($n \geq 3$) admitting a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor $\overline{R}$ is constant.*

## 5 Sufficient conditions for a compact orientable Riemannian manifold admitting a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor $\overline{R}$ to be (a) conformal to a sphere in $E_{n+1}$ and (b) isometric to a sphere

First we give the definition of conformality between two Riemannian manifolds.

Let $(M^n, g)$ and $(\widetilde{M}^n, \widetilde{g})$ be two Riemannian manifolds. If there exists a one-one differentiable mapping $(M^n, g) \longrightarrow (\widetilde{M}^n, \widetilde{g})$ such that the angle between any two vectors at a point p of $M$ is always equal to that of the corresponding two vectors at the corresponding point $\widetilde{p}$ of $\widetilde{M}$, then $(M^n, g)$ is said to be conformal to $(\widetilde{M}^n, \widetilde{g})$. A sufficient condition was given by Y.Watanabe[12] as follows:

Let $M^n$ $(n \geq 3)$ be a Riemannian manifold, if there exists a non parallel vector field X such that the condition

$$\int_M S(X,X)\,dv = \frac{1}{2}\int_M |dX|^2\,dv + \frac{(n-1)}{n}\int_M (\partial X)^2\,dv \tag{52}$$

holds, then $M^n$ is conformal to a sphere in $E_{n+1}$, where $dv$ is the volume element of $M$ and $dX$ and $\partial X$ are curl and divergence of $X$ respectively.

In this section we consider a compact orientable Riemannian manifold $M^n$ $(n \geq 3)$ admitting a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor $\overline{R}$ without boundary having generator $\rho$, where $\rho$ is a unit vector field defined by $g(X, \rho) = \pi(X)$.
Substituting $X = \rho$ in (52) and making use (48), we obtain

$$\int_M \lambda\,dv = \frac{1}{2}\int_M |d\rho|^2\,dv + \frac{(n-1)}{n}\int_M (\partial\rho)^2\,dv. \tag{53}$$

From (47), we get

$$S(X,\rho) = \lambda\pi(X), \tag{54}$$

Suppose $\rho$ is a parallel vector field. Then $\nabla_X \rho = 0$.
Therefore by Ricci identity we have

$$R(X,Y)\rho = 0. \tag{55}$$

Contracting $X$ in (55), we get

$$S(Y,\rho) = 0. \tag{56}$$

Since $\lambda \neq 0$ and $\pi(X) \neq 0$, then from (54) we obtain $S(X,\rho) \neq 0$. Hence $\rho$ cannot be parallel vector field.

If a compact orientable Riemannian manifold $M^n$ admits a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor $\overline{R}$ without boundary, the vector field $\rho$ is a non-parallel vector field. If in such a case the condition (53) is satisfied, then by Watanabe's condition (52) $(M^n, g)$ $(n \geq 3)$ is conformal to a sphere in $E_{n+1}$.

Hence we can state the following:

**Theorem 4.** *If a compact orientable Riemannian manifold* $(M^n,\ g)$ $(n \geq 3)$ *without boundary admitting a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor* $\overline{R}$ *without boundary, satisfies the condition (53), then the manifold* $(M^n,\ g)$ $(n \geq 3)$ *is conformal to a sphere immersed in* $E_{n+1}$.

Further, we suppose that a compact orientable Riemannian manifold ($M^n$, g) admits a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor $\overline{R}$ without boundary, the unit vector field $\rho$ under consideration admits a non-isometric conformal motion generated by a vector X. Since $l^2$ is constant by Lemma(1), it follows that

$$\pounds_X l^2 = 0, \tag{57}$$

where $\pounds_X$ denotes the Lie differentiation with respect to X. Also from (46) we see that the scalar curvature $r$ is constant.

Now, it is known [13] that if a compact Riemannian manifold $M$ of dimension $n \geq 3$ with constant scalar curvature admits an infinitesimal non-isometric conformal transformation $X$ such that $\pounds_X l^2 = 0$, then $M$ is isometric to a sphere.

This leads to the following:

**Theorem 5.** *If a compact orientable Riemannian manifold* $(M^n,\ g)$ $(n \geq 3)$ *without boundary admits a projective semi-symmetric connection with recurrent torsion tensor and vanishing curvature tensor* $\overline{R}$ *and a non-isometric conformal transformation* $X$*, then the manifold is isometric to a sphere.*

**Acknowledgement.** The author is very thankful to the referee for his/her valuable suggestions towards the improvement of the paper.

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