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CLASSIFICATIONS OF K-CONTACT MANIFOLDS SATISFYING CERTAIN CURVATURE CONDITIONS

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Abstract

The object of the present paper is to classify K-contact metric manifolds satisfying certain curvature conditions on projective and concircular curvature tensors. K-contact manifolds satisfying $P.\tilde{C} = 0$, $\tilde{C}.P = 0$ and $\tilde{C}.S = 0$ are considered, where $\tilde{C}$ and P denote the concircular and projective curvature tensors respectively. Finally, we study K-contact manifold satisfying $S.R = 0$. It is shown that in all the cases the compact K-contact manifold becomes Sasakian.

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1 Introduction

A complete regular contact metric manifold M^{2n+1} carries a K-contact structure (ϕ, ξ, η, g), defined in terms of the almost Kähler structure (J, G) of the base manifold M^{2n}. Here the K-contact structure (ϕ, ξ, η, g) is Sasakian if and only if the base manifold (M^{2n}, J, G) is Kählerian. If (M^{2n}, J, G) is only almost Kähler, then (ϕ, ξ, η, g) is only K-contact [3]. In a Sasakian manifold the Ricci operator Q commutes with ϕ, that is, $Q\phi = \phi Q$. In [12] it has been shown that there exist K-contact manifolds with $Q\phi = \phi Q$ which are not Sasakian. It is to be noted that a K-contact manifold is intermediate between a contact metric manifold and a Sasakian manifold. K-contact and Sasakian manifolds have been studied by several authors such as ([2], [6], [7], [8], [9], [10], [15], [16], [20], [21], [25], [26], [27]) and many others. It is well known that every Sasakian manifold is K-contact, but the converse ia not true, in general. However a three-dimensional K-contact manifold is Sasakian [11]. The nature of a manifold mostly depends on its curvature tensor. Using the tools of conformal transformation, geometers have deduced

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conformal curvature tensor. In a similar way, with the help of projective transformation the notion of projective curvature has been defined [19]. Apart from conformal curvature tensor, the projective curvature tensor is another important tensor from the differential geometric point of view. A Riemannian manifold is said to be semisymmetric if its curvature tensor R satisfies $R(X,Y).R = 0$, where $R(X,Y)$ acts on R as a derivation [13].
The object of the present paper is to enquire under what conditions will a K contact manifold be a Sasakian manifold.
The present paper is organized as follows:
After a brief introduction in Section 2, we discuss about some preliminaries that will be used in the later sections. In section 3, we consider K-contact manifolds satisfying $P.\tilde{C} = 0$. Section 4 is devoted to the study of K-contact manifolds satisfying $\tilde{C}.P = 0$ and to prove that the manifold is Sasakian. In section 5, we consider K-contact manifolds satisfying $\tilde{C}.S = 0$. Section 6 deals with K-contact manifolds satisfying $S.R = 0$.

2 Preliminaries

An odd dimensional smooth manifold M^{2n+1} $(n \geq 1)$ is said to admit an almost contact structure, sometimes called a (ϕ, ξ, η)-structure, if it admits a tensor field ϕ of type $(1,1)$, a vector field ξ and a 1-form η satisfying ([3], [4])

$$\phi^2 = -I + \eta \otimes \xi, \;\; \eta(\xi) = 1, \;\; \phi\xi = 0, \;\; \eta \circ \phi = 0. \tag{1}$$

The first and one of the remaining three relations in (1) implies the other two relations in (1). An almost contact structure is said to be normal if the induced almost complex structure J on $M^n \times \mathbb{R}$ defined by

$$J(X, f\frac{d}{dt}) = (\phi X - f\xi, \eta(X)\frac{d}{dt}) \tag{2}$$

is integrable, where X is tangent to M, t is the coordinate of $\mathbb{R}$ and f is a smooth function on $M^n \times \mathbb{R}$. Let g be a compatible Riemannian metric with (ϕ, ξ, η), structure, that is,

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{3}$$

or equivalently,

$$g(X, \phi Y) = -g(\phi X, Y) \tag{4}$$

and

$$g(X, \xi) = \eta(X),$$

for all vector fields X, Y tangent to M. Then M becomes an almost contact metric manifold equipped with an almost contact metric structure (ϕ, ξ, η, g).
An almost contact metric structure becomes a contact metric structure if

$$g(X, \phi Y) = d\eta(X,Y), \tag{5}$$

for all X, Y tangent to M. The 1-form η is then a contact form and ξ is its characteristic vector field.
If ξ is a Killing vector field, then M^{2n+1} is said to be a K-contact manifold ([3], [17]). A contact metric manifold is Sasakian if and only if

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y. \tag{6}$$

Every Sasakian manifold is K-contact, but the converse need not be true, except in dimension three [11]. K-contact manifolds are not too well known, because there is no such a simple expression for the curvature tensor as in the case of Sasakian manifolds.

For details we refer to ([1], [3], [17], [18]).
Besides the above relations in K-contact manifold the following relations hold ([1], [3], [17]):

$$\nabla_X \xi = -\phi X. \tag{7}$$

$$\tilde{R}(\xi, X, Y, \xi) = \eta(R(\xi, X)Y) = g(X,Y) - \eta(X)\eta(Y). \tag{8}$$

$$R(\xi, X)\xi = -X + \eta(X)\xi. \tag{9}$$

$$S(X, \xi) = 2n\eta(X). \tag{10}$$

$$(\nabla_X \phi)Y = R(\xi, X)Y, \tag{11}$$

for any vector fields X, Y.
Again a K-contact manifold is called Einstein if the Ricci tensor S is of the form $S = \lambda g$, where λ is a constant and η- Einstein if the Ricci tensor S is of the form $S = ag + b\eta \otimes \eta$, where a, b are smooth functions on M. It is well known [11] that in a K-contact manifold a and b are constants. Also it is known [5] that a compact η-Einstein K-contact manifold is Sasakian provided $a \geq$-2.
A transformation of a $(2n+1)$-dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle, is called a concircular transformation ([14], [23]). A concircular transformation is always a conformal transformation [14]. Here geodesic circle means a curve in M whose first curvature is constant and whose second curvature is identically zero. Thus, the geometry of concircular transformations, i.e., the concircular geometry, is generalization of inversive geometry in the sense that the change of metric is more general than that induced by a circle preserving diffeomorphism . An interesting invariant of a concircular transformation is the concircular curvature tensor $\tilde{C}$. It is defined by ([22], [24])

$$\tilde{C}(X,Y)W = R(X,Y)W - \frac{r}{2n(2n+1)}[g(Y,W)X - g(X,W)Y], \tag{12}$$

where $X, Y, W \in T(M)$. Riemannian manifolds with vanishing concircular curvature tensor are of constant curvature. Thus the concircular curvature tensor is a measure of the failure of a Riemannian manifold to be of constant curvature. If there exists a one-to-one correspondence between each coordinate neighborhood of M and a domain in Euclidian space such that any geodesic of the Riemannian manifold corresponds to a straight line in the Euclidean space, then M is said to be locally projectively flat. For $n \geq 1$, M is locally projectively flat if and only if the well known projective curvature tensor P vanishes. Here P is defined by [19]

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - S(X,Z)Y], \tag{13}$$

for all X, Y, $Z \in T(M)$, where R is the curvature tensor and S is the Ricci tensor. In fact M is projectively flat if and only if it is of constant curvature [24]. Thus the projective curvature tensor is the measure of the failure of a Riemannian manifold to be of constant curvature.

Lemma 1. *[5] A compact K-contact Einstein manifold is Sasakian.*

3 K-contact manifolds satisfying $P.\tilde{C} = 0$

In this section we consider K-contact manifolds satisfying $P.\tilde{C} = 0$. Therefore

$$(P(X,Y).\tilde{C})(U,V)W = 0. \tag{14}$$

This implies

$$\begin{aligned} P(X,Y)\tilde{C}(U,V)W - \tilde{C}(P(X,Y)U,V)W &- \tilde{C}(U,P(X,Y)V)W \\ &- \tilde{C}(U,V)P(X,Y)W = 0. \end{aligned} \tag{15}$$

Putting $V = W = \xi$ in (15) we have

$$\begin{aligned} P(X,Y)\tilde{C}(U,\xi)\xi - \tilde{C}(P(X,Y)U,\xi)\xi &- \tilde{C}(U,P(X,Y)\xi)\xi \\ &- \tilde{C}(U,\xi)P(X,Y)\xi = 0. \end{aligned} \tag{16}$$

Now,

$$\begin{aligned} P(X,Y)\tilde{C}(U,\xi)\xi &= P(X,Y)\{R(X,\xi)\xi - \frac{r}{2n(2n+1)}[U - \eta(U)\xi]\} \\ &= P(X,Y)(1 - \frac{r}{2n(2n+1)})\{(U - \eta(U)\xi\} \\ &= (1 - \frac{r}{2n(2n+1)})\{P(X,Y)U - \eta(U)P(X,Y)\xi\}. \end{aligned} \tag{17}$$

Similarly,

$$\tilde{C}(P(X,Y)U,\xi)\xi = (1 - \frac{r}{2n(2n+1)})\{P(X,Y)U - \eta(P(X,Y)U)\xi\}. \tag{18}$$

$$\begin{aligned} \tilde{C}(U,P(X,Y)\xi)\xi &= \tilde{C}(U,R(X,Y)\xi - \frac{1}{2n}[S(Y,\xi)X - S(X,\xi)Y]\xi \\ &= \tilde{C}(U - R(X,Y)\xi - \{\eta(Y)X - \eta(X)Y\})\xi. \end{aligned} \tag{19}$$

and

$$\tilde{C}(U,\xi)P(X,Y)\xi = \tilde{C}(U,\xi)[R(X,Y)\xi - \{\eta(Y)X - \eta(X)Y\}]. \tag{20}$$

Using (17), (18), (19), (20) in (16) we have

$$\begin{aligned}
& (1 - \frac{r}{2n(2n+1)})\{P(X,Y)U - \eta(U)P(X,Y)\xi\} \\
- & (1 - \frac{r}{2n(2n+1)})\{P(X,Y)U - \eta(P(X,Y)U)\xi\} \\
- & \tilde{C}(U - R(X,Y)\xi - \{\eta(Y)X - \eta(X)Y\})\xi \\
- & \tilde{C}(U,\xi)[R(X,Y)\xi - \{\eta(Y)X - \eta(X)Y\}] = 0.
\end{aligned} \tag{21}$$

Putting $Y = \xi$ in (21) we have

$$(1 - \frac{r}{2n(2n+1)})\eta(P(X,\xi)U)\xi = 0. \tag{22}$$

Therefore either $r = 2n(2n+1)$ or $\eta(P(X,\xi)U) = 0$.
Now,

$$\begin{aligned}
\eta(P(X,\xi)U) &= g(P(X,\xi)U,\xi) \\
&= g(R(X,\xi)U - \frac{1}{2n}\{S(\xi,U)X - S(X,U)\xi\},\xi) \\
&= g(R(X,\xi)U,\xi) - \frac{1}{2n}\{2n\eta(V)\eta(X) - S(X,U)\} \\
&= -g(R(X,\xi)\xi,\xi) - \eta(U)\eta(X) + \frac{1}{2n}S(X,U) \\
&= -g(X - \eta(X)\xi, U) - \eta(U)\eta(X) + \frac{1}{2n}S(X,U) \\
&= -g(X,U) + \eta(X)\eta(U) - \eta(U)\eta(X) + \frac{1}{2n}S(X,U) \\
&= -g(X,U) + \frac{1}{2n}S(X,U).
\end{aligned} \tag{23}$$

Thus $\eta(P(X,\xi)U) = 0$ implies $S(X,U) = 2ng(X,U)$. Therefore the manifold is an Einstein manifold. Thus we can state the following:

Theorem 1. *A K-contact manifold satisfying $P.\tilde{C} = 0$ is either an Einstein manifold or the manifold is of constant scalar curvature $2n(2n+1)$.*

It is known that [5] a compact K-contact Einstein manifold is Sasakian. Thus we get the following:

Corollary 1. *A compact K-contact manifold satisfying $P.\tilde{C} = 0$ is Sasakian or the manifold is of constant scalar curvature $2n(2n+1)$.*

4 K-contact manifolds satisfying $\tilde{C}.P = 0$

In this section we consider a K contact manifold satisfying $\tilde{C}.P = 0$. Therefore we have

$$(\tilde{C}(X,Y).P)(U,V)W = 0. \tag{24}$$

This implies

$$\begin{aligned}\tilde{C}(X,Y)P(U,V)W - P(\tilde{C}(X,Y)U,V)W &- P(U,\tilde{C}(X,Y)V)W \\ &- P(U,V)\tilde{C}(X,Y)W = 0.\end{aligned} \tag{25}$$

Now from the above equation with the help of (10) and (13) we get

$$P(\xi,V)\xi = P(V,\xi)\xi = 0. \tag{26}$$

Putting $V = W = \xi$ in (25) we have

$$\begin{aligned}\tilde{C}(X,Y)P(U,\xi)\xi - P(\tilde{C}(X,Y)U,\xi)\xi &- P(U,\tilde{C}(X,Y)\xi)\xi \\ &- P(U,\xi)\tilde{C}(X,Y)\xi = 0.\end{aligned} \tag{27}$$

Now,

$$\tilde{C}(X,Y)P(U,\xi)\xi = 0. \tag{28}$$

$$P(\tilde{C}(X,Y)U,\xi)\xi = 0. \tag{29}$$

$$P(U,\tilde{C}(X,Y)\xi)\xi = P(U,R(X,Y)\xi - \frac{r}{2n(2n+1)}\{\eta(Y)X - \eta(X)Y\}. \tag{30}$$

$$P(U,\xi)\tilde{C}(X,Y)\xi = P(U,\xi)\{R(X,Y)\xi - \frac{r}{2n(2n+1)}\{\eta(Y)X - \eta(X)Y\}. \tag{31}$$

Using (28), (29), (30) and (31) in (27) we have

$$\begin{aligned}&- P(U,R(X,Y)\xi - \frac{r}{2n(2n+1)}\{\eta(Y)X - \eta(X)Y\} \\ &- P(U,\xi)\{R(X,Y)\xi - \frac{r}{2n(2n+1)}\{\eta(Y)X - \eta(X)Y\} = 0.\end{aligned} \tag{32}$$

Putting $Y = \xi$ in (32) we obtain

$$\begin{aligned}&- P(U,R(X,\xi)\xi - \frac{r}{2n(2n+1)}\{X - \eta(X)\xi\} \\ &- P(U,\xi)\{R(X,\xi)\xi - \frac{r}{2n(2n+1)}\{X - \eta(X)\xi\} = 0.\end{aligned} \tag{33}$$

$$-(1-\frac{r}{2n(2n+1)})P(U,X)\xi-(1-\frac{r}{2n(2n+1)})P(U,\xi)X=0. \tag{34}$$

This implies

$$(1-\frac{r}{2n(2n+1)}(P(U,X)\xi+P(U,\xi)X\}=0. \tag{35}$$

Therefore either $r=2n(2n+1)$ or $P(U,X)\xi+P(U,\xi)X=0$.
Now,

$$P(U,X)\xi+P(U,\xi)X=0. \tag{36}$$

This implies

$$\begin{aligned} R(U,X)\xi &- \frac{1}{2n}\{S(X,\xi)U-S(U,\xi)X\}+R(U,\xi)X \\ &- \frac{1}{2n}(S(\xi,X)U-S(U,X)\xi)=0. \end{aligned} \tag{37}$$

With the help of (10) and (37) we have

$$R(U,X)\xi+R(U,\xi)X-2\eta(X)U+\eta(U)X+S(U,X)\xi=0. \tag{38}$$

Interchanging X and U in (38) we have

$$R(X,U)\xi+R(X,\xi)U-2\eta(U)X+\eta(X)U+S(X,U)\xi=0. \tag{39}$$

Subtracting (39) from (38) we have

$$R(U,X)\xi+R(U,\xi)X-R(X,U)\xi-R(X,\xi)U-3\eta(X)U+3\eta(U)X=0. \tag{40}$$

Using Bianchi identity we get from the above equation

$$3R(U,X)\xi=3\eta(X)U-3\eta(U)X, \tag{41}$$

or,

$$R(U,X)\xi=\eta(X)U-\eta(U)X, \tag{42}$$

Hence the manifold is a Sasakian manifold. Now we are in a position to state the following:

Theorem 2. *A K-contact manifold satisfying $\tilde{C}.P=0$ is a Sasakian manifold or the manifold is of constant scalar curvature $2n(2n+1)$.*

5 K-contact manifolds satisfying $\tilde{C}.S = 0$

In this section we consider K-contact manifolds satisfying $\tilde{C}.S = 0$, Therefore we have

$$(\tilde{C}(X,Y).S)(U,V) = 0. \tag{43}$$

This implies

$$S(\tilde{C}(X,Y)U,V) + S(U,\tilde{C}(X,Y)V) = 0. \tag{44}$$

Putting $Y = U = \xi$ in (44) we have

$$S(\tilde{C}(X,\xi)\xi,V) + S(\xi,\tilde{C}(X,\xi)V) = 0. \tag{45}$$

$$S((1 - \frac{r}{2n(2n+1)})(X - \eta(X)\xi),V) + 2ng(\tilde{C}(X,\xi)V,\xi) = 0. \tag{46}$$

$$(1 - \frac{r}{2n(2n+1)})\{S(X,V) - 2n\eta(X)\eta(Y)\} - 2ng(\tilde{C}(X,\xi)\xi,V) = 0. \tag{47}$$

This implies

$$\begin{aligned}&(1 - \frac{r}{2n(2n+1)})\{S(X,V) - 2n\eta(X)\eta(V)\}\\ &-2n(1 - \frac{r}{2n(2n+1)})\{g(X,V) - \eta(X)\eta(V)\}) = 0.\end{aligned} \tag{48}$$

Therefore

$$(1 - \frac{r}{2n(2n+1)})\{S(X,V) - 2ng(X,V)\} = 0. \tag{49}$$

Therefore, either $r = 2n(2n+1)$ or, $S(X,V) = 2ng(X,V)$. Hence either $r = 2n(2n+1)$ or, the manifold is an Einstein manifold.
Conversely, let the manifold be an Einstein manifold, that is, $S(X,V) = 2ng(X,V)$. Therefore,

$$\begin{aligned}& S(\tilde{C}(X,Y)U,V) + S(U,\tilde{C}(X,Y)V)\\ =\ & 2n[g(\tilde{C}(X,Y)U,V) + g(U,\tilde{C}(X,Y)V)]\\ =\ & 2n[g(\tilde{C}(X,Y)U,V) - g(\tilde{C}(X,Y)U,V)]\\ =\ & 0.\end{aligned} \tag{50}$$

Therefore we can state the following:

Theorem 3. *A (2n+1)-dimensional K-contact manifold satisfies $\tilde{C}.S = 0$ if and only if the manifold is Einstein provided $r \neq 2n(2n+1)$.*

By the Lemma 1 and the Theorem 3 we can state the following:

Corollary 2. *A (2n+1)-dimensional compact K-contact manifold satisfies $\tilde{C}.S = 0$ is Sasakian provided $r \neq 2n(2n+1)$.*

6 K-contact manifolds satisfying $S.R = 0$

In this section we consider K-contact satisfying $S.R = 0$. Therefore

$$(S(X,Y).R)(U,V)W = 0. \tag{51}$$

This implies

$$\begin{aligned}(X \wedge_S Y)R(U,V)W &+ R((X \wedge_S Y)U,V)W + R(U,(X \wedge_S Y)V)W \\ &+ R(U,V)(X \wedge_S Y)W = 0,\end{aligned} \tag{52}$$

where the endomorphism $X \wedge_S Y$ is given by

$$(X \wedge_S Y)W = S(Y,W)X - S(X,W)Y. \tag{53}$$

Using (53) in (52) we have

$$\begin{aligned}&S(Y,R(U,V)W)X - S(X,R(U,V)W)Y + S(Y,U)R(X,V)W \\ -\ &S(X,U)R(Y,V)W + S(Y,V)R(U,X)W - S(X,V)R(U,Y)W \\ +\ &S(Y,W)R(U,V)X - S(X,W)R(U,V)Y = 0.\end{aligned} \tag{54}$$

Putting $X = V = W = \xi$ in (54) we have

$$\begin{aligned}&S(Y,R(U,\xi)\xi)\xi - S(\xi,R(U,\xi)\xi)Y + S(Y,U)R(\xi,\xi)\xi \\ -\ &S(\xi,U)R(Y,\xi)\xi + S(Y,\xi)R(U,\xi)\xi - S(\xi,\xi)R(U,Y)\xi \\ +\ &S(Y,\xi)R(U,\xi)\xi - S(\xi,\xi)R(U,\xi)Y = 0.\end{aligned} \tag{55}$$

This implies

$$\begin{aligned}&S(Y,U)\xi - \eta(U)S(Y,\xi)\xi - S(\xi,U)Y + \eta(U)S(\xi,\xi)Y \\ -\ &S(U,\xi)Y + \eta(Y)S(U,\xi)\xi + S(Y,\xi)U - \eta(u)S(Y,\xi)\xi \\ -\ &S(\xi,\xi)R(U,Y)\xi + S(Y,\xi)U - \eta(U)S(Y,\xi)\xi - S(\xi,\xi)R(U,\xi)Y \\ -\ &S(\xi,U)Y + \eta(U)S(\xi,\xi)Y = 0.\end{aligned} \tag{56}$$

Using $S(X,\xi) = 2n\eta(X)$ in (56) we have

$$S(Y,U) + 2n\eta(Y)\eta(U) - 2n\eta(R(U,\xi)Y) = 0. \tag{57}$$

Now,

$$\begin{aligned}\eta(R(U,\xi)Y) &= g(R(U,\xi)Y,\xi) \\ &= -g(R(U,\xi)\xi,Y) \\ &= -g(U-\eta(U)\xi,Y) \\ &= \eta(U)\eta(Y) - g(U,Y).\end{aligned} \tag{58}$$

Using (58) in (57) yields

$$S(Y,U) = -2ng(Y,U). \tag{59}$$

Therefore the manifold is an Einstein manifold. Thus we can state the following:

Theorem 4. *A K-contact manifold satisfying $S.R = 0$ is an Einstein manifold.*

It is known that [5] a compact K-contact Einstein manifold is Sasakian. Thus we get the following:

Corollary 3. *A compact K-contact manifold satisfying $S.R = 0$ is Sasakian.*

Observations:
It is easy to see that $R.\tilde{C} \equiv R.R$ provided $r = 2n(2n+1)$, that is $R.\tilde{C} = 0$ and $R.R = 0$ are equivalent. In [20], Tanno proved that a K-contact manifold satisfying $R.R = 0$ is a manifold of constant curvature and hence Einstein manifold. Thus we can state the following:

Remark 1. *A K-contact manifold satisfying $R.\tilde{C} = 0$ is an Einstein manifold provided $r = 2n(2n+1)$.*

Also, if $r = 2n(2n+1)$, then $\tilde{C}.\tilde{C} \equiv R.R$. Therefore we can state the following:

Remark 2. *A K-contact manifold satisfying $\tilde{C}.\tilde{C} = 0$ is an Einstein manifold provided $r = 2n(2n+1)$.*

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References

[1] Arslan, K., Murathan, C. and *Özgür*, C., *On ϕ-conformally flat contact metric manifolds*, Balkan J. Geom. Appl. (BJGA), **5(2)**(2000), 1-7.

[2] Binh, T. Q., De, U. C., Tamassy, L., *On partially pseudo symmetric K-contact Riemannian manifolds*, Acta Math. Acad. Paedagogicae Nyíregyháziensis, **18**(2002), 19-25.

[3] Blair, D. E., *Contact manifold in Riemannian geometry*, Lecture notes on mathematics, 509, Springer-Verlag, Berlin, 1976.

[4] Blair, D. E., *Riemannian geometry on contact and symplectic manifolds*, Progress in Math., Vol. 203, 2001, Berlin.

[5] Boyer, C. P. and Galicki, K., *Einstein manifold and contact geometry*, Proc. Amer. Math. Soc., **129**(2001), 2419-2430.

[6] Chaki, M. C. and Ghosh, D., *On a type of K-contact Riemanian manifold*, J. Australian Math. soc., **13**(1972), 447-450.

[7] De, U. C. and Biswas, S., *On K-contact η-Einstein maifolds*, Bull. Math. Soc. Sc. Math. Roumanie. Tome. **48**(96)(2005), 295-301.

[8] De, U. C. and Ghosh, S., *On a class of K-contact manifolds*, SUT. J. Math. **45**(2)(2009), 103-118.

[9] De, U. C. and De, A., *On some curvature properties of K-contact manifolds*, Extracta Mathematicae **27**(2012), 125-134.

[10] Endo, H., *On K-contact Riemannian manifolds with vanishing E-contact Bochner curvature tensor*, Colliq. Math. **62**(1991), 293-297.

[11] Jun, J. B. and Kim, U. K., *On* 3*-dimentional almost contact metric manifold*, Kyungpook Math. J., **34**(1994), 293-301.

[12] Koufogiorgos, T., *Contact metric manifolds*, Ann. Global Anal. Geom. **11**(1993), 25-34.

[13] Kowalaski, O., *An explicit classification of* 3*-dimensional Riemannian spaces satisfying* $R(X,Y).R = 0$, Czechoslovak Math. J. **46**(121)(1996), 427-474.

[14] Kuhnel, W., *Conformal transformations between Einstein spaces, Conformal geometry* (Bonn,1985/1986), 105-146, Aspects Math., E 12, Vieweg, Braunschweig, 1988.

[15] Majhi, P., and De, U. C., *Concircular curvature tensor on K-contact manifolds*, Acta Math. Acad. Paed. Nyir(AMAPN), **29**(2013), 89-99.

[16] Majhi, P., Matsuyama, Y. and De, U. C., *On a class of K-contact manifolds*, Rev. Acad. canar. cienc., xxvl, 9-20 (2014) (publicado en julio de 2015).

[17] Sasaki, S., *Lecture notes on almost contact manifold*, Part-1. Tohoku University, 1965.

[18] Sasaki, S., *Almost contact manifolds, Lecture notes*, Tohoku Univ. Vol. 1, 1965; Vol.2, 1967; Vol.3, 1968.

[19] Sooś, S., *Über die geodätischen Abbildungen von Riemannschen R̈umen auf projektiv symmetrische Riemannsche R̈ume*, Acta Math. Acad. Sci. Hungarica, **9**(1958), 359-361.

[20] Tanno, S., *Locally symmetric K-contact manifolds*, Proc. Japan Acad., **43(7)**(1967), 581-583.

[21] Tanno, S., *A remark on transformations of a K-contact manifold*, Tohoku Math. J., **16(2)**(1964), 173-175.

[22] Tanno, S., *Hypersurfaces satisfying a certain condition on Ricci tensor* Tohoku Math. J., **21**(1969), 297-303.

[23] Yano, K., *Concircular geometry I. concircular transformations*, Proc. Imp. Acad. Tokyo **16**(1940), 195-200.

[24] Yano, K. and Bochner, S., *Curvature and Betti numbers Annals of Mathematics Studies* 32, Princeton University Press, 1953.

[25] Zhen, G., *On conformal symmetric K-contact manifolds*, Chinese Quart. J. Math., **7**(1992), 5-10.

[26] Zhen, G., Cabrerizo, J. L., Fernandez, L. M. and Fernandez, M., *On ξ-conformally flat K-contact metric manifolds*, Indian J. Pure Appl. Math., **28**(1997), 725-734.

[27] Zhen, G., Cabrerizo, J. L., Fernandez, L. M. and Fernandez, M., *The structure of a class of K-contact manifolds*, Acta Math. Hungar, **82(4)**(1999), 331-340.