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IDEALS OF A COMMUTATIVE ROUGH SEMIRING

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Abstract

In this paper, we proved that for every subset X of U there is a corresponding ideal J_X of the rough semiring (T, Δ, ∇) also we gave a characterization theorem for the ideals in the rough semiring (T, Δ, ∇) by proving every ideal in (T, Δ, ∇) will be of the form J_X for some subset X of U and the properties of these ideals are discussed with suitable examples.

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1 Introduction

Fundamentals of semigroups were discussed by J. M. Howie [14] in his classical book in 2003. Z.Pawlak [23] introduced the concept of rough set theory in 1982 to process incomplete information in the information system and it is defined as a pair of sets called lower and upper approximation. Rough sets can be applied in many fields like data analysis, pattern recognition, remove redundancies and generate decision rules. Also rough set theory will be applied in several fields like computational intelligence such as machine learning, intelligent systems, knowledge discovery, expert systems and others [27],[22],[5],[1],[7]. Praba and Mohan [26] discussed the concept of rough lattice. In this paper the authors considered an information system $I = (U, A)$. A partial ordering relation was defined on $T = \{RS(X) \mid X \subseteq U\}$. The least upper bound and greatest lower bound were established using the operation $Praba\ \Delta$ and $Praba\ \nabla$. Praba et al.[24] discussed a commutative regular monoid on rough sets under the operation $Praba\ \Delta$ in 2013. In this paper the authors dealt with the rough ideals on (T, Δ). Manimaran et al.[20] studied the notion of a

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regular rough ∇ monoid of idempotents under $Praba\ \nabla$ in 2014. Praba et al. [25] dealt with semiring on the set of all rough sets also the authors discussed the pivot rough set as rough ideals on rough semiring in 2014. Manimaran et al.[21] discussed the characterization of rough semiring in 2017. Also, we defined the concept of rough homomorphism between two rough semirings (T, Δ, ∇) and (T', Δ_1, ∇_1). N. Kuroki and P. P. Wang [17] discussed some properties of lower and upper approximations with respect to the normal subgroup. R. Biswas and S. Nanda [2] introduced the notion of rough groups and rough subgroups. The concept of rough ideal semigroup was introduced by Kuroki [18] in 1997. M. Kondo [16] described the notion of the structure on generalized rough sets in 2006. Changzhong Wang and Degang Chen [4] discussed about a short note on some properties of rough groups and the authors studied the image and inverse image of rough approximations of a subgroup with respect to a homomorphism between two groups in 2010. Zadeh [28] introduced the concept of fuzzy sets in his paper. Golan [11] described the concept of ideals in semirings in 1999.

Yonghong Liu [19] dealt with the concepts of special lattice of rough algebras in 2011. Ronnason Chinram [6] introduced the concept of rough prime ideals and rough fuzzy prime ideals in gamma semigroups in 2009. Also the authors T. B. Iwinski [15] and Z. Bonikowaski [3] studied algebraic properties of rough sets. The concept of rough fuzzy sets and fuzzy rough sets was introduced by D. Dubois, H. Parade [8]. Nick C. Fiala [9] discussed about semigroup, monoid and group models of groupoid identities in his paper. Gupta and Chaudhari [12] described that an ideal is a partitioning ideal if and only if it is a subtractive ideal. They also proved that a monic ideal is a partitioning ideal if and only if it is a substractive ideal. Hong et al. [13] dealt with some resultants over commutative idempotent semirings in 2017.

In this paper we discuss the ideals of a rough semiring (T, Δ, ∇) and we give a relation between the principal rough ideal of a commutative regular rough monoid of idempotent (T, ∇) and the rough semiring (T, Δ, ∇) for the given information system $I = (U, A)$ where the information system is defined by using the universal set U and a nonempty set of fuzzy attributes A. The paper is organized as follows.

In section 2, we give the necessary definitions related to rough set theory.

In section 3, we deal with the ideals of a rough semiring (T, Δ, ∇) and a relation between the principal rough ideal of a commutative regular rough monoid of idempotent (T, ∇) and the ideals of a rough semiring (T, Δ, ∇).

Section 4 deals with the properties of the ideals of rough semiring.

Section 5 gives the conclusion.

2 Preliminaries

In this section we present some preliminaries in rough sets and monoids.

2.1 Rough sets

An information system is a pair $I = (U, A)$ where U is a non empty finite set of objects, called universal set and A is a nonempty set of fuzzy attributes defined by $\mu_a : U \to [0, 1]$, $a \in A$, is a fuzzy set. *Indiscernibility* is a core concept of rough set theory and it is defined as an equivalence between objects. Objects in the information system about which we have the same knowledge forms an equivalence relation.

Formally any set $P \subseteq A$, there is an associated equivalence relation called $P - Indiscernibility$ relation defined as follows,

$$IND(P) = \{(x, y) \in U^2 \mid \forall a \in P, \mu_a(x) = \mu_a(y)\}.$$

The partition induced by $IND(P)$ consists of equivalence classes defined by

$$[x]_p = \{y \in U \mid (x, y) \in IND(P)\}.$$

For any $X \subseteq U$, define the lower approximation space $\underline{P}(X) = \{x \in U \mid [x]_p \subseteq X\}$.
Also, define the upper approximation space $\overline{P}(X) = \{x \in U \mid [x]_p \cap X \neq \phi\}$.

Let $I = (U, A)$ be an information system, where U is a non empty finite set of objects, called the universe, A is a non empty finite fuzzy set of attributes and $T = \{RS(X) \mid X \subseteq U\}$ denotes the set of all rough sets.

Definition 2.1 (Rough set)**.** *A rough set corresponding to X, where X is an arbitrary subset of U in the approximation space P, we mean the ordered pair $RS(X) = (\underline{P}(X), \overline{P}(X))$.*

Example 2.1. *[26] Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $A = \{a_1, a_2, a_3, a_4\}$ where each a_i $(i = 1$ to $4)$ is a fuzzy set whose membership values are shown in Table 1.*

Table 1:

A/U	a_1	a_2	a_3	a_4
x_1	0	0.1	0.3	0.2
x_2	1	0.6	0.7	0.3
x_3	0	0.1	0.3	0.2
x_4	1	0.6	0.7	0.3
x_5	0.8	0.5	0.2	0.4
x_6	1	0.6	0.7	0.3

Let $X = \{x_1, x_3, x_5, x_6\}$ and $P = A$. Then the equivalence classes induced by the $P - Indiscernibility$ are given below.

$$X_1 = [x_1]_p = \{x_1, x_3\} \quad (1)$$
$$X_2 = [x_2]_p = \{x_2, x_4, x_6\} \quad (2)$$
$$X_3 = [x_5]_p = \{x_5\} \quad (3)$$

$$\text{Hence,}\quad \underline{P}(X) = \{x_1, x_3, x_5\} \text{ and}$$
$$\overline{P}(X) = \{x_1, x_2, x_3, x_4, x_5, x_6\}.$$
$$\text{Therefore}\quad RS(X) = (\{x_1, x_3, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}).$$

Note that the upper approximation space consists of those objects that are possibly members of the target set X.

Definition 2.2. *[26] If $X \subseteq U$, then the number of equivalence classes (Induced by IND(P)) contained in X is called as the Ind. weight of X. It is denoted by $IW(X)$.*

Example 2.2. *[26] Let $U = \{x_1, x_2, \cdots, x_6\}$ as in Table 1 and from equations (1),(2) & (3). The equivalence classes induced by $IND(P)$ are*

$$\begin{aligned} [x_1]_p &= \{x_1, x_3\} \\ [x_2]_p &= \{x_2, x_4, x_6\} \\ [x_5]_p &= \{x_5\} \end{aligned}$$

Let $X = \{x_1, x_4, x_5\} \subseteq U$ then by definition, Ind. weight of $X = IW(X) = 1$ (since there is only one equivalence class $[x_5]_p = \{x_5\}$ present in X).

Definition 2.3. *[26] Let $X, Y \subseteq U$. The Praba Δ is defined as*
$X\Delta Y = X \cup Y$, if $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$.
If $IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y)$, then identify the equivalence class obtained by the union of X and Y. Then delete the elements of that class belonging to Y. Call the new set as Y. Now, obtain $X\Delta Y$. Repeat this process until $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$.

Example 2.3. *[26] Let $U = \{x_1, x_2, \ldots, x_6\}$ as in Table 1.*
Let $X = \{x_2, x_4, x_5\}, Y = \{x_1, x_6\} \subseteq U$ then by definition,

$$IW(X) = 1;\ IW(Y) = 0;\ IW(X \cup Y) = 2;\ IW(X \cap Y) = 0$$

Here,

$$IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y).$$

The new equivalence class formed in $X \cup Y$ is $[x_2]_p$. As $x_6 \in Y$ and x_6 is an element of $[x_2]_p$, delete x_6 from Y. Now the new Y is $\{x_1\}$. Now for $X = \{x_2, x_5, x_6\}$ and $Y = \{x_1\}$. Finding $IW(X \cup Y)$,

$$IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y).$$

Therefore, $X\Delta Y = X \cup Y = \{x_1, x_2, x_4, x_5\}$.

Definition 2.4. *[26] If $X, Y \subseteq U$ then an element $x \in U$ is called a Pivot element, if $[x]_p \nsubseteq X \cap Y$, but $[x]_p \cap X \neq \phi$ and $[x]_p \cap Y \neq \phi$*

Definition 2.5. *[26] If $X, Y \subseteq U$ then the set of Pivot elements of X and Y is called the Pivot set of X and Y and it is denoted by $P_{X \cap Y}$.*

Definition 2.6. *[26] Praba ∇ of X and Y is denoted by $X\nabla Y$ and it is defined as*

$$X\nabla Y = \{x \mid [x]_p \subseteq X \cap Y\} \cup P_{X\cap Y} \text{ where } X, Y \subseteq U.$$

Note that each Pivot element in $P_{X\cap Y}$ is the representative of that particular class.

Example 2.4. *[26] Let $U = \{x_1, x_2, \ldots, x_6\}$ as in Table 1.*
and let $X = \{x_1, x_2, x_4, x_5\}$ and $Y = \{x_3, x_5, x_6\} \subseteq U$ then $X \cap Y = \{x_5\}$
Here, $[x_1]_p \nsubseteq X \cap Y$, but $[x_1]_p \cap X \neq \phi$ and $[x_1]_p \cap Y \neq \phi$. Therefore x_1 is a pivot element
Similarly x_2 is a pivot element. Also pivot set $P_{X\cap Y} = \{x_1, x_2\}$. Therefore $X\cap Y = \{x_1, x_2, x_5\}$.
Similarly $Y\nabla X = \{x_3, x_5, x_6\}$
$\therefore X\nabla Y \neq Y\nabla X$
$RS(X\nabla Y) = ([x_5]_p, [x_1]_p \cup [x_2]_p \cup [x_5]_p)$ *and* $RS(Y\nabla X) = ([x_5]_p, [x_1]_p \cup [x_2]_p \cup [x_5]_p)$
$\therefore RS(X\nabla Y) = RS(Y\nabla X)$.

Definition 2.7 (Binary operation as Δ)**.** *[24] Let T be the collection of rough sets and let $\Delta : T \times T \to T$ such that $\Delta(RS(X), RS(Y)) = RS(X\Delta Y)$.*

Theorem 2.1. *[24] Let $I = (U, A)$ be an information system where U is the universal (finite) set and A is the set of attributes and T is the set of all rough sets then (T, Δ) is a commutative monoid of idempotents.*

Theorem 2.2. *[24] (T, Δ) is a regular rough monoid of idempotents.*

Definition 2.8 (Binary operation as ∇)**.** *[20] Let T be the collection of rough sets and let $\nabla : T \times T \to T$ such that $\nabla(RS(X), RS(Y)) = RS(X\nabla Y)$.*

Theorem 2.3. *[20] Let $I = (U, A)$ be an information system where U is the universal (finite) set and A is the set of attributes and T is the set of all rough sets then (T, ∇) is a monoid of idempotents and it is called rough monoid of idempotents.*

Theorem 2.4. *[20] (T, ∇) is a commutative rough ∇ monoid of idempotents.*

Theorem 2.5. *[20] (T, ∇) is a commutative regular rough ∇ monoid of idempotents.*

Theorem 2.6. *[25] (T, Δ, ∇) is a rough semiring.*

Theorem 2.7. *For any subset X of U the principal ideal generated by $RS(X)$ in T (with respect to Δ) is given by $RS(X)\Delta T = T_1$ where $T_1 = \{RS(Y) \mid Y \in (X \cup P(E\setminus E_X) \cup P(P_{\overline{X}}))\}$, $E = \{X_1, X_2, ...X_n\}$ is the equivalence classes induced by $Ind(P)$ and E_X is the set of all equivalence classes contained in X.*

Theorem 2.8. *For any subset X of U, the principal ideal generated by $RS(X)$ in T (with respect to ∇) is given by $RS(X)\nabla T = T_2$ where $T_2 = \{RS(Y) \mid Y \in (P(E_X) \cup P(Z_X))\}$ where $Z_X = \{x \in U \mid [x]_p \cap X \neq \phi\}$, $P(E_X)$ is the power set of E_X and $P(Z_X)$ is the power set of Z_X.*

In the following section, we discuss the ideals of a commutative rough semiring (T, Δ, ∇), the principal ideals of a regular rough monoid of idempotents (T, ∇) and relation between them with their properties.

3 Ideals of a rough semiring and principal ideals of a commutative regular rough ∇ monoid of idempotents

In this section, we consider an information system $I = (U, A)$. Now for any $X \subseteq U$, $RS(X) = (\underline{P}(X), \overline{P}(X))$ be the rough set and let $T = \{RS(X) | X \subseteq U\}$ be the set of all rough sets and let $E = \{X_1, X_2, ...X_n\}$ be the equivalence classes induced by $Ind(P)$

For any subset X of U, let E_X be the set of equivalence classes contained in X, P_X be the set of pivot elements of X and $P(X)$ be the power set of X which is a subset of U.

Theorem 3.1. *For any $X \subset U$, let $J_X = \{RS(Y) \mid Y \in P(X)\}$ then J_X is an ideal of (T, Δ, ∇).*

Proof. Case 1: Let $RS(Y), RS(Z) \in J_X$ where $Y, Z \in P(X)$ and $X \subset U$ implies that $Y \Delta Z \subset X$ then $RS(Y \Delta Z) \in J_X$.
Case 2: Let $RS(Y) \in J_X$ and $RS(Z) \in T$.
Subcase 1: If $Z \subset X$ then $RS(Y \nabla Z) \in J_X$.
Subcase 2: If $Z \not\subset X$ and $Z \cap X \neq \phi$ then $Y \nabla Z = Y \nabla (Z \cap X) \subset X$ implies $RS(Y \nabla Z) \in J_X$.
Subcase 3: If $Z \not\subset X$ and $Z \cap X = \phi$ then $RS(Y \nabla Z) = RS(\phi) \in J_X$.
Therefore J_X is an ideal. □

Theorem 3.2. *Let $J = \{J_X \mid X \subseteq U\}$ and $R = \{< RS(X) > \mid X \subseteq U\}$ then $J = R$.*

Proof. Let $X \subseteq U$ and consider the ideal generated by $RS(X)$ where $< RS(X) >= RS(X) \nabla T = \{RS(Y) \mid Y \in P(E_X) \cup P(Z_X)\}$ and $J_X = \{RS(Y) \mid Y \in P(X)\}$. To prove that $< RS(X) >= J_X$. Let $RS(Y) \in RS(X) \nabla T$ then $Y \in P(E_X) \cup P(Z_X) \subseteq X$ where $Z_X = \{x \in U \mid [x]_p \cap X \neq \phi\}$ implies that $RS(Y) \in J_X$. Conversely, if $RS(Y) \in J_X$ then $Y \in P(X)$ implies that $Y \subseteq X$ implies that $Y = Y \cap X$. Since $X \nabla Y = \{x \mid [x]_p \subseteq X \cap Y\} \cup P_{X \cap Y} = \{x \mid [x]_p \subseteq Y\} \cup P_Y = Y$. Therefore $RS(Y) = RS(X \nabla Y) = RS(X) \nabla RS(Y) \in RS(X) \nabla T$ implies that $RS(Y) \in RS(X) \nabla T$. Hence $J = R$. □

Theorem 3.3 (Characterization theorem for the rough ideals in rough semiring (T, Δ, ∇))**.** *Let (T, Δ, ∇) be a rough semiring and let J_1 be a rough ideal in T then $J_1 = J_X$ for some subset X of U.*

Proof. As U is finite, $|T|$ is also finite and $J_1 \subseteq T$ implies that $|J_1|$ is also finite. Let $J_1 = \{RS(Y_1), RS(Y_2), ...RS(Y_k)\}$ where $k \leq |T|$ then we have to prove that $J_1 = J_X$ for some subset X of U. Let $X = Y_1\Delta Y_2\Delta \cdots \Delta Y_k$ then $J_X = \{RS(Z) \mid Z \in P(Y_1\Delta Y_2\Delta \cdots \Delta Y_k)\}$. Let $RS(Y_j) \in J_1$ then $RS(Y_j) \in J_X$. Conversely, let $RS(Z) \in J_X$ implies that $Z \in P(X)$ implies that $Z \in P(Y_1\Delta Y_2\Delta \cdots \Delta Y_k)\}$ where $Z = E_Z \cup P_Z = E_Z\Delta P_Z$. Let $Y_1, Y_2, \cdots Y_r$ be the subsets containing the equivalence classes that are completely contained in Z and let P_Z is a subset of $Y_{t_1}\Delta Y_{t_2}\Delta \cdots \Delta Y_{t_j}$ where $t_1, t_2, t_3, \cdots t_j \in \{1, 2, 3, \cdots r\}$. Therefore $RS(Z) = RS(E_Z\Delta P_Z) = RS(E_Z)\Delta RS(P_Z) = (RS(Y_1\Delta Y_2\Delta \cdots \Delta Y_r)\nabla RS(E_Z))\Delta(RS(Y_{t_1}\Delta Y_{t_2}\Delta \cdots \Delta Y_{t_j})\nabla RS(P_Z))$ since $RS(Y_1\Delta Y_2 \cdots \Delta Y_r) \in J_1$, $RS(E(Z) \in T$ and J_1 is an ideal in T. Therefore $RS(Y_1\Delta Y_2\Delta \cdots \Delta Y_r)\nabla RS(E_Z) \in J_1$ similarly $RS(Y_{t_1}\Delta Y_{t_2}\Delta \cdots \Delta Y_{t_j}) \in J_1$, $RS(P_Z) \in T$ and J_1 is an ideal in T. Therefore $RS(Y_{t_1}\Delta Y_{t_2}\Delta \cdots \Delta Y_{t_j})\nabla RS(P_Z) \in J_1$ and J_1 is closed under Δ. Hence $(RS(Y_1\Delta Y_2\Delta \cdots \Delta Y_r)\nabla RS(E_Z))\Delta(RS(Y_{t_1}\Delta Y_{t_2}\Delta \cdots \Delta Y_{t_j})\nabla RS(P_Z)) \in J_1$. Therefore $RS(Z) \in J_1$. Hence $J_1 = J_X$. □

3.1 Examples

Example 3.1. *From example* 2.1, *let* $X = \{x_1, x_2, x_5\}$ *then*

$$P(X) = \{\phi, \{x_1\}, \{x_2\}, \{x_5\}, \{x_1, x_2\}, \{x_1, x_5\}, \{x_2, x_5\}, \{x_1, x_2, x_5\}\}$$

and from equations (1),(2) & (3), we have, $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_3), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_3\}), RS(\{x_2\} \cup X_3\}), RS(\{x_1\} \cup \{x_2\} \cup X_3\})\}$ *where*

$T = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_1), RS(X_2), RS(X_3), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_2), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(U)\}$

Table 2:

Δ	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(\phi)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(X_3)$	$RS(X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$
$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\{x_1\})\cup\{x_2\}\cup X_3$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$

From Table 2, it is clear that J_X is closed under Δ

Table 3:

∇	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1)$	$RS(X_2)$	$RS(X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(X_1\cup\{x_2\})$	$RS(\{x_1\}\cup X_2)$	$RS(X_1\cup X_2)$	$RS(X_1\cup X_3)$	$RS(X_2\cup X_3)$	$RS(\{x_1\}\cup X_2\cup X_3)$	$RS(X_1\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(U)$
$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$
$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$
$RS(\{x_2\})$	$RS(\phi)$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$
$RS(X_3)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(X_3)$	$RS(\phi)$	$RS(X_3)$	$RS(X_3)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(X_3)$	$RS(X_3)$	$RS(X_3)$	$RS(X_3)$	$RS(X_3)$	$RS(X_3)$
$RS(\{x_1\}\cup\{x_2\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$
$RS(\{x_1\}\cup X_3)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(X_3)$	$RS(\{x_1\})$	$RS(\{x_1\}\cup X_3)$	$RS(X_3)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\}\cup X_3)$	$RS(X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_1\}\cup X_3)$
$RS(\{x_2\}\cup X_3)$	$RS(\phi)$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(X_3)$	$RS(\{x_2\})$	$RS(X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$
$RS(\{x_1\})\cup\{x_2\}\cup X_3)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$

From Table 3, it is clear that for all $RS(X) \in J_X$ and $RS(Y) \in T$ such that $RS(X)\nabla RS(Y) \in J_X$. Thus J_X is an ideal of T. Also, for $X = \{x_1, x_2, x_5\}$ we have $< RS(X) >= RS(X)\nabla T = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_3), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3)\} = J_X$.

Example 3.2. *From example* 2.1*, let* $X = \{x_1, x_2\}$ *then* $P(X) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ *and* $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$ *where*
$T = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_1), RS(X_2), RS(X_3), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_2), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(U)\}$

Table 4:

Δ	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\phi)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$

From Table 4, it is clear that J_X is closed under Δ

Table 5:

∇	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1)$	$RS(X_2)$	$RS(X_3)$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup X_3)$	$RS(\{x_2\} \cup X_3)$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(X_1 \cup X_2)$	$RS(X_1 \cup X_3)$	$RS(X_2 \cup X_3)$	$RS(\{x_1\} \cup X_2 \cup X_3)$	$RS(X_1 \cup \{x_2\} \cup X_3)$	$RS(\{x_1\} \cup \{x_2\} \cup X_3)$	$RS(U)$
$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$
$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$
$RS(\{x_2\})$	$RS(\phi)$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$
$RS(\{x_1\} \cup \{x_2\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$

From Table 5, it is clear that for all $RS(X) \in J_X$ and $RS(Y) \in T$ such that $RS(X) \nabla RS(Y) \in J_X$. Thus J_X is an ideal of T.
Also, for $X = \{x_1, x_2\}$ we have $< RS(X) >= RS(X) \nabla T = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\} = J_X$.

Example 3.3. *From example* 2.1*, let* $X = \{x_1, x_3\}$ *then* $P(X) = \{\phi, \{x_1\}, \{x_3\}, \{x_1, x_3\}\}$ *and* $J_X = \{RS(\phi), RS(\{x_1\}), RS(X_1)\}$ *where* $T = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_1), RS(X_2), RS(X_3), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(X_1 \cup \{x_2\}), RS(\{x_1\} \cup X_2), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3), RS(U)\}$

Table 6:

Δ	$RS(\phi)$	$RS(\{x_1\})$	$RS(X_1)$
$RS(\phi)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(X_1)$
$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(X_1)$
$RS(X_1)$	$RS(X_1)$	$RS(X_1)$	$RS(X_1)$

From Table 6, it is clear that J_X is closed under Δ

Table 7:

∇	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1)$	$RS(X_2)$	$RS(X_3)$	$RS(\{x_1\}\cup\{x_2\})$	$RS(\{x_1\}\cup X_3)$	$RS(\{x_2\}\cup X_3)$	$RS(X_1\cup\{x_2\})$	$RS(\{x_1\}\cup X_2)$	$RS(X_1\cup X_2)$	$RS(X_1\cup X_3)$	$RS(X_2\cup X_3)$	$RS(\{x_1\}\cup X_2\cup X_3)$	$RS(X_1\cup\{x_2\}\cup X_3)$	$RS(\{x_1\}\cup\{x_2\}\cup X_3)$	$RS(U)$
$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$
$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$
$RS(X_1)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(X_1)$	$RS(\phi)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(X_1)$	$RS(\{x_1\})$	$RS(X_1)$	$RS(X_1)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(X_1)$	$RS(\{x_1\})$	$RS(X_1)$

From Table 7, it is clear that for all $RS(X) \in J_X$ and $RS(Y) \in T$ such that $RS(X) \nabla RS(Y) \in J_X$. Thus J_X is an ideal of T. Also, for $X = \{x_1, x_2, x_5\}$ we have $< RS(X) >= RS(X) \nabla T = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_3), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3)\} = J_X$.

4 Properties

Lemma 4.1. *If $X \subseteq Y$ then $J_X \subseteq J_Y$ where X and $Y \subseteq U$.*

Proof. Let $X \subseteq Y$ where $X, Y \subseteq U$ then $X \cap Y = X$ implies that $RS(X) = RS(X \nabla Y) = RS(X)\nabla RS(Y) \in RS(X)\nabla T$. Let $RS(Z) \in J_X$ then $Z \in P(X) \subseteq X \subseteq Y$ implies that $Z \in P(Y)$ then $RS(Z) \in J_Y$. Hence $J_X \subseteq J_Y$. □

Example 4.1. *From example* 2.1*, let* $X = \{x_1, x_2\}$ *and* $Y = \{x_1, x_2, x_4, x_6\}$ *where* $X \subseteq Y$ *then* $P(X) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ *and* $P(Y) = \{\phi, \{x_1\}, \{x_2\}, \{x_4\}, \{x_6\}, \{x_1, x_2\}, \{x_1, x_4\}, \{x_1, x_6\}, \{x_2, x_4\}, \{x_2, x_6\}, \{x_4, x_6\}, \{x_1, x_2, x_4\}, \{x_1, x_2, x_6\}, \{x_2, x_4, x_6\}, \{x_1, x_4, x_6\}, \{x_1, x_2, x_4, x_6\}\}$ *also* $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$ *and* $J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_2), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_2)\}$ *clearly if* $X \subseteq Y$ *then* $J_X \subseteq J_Y$.

Lemma 4.2. *For any two subsets X and Y of U, $J_X \cap J_Y = J_{X\nabla Y}$.*

Proof.

$$
\begin{aligned}
\text{Let} \quad & RS(Z) \in J_X \cap J_Y \\
\Leftrightarrow \quad & Z \in \{P(E_X \cup P(Z_X)\} \cap \{P(E_Y) \cup P(Z_Y)\} \\
\Leftrightarrow \quad & Z \in \{P(E_X \cap P(E_Y)\} \cup \{P(Z_X) \cap P(Z_Y)\} \\
\Leftrightarrow \quad & Z \in \{P(E_{X\cap Y}) \cup P(Z_{X\cap Y})\} \\
\Leftrightarrow \quad & \therefore \;\; Z \in P(X\nabla Y) \;\; i.e., \;\; RS(Z) \in J_{X\nabla Y} \\
\Leftrightarrow \quad & \text{Hence } \; J_X \cap J_Y = J_{X\nabla Y}.
\end{aligned}
$$

□

Example 4.2. *From example* 2.1*, let* $X = \{x_1, x_4\}$ *and* $Y = \{x_1, x_2\}$ *then* $X\nabla Y = \{x_1, x_2\}$, $P(X\nabla Y) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ *then* $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$, $J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$, $J_X \cap J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$ *and* $J_{X\nabla Y} = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\} = J_X \cap J_Y$.

Example 4.3. *From example* 2.1*, let* $X = \{x_1, x_2\}$ *and* $Y = \{x_3, x_5\}$ *then* $X\nabla Y = \{x_1\}$, $P(X\nabla Y) = \{\phi, \{x_1\}\}$ *then* $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$, $J_Y = \{RS(\phi), RS(\{x_1\}), RS(X_3), RS(\{x_1\} \cup X_3)\}$, $J_X \cap J_Y = \{RS(\phi), RS(\{x_1\})\}$ *and* $J_{X\nabla Y} = \{RS(\phi), RS(\{x_1\})\} = J_X \cap J_Y$.

Example 4.4. *From example* 2.1*, let* $X = \{x_1, x_2\}$ *and* $Y = \{x_5\}$ *then* $X\nabla Y = \{\phi\}$, $P(X\nabla Y) = \{\phi\}$ *and* $P(X) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ *also* $P(Y) = \{\phi, \{x_5\}\}$ *then* $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$ *and* $J_Y = \{RS(\phi), RS(X_3)\}$ *also* $J_X \cap J_Y = \{RS(\phi)\}$ *then* $J_{X\nabla Y} = \{RS(\phi)\} = J_X \cap J_Y$.

Lemma 4.3. *If $X \cap Y \neq \phi$ then $J_X \cap J_Y = J_{P_{X\cap Y}}$.*

Proof.

$$\begin{array}{ll}
\text{Let} & RS(Z) \in J_X \cap J_Y \\
\Leftrightarrow & Z \in \{P(E_X \cup P(Z_X)\} \cap \{P(E_Y) \cup P(Z_Y)\} \\
\Leftrightarrow & Z \in \{P(E_X \cap P(E_Y)\} \cup \{P(Z_X) \cap P(Z_Y)\} \\
\Leftrightarrow & If \;\; Z \in \{P(E_X \cap P(E_Y)\} \;\; then \;\; X \cap Y \neq \phi \\
 & \qquad\qquad which \;\; is \;\; not \;\; possible \;\; so \;\; Z \in P(Z_X) \cap P(Z_Y) \\
\Leftrightarrow & \text{Hence } \; Z \in P(P_{X\cap Y}) \;\; i.e., \;\; RS(Z) \in J_{P_{X\cap Y}}
\end{array}$$

□

Example 4.5. *From example 2.1, let $X = \{x_1, x_2\}$ and $Y = \{x_3, x_4, x_5\}$ where $X \cap Y = \phi$ then $P(X) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ and $P(Y) = \{\phi, \{x_3\}, \{x_4\}, \{x_5\}, \{x_3, x_4\}, \{x_3, x_5\}, \{x_4, x_5\}, \{x_3, x_4, x_5\}\}$ also $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$ and $J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(X_3), RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup \{x_2\} \cup X_3)\}$ then $J_X \cap J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$. Now $P_{X\cap Y} = \{x_1, x_2\}$ and $J_{P_{X\cap Y}} = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$. Therefore $J_X \cap J_Y = J_{P_{X\cap Y}}$.*

Remarks 4.1.
1. *For any two subsets X and Y of U, $X \neq Y$ does not imply that $J_X \neq J_Y$.*
2. *For any two subsets X and Y of U, $X \cap Y = \phi$ does not imply that $J_X \neq J_Y$.*

Example 4.6. *From example 2.1, let $X = \{x_1, x_4\}$ and $Y = \{x_2, x_3\}$ since $X \neq Y$ then $P(X) = \{\phi, \{x_1\}, \{x_4\}, \{x_1, x_4\}\}$ and $P(Y) = \{\phi, \{x_2\}, \{x_3\}, \{x_2, x_3\}\}$ also $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\},$
$RS(\{x_1\} \cup \{x_2\})\}$ and $J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}, RS(\{x_1\} \cup \{x_2\})\}$ where $J_X = J_Y$. This shows that for any two subsets X and Y of U, $X \neq Y$ does not imply that $J_X \neq J_Y$ and $X \cap Y = \phi$ does not imply that $J_X \neq J_Y$.*

Remarks 4.2. *If $X \cap Y \neq \phi$ then $J_X \cap J_Y$ need not be equal to $J_{X\cap Y}$.*

Example 4.7. *From example 2.1, let $X = \{x_1, x_2\}$ and $Y = \{x_1, x_6\}$ then $X \cap Y = \{x_1\}$ and $P(X \cap Y) = \{\phi, \{x_1\}\}$ then $P(X) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ and $P(Y) = \{\phi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$ then $J_X = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$, $J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$, $J_X \cap J_Y = \{RS(\phi), RS(\{x_1\}), RS(\{x_2\}), RS(\{x_1\} \cup \{x_2\})\}$ and $J_{X\cap Y} = \{RS(\phi), RS(\{x_1\})\}$. Thus we can conclude if $X \cap Y \neq \phi$ then $J_X \cap J_Y$ need not be equal to $J_{X\cap Y}$.*

5 Conclusion

In this paper, we discussed the ideals of a commutative rough semiring (T, Δ, ∇) and we gave a characterization for the ideals of a rough semiring (T, Δ, ∇) in terms of the principal ideals of the rough monoid (T, ∇) for a given information system $I = (U, A)$. We present some properties related to these concepts and the same concepts are illustrated through examples.

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