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SOME MORE CURVATURE PROPERTIES OF A QUARTER SYMMETRIC METRIC CONNECTION IN A LP-SASAKIAN MANIFOLD

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Abstract

The object of the present article is to study a quarter symmetric metric connection in a LP-Sasakian manifold whose curvature tensor admits the conditions $\tilde{W}(\xi, U) \cdot \tilde{R} = 0$, $\tilde{R}(\xi, U) \cdot \tilde{W} = 0$ and $\tilde{W}(\xi, X) \cdot \tilde{S} = 0$.

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1 Introduction

Recently, in tune with Yano and Sawaki [26], the authors in [2] have introduced and studied *generalized quasi-conformal curvature tensor* in the frame of $N(k, \mu)$-manifold. The *generalized quasi-conformal curvature tensor* is defined for an n-dimensional manifold as

$$\begin{aligned} W(X,Y)Z &= \frac{n-2}{n}[(1+(n-1)a-b)-\{1+(n-1)(a+b)\}c]C(X,Y)Z \\ &+[1+(n-1)a-b]E(X,Y)Z+(n-1)(b-a)P(X,Y)Z \\ &+\frac{n-2}{n}(c-1)\{1+2n(a+b)\}L(X,Y)Z \end{aligned} \quad (1)$$

for all X, Y & $Z \in \chi(M)$, the set of all vector field of the manifold M, where scalar triples (a, b, c) are real constants. The beauty of such curvature tensor lies in the fact that it has the flavour of Riemann curvature tensor R if the scalar triples (a, b, c) $\equiv (0,0,0)$, conformal curvature tensor C [6] if (a, b, c) $\equiv (-\frac{1}{n-2}, -\frac{1}{n-2}, 1)$, conharmonic curvature tensor L [10] if (a, b, c) $\equiv (-\frac{1}{n-2}, -\frac{1}{n-2}, 0)$, concircular curvature tensor E ([24], p. 84) if (a, b, c) $\equiv (0,0,1)$, projective curvature tensor

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P ([24], p. 84) if $(a,\ b,\ c) \equiv (-\frac{1}{n-1}, 0, 0)$ and m-projective curvature tensor H [15], fi $(a,\ b,\ c) \equiv (-\frac{1}{2n-2}, -\frac{1}{2n-2}, 0)$. The equation (1) can also be written as

$$\begin{aligned} & W(X,Y)Z \\ = \ & R(X,Y)Z + a[S(Y,Z)X - S(X,Z)Y] + b[g(Y,Z)QX - g(X,Z)QY] \\ & -\frac{cr}{n}\left(\frac{1}{n-1} + a + b\right)[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{2}$$

In the theory of Riemannian geometry, Golab [8] has defined and studied quarter-symmetric connection in differentiable manifolds with affine connections. A liner connection $\bar{\nabla}$ on an n-dimensional Riemannian manifold (M^n, g) is called a quarter-symmetric connection [8] if its torsion tensor T of the connection $\bar{\nabla}$

$$T(X;Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X,Y]$$

satisfies

$$T(X,Y) = \eta(Y)\phi X - \eta(X)\phi Y, \tag{3}$$

where η is a 1-form and ϕ is a $(1,1)$ tensor field. If moreover, a quarter-symmetric connection $\bar{\nabla}$ satisfies the condition

$$(\bar{\nabla}_X g)(Y,Z) = 0$$

for all $X, Y, Z \in \chi(M)$, then $\bar{\nabla}$ is said to be a quarter-symmetric metric connection, otherwise it is said to be a quarter-symmetric non-metric connection. In particular, if $\phi X = X$ for all X, then the quarter-symmetric connection reduces to the semi-symmetric connection [7]. Thus the notion of the quarter-symmetric connection generalizes the notion of the semi-symmetric connection. After Golab [8] and Rastogi ([17], [18]), the systematic study of quarter-symmetric metric connection have been carried out by R. S. Mishra and S. N. Pandey [13], K. Yano and T. Imai [25], S. Mukhopadhyay, A. K. Roy and B. Barua [14], Haseeb, Prakash and Siddiqi [9], Ahmad, Haseeb, Jun and Shahid [1], Singh and Pandey [23] and the references therein.

Our paper is organized as follows: In Section 2, we give a brief account of LP-Sasakian manifolds. LP-Sasakian manifold with $\tilde{W}(\xi,U)\cdot\tilde{R} = 0$ is investigated in Section 3. And it is obtained that in such a LP-Sasakian manifold, for each of $\bar{C}(\xi,U)\cdot\tilde{R} = 0$, $\bar{L}(\xi,U)\cdot\tilde{R} = 0$, $\bar{P}(\xi,U)\cdot\tilde{R} = 0$ and $\bar{H}(\xi,U)\cdot\tilde{R} = 0$, ξ is semi-torse forming vector field with respect to $\bar{\nabla}$. Section 4, is concerned with a LP-Sasakian manifold admitting $\tilde{R}(\xi,U)\cdot\tilde{W} = 0$. We observed that for each of $\tilde{R}(\xi,U)\cdot\bar{C} = 0$, $\tilde{R}(\xi,U)\cdot\bar{L} = 0$, $\tilde{R}(\xi,U)\cdot\bar{P} = 0$ and $\tilde{R}(\xi,U)\cdot\bar{H} = 0$, ξ is semi-torse forming vector field with respect to ∇. Finally, In Section 5, we consider a LP-Sasakian manifold satisfying $\tilde{W}(\xi,U)\cdot\tilde{S} = 0$ and for found that each of $\bar{C}(\xi,U)\cdot\tilde{S} = 0$, $\bar{L}(\xi,U)\cdot\tilde{S} = 0$, $\bar{P}(\xi,U)\cdot\tilde{S} = 0$ and $\bar{H}(\xi,U)\cdot\tilde{S} = 0$, the Ricci tensor is of the form $S(X,Z) = -(n-1)\eta(X)\eta(Z)$.

2 LP-Sasakian manifold and some known results

In 1989 K. Matsumoto ([11]) introduced the notion of Lorentzian para-Sasakian (LP-Sasakian for short) manifold. In 1992, Mihai and Rosca ([12]) defined the same notion independently. This type of manifold is also discussed in ([19], **[20])**

An n-dimensional differentiable manifold M is said to be a LP-Sasakian manifold [11] if it admits a $(1,1)$ tensor field ϕ, a unit timelike contravarit vector field ξ, a 1-form η and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \quad g(X,\xi) = \eta(X), \quad \phi^2 X = X + \eta(X)\xi, \tag{4}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \qquad \nabla_X \xi = \phi X, \tag{5}$$

$$(\nabla_X \phi)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{6}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g. It can be easily seen that in an LP-Sasakian manifold, the following relations hold :

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \text{Rank } \phi = n. \tag{7}$$

Again, if we put

$$\Omega(X,Y) = g(X, \phi Y)$$

for any vector fields X, Y then the tensor field $\Omega\Big(X,Y\Big)$ is a symmetric $(0,2)$ tensor field ([12]). Also, since the vector field η is closed in an LP-Sasakian manifold, we have ([11], [12])

$$(\nabla_X \eta)(Y) = \Omega(X,Y), \qquad \Omega(X,\xi) = 0 \tag{8}$$

for any vector fields X and Y.

Let M be an n-dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g). Then the following relations hold ([11], [12]) :

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y), \tag{9}$$

$$R(\xi, X)Y = g(X,Y)\xi - \eta(Y)X, \tag{10}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{11}$$

$$S(X,\xi) = (n-1)\eta(X), \tag{12}$$

$$S(\phi X, \phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{13}$$

for any vector fields X, Y, Z where R is the Riemannian curvature tensor of the manifold.

Let $\bar{\nabla}$ be the linear connection and ∇ be Riemannian connection of an almost contact metric manifold such that

$$\bar{\nabla}_X Y = \nabla_X Y + H(X,Y) \tag{14}$$

where H is the tensor field of type $(1,1)$. For $\bar{\nabla}$ to be a quarter-symmetric metric connection in M^n, we have

$$H(X,Y)=\frac{1}{2}[\bar{T}(X,Y)+\bar{T}'(X,Y)+\bar{T}'(Y,X) \tag{15}$$

and

$$g(\bar{T}'(X,Y),Z)=g(\bar{T}(Z,X),Y). \tag{16}$$

In view of equations (3), (16) and (15), we get

$$H(X,Y)=\eta(Y)\phi X-g(\phi X,Y)\xi. \tag{17}$$

Hence, the relation between quarter-symmetric metric connection and the Levi-Civita connection in a LP-Sasakian manifold is given by

$$\bar{\nabla}_X Y=\nabla_X Y+\eta(Y)\phi X-g(\phi X,Y)\xi. \tag{18}$$

The curvature tensor $\bar{R}$ of M^n with respect to quarter-symmetric metric connection $\bar{\nabla}$ is defined by

$$\tilde{R}(X,Y)Z=\bar{\nabla}_X\bar{\nabla}_Y Z-\bar{\nabla}_Y\bar{\nabla}_X Z-\bar{\nabla}_{[X,Y]}Z. \tag{19}$$

Making use of (18) in (19) we have

$$\begin{aligned}\tilde{R}(X,Y)Z \;=\;& R(X,Y)Z+g(\phi X,Z)\phi Y-g(\phi Y,Z)\phi X+\eta(Y)\eta(Z)X\\ &-\eta(X)\eta(Z)Y+\{g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\}\xi,\end{aligned} \tag{20}$$

where $\tilde{R}$ and R are the Riemannian curvature tensor with respect to $\bar{\nabla}$ and ∇ respectively.

From equation (20), we can easily bring out the followings

$$\tilde{S}(Y,Z)=S(Y,Z)+(n-1)\eta(Y)\eta(Z), \tag{21}$$

$$\tilde{r}=r+n(n-1), \tag{22}$$

$$\tilde{Q}X=QX+(n-1)\eta(X)\xi. \tag{23}$$

In view of (2), (20), (21), (22) and (23), we have

$$\begin{aligned}&\tilde{W}(X,Y)Z\\ =\;& R(X,Y)Z+a[S(Y,Z)X-S(X,Z)Y]+b[g(Y,Z)QX-g(X,Z)QY]\\ &-\frac{cr}{n}\left(\frac{1}{n-1}+a+b\right)[g(Y,Z)X-g(X,Z)Y]+g(\phi X,Z)\phi Y\\ &-g(\phi Y,Z)\phi X+\{1+a(n-1)\}[\eta(Y)\eta(Z)X-\eta(X)\eta(Z)Y]\\ &+\{1+b(n-1)\}[g(Y,Z)\eta(X)\xi-g(X,Z)\eta(Y)\xi]\\ &-\frac{c(n-1)}{n}\left(\frac{1}{n-1}+a+b\right)[g(Y,Z)X-g(X,Z)Y].\end{aligned} \tag{24}$$

Definition 1. *A vector field ξ is called semi-torse forming vector field [16] for (M,g) if, for all vector fields X*

$$R(X,\xi)\xi=0.$$

3 LP-Sasakian manifold with $\tilde{W}(\xi, U) \cdot \tilde{R} = 0$

Let us consider a LP-Sasakian manifold with the following identity

$$\tilde{W}(\xi, U) \cdot \tilde{R}(X, Y)\xi = 0 \tag{25}$$

which is equivalent to

$$\tilde{W}(\xi, U)\tilde{R}(X, Y)\xi - \tilde{R}(\tilde{W}(\xi, U)X, Y)\xi - \tilde{R}(X, \tilde{W}(\xi, U)Y)\xi - \tilde{R}(X, Y)\tilde{W}(\xi, U)\xi = 0. \tag{26}$$

As a consequence of (24) and (20), one can easily bring out the followings

$$\tilde{W}(\xi, U)\tilde{R}(X, Y)\xi = 0, \tag{27}$$

$$\tilde{R}(\tilde{W}(\xi, U)X, Y)\xi = 0, \tag{28}$$

$$\tilde{R}(X, \tilde{W}(\xi, U)Y)\xi = 0 \tag{29}$$

and

$$\begin{aligned} & \tilde{R}(X, Y)\tilde{W}(\xi, U)\xi \\ = \; & [b(n-1) - \frac{cr}{n}\left(\frac{1}{n-1} + a + b\right) - \frac{c(n-1)}{n}\left(\frac{1}{n-1} + a + b\right)] \times \\ & [R(X, Y)U + g(\phi X, U)\phi Y - g(\phi Y, U)\phi X + \eta(Y)\eta(U)X - \eta(X)\eta(U)Y \\ & + \{g(Y, Z)\eta(X) - g(X, Z)\eta(Y)\}\xi]. \end{aligned} \tag{30}$$

Using (27), (28), (29) and (30) in (26), we get

$$\begin{aligned} & [b(n-1) - \frac{cr}{n}\left(\frac{1}{n-1} + a + b\right) - \frac{c(n-1)}{n}\left(\frac{1}{n-1} + a + b\right)] \times \\ & [R(X, Y)U + g(\phi X, U)\phi Y - g(\phi Y, U)\phi X + \eta(Y)\eta(U)X - \eta(X)\eta(U)Y \\ & + \{g(Y, Z)\eta(X) - g(X, U)\eta(Y)\}\xi] = 0. \end{aligned} \tag{31}$$

$$i.e, [b(n-1) - \frac{c(r+n-1)}{n}\left(\frac{1}{n-1} + a + b\right)]\tilde{R}(X, Y)U = 0. \tag{32}$$

This leads to the following:

Theorem 1. *Let (M^n, g), $(n > 2)$ be a LP-Sasakian manifold. Then for each of $\bar{C}(\xi, U) \cdot \tilde{R} = 0$, $\bar{L}(\xi, U) \cdot \tilde{R} = 0$, $\bar{P}(\xi, U) \cdot \tilde{R} = 0$ and $\bar{H}(\xi, U) \cdot \tilde{R} = 0$, ξ is semi-torse forming vector field with respect to $\bar{\nabla}$.*

4 LP-Sasakian manifold with $\tilde{R}(\xi, U) \cdot \tilde{W} = 0$

In this section, we investigate the curvature properties of LP-Sasakian manifold satisfying

$$\tilde{R}(\xi, U) \cdot \tilde{W}(X, Y)\xi = 0. \tag{33}$$

This implies that

$$\tilde{R}(\xi, U)\tilde{W}(X, Y)\xi - \tilde{W}(\tilde{R}(\xi, U)X, Y)\xi - \tilde{W}(X, \tilde{R}(\xi, U)Y)\xi - \tilde{W}(X, Y).\tilde{R}(\xi, U)\xi = 0. \tag{34}$$

In view of (20), we have

$$\begin{aligned} \tilde{R}(\xi, X)Y &= 0, && (35) \\ \tilde{R}(X, Y)\xi &= 0, && (36) \end{aligned}$$

Putting $Z = \xi$ in (24) and using (10), (11) and (12), we get

$$\begin{aligned} &\tilde{W}(X, Y)\xi \\ = &\left\{ b(n-1) - c\left(\frac{r}{n} + n - 1\right)\left(\frac{1}{n-1} + a + b\right)\right\}[\eta(Y)X - \eta(X)Y]. \end{aligned} \tag{37}$$

In view of (35) and (36), (34) becomes

$$\left\{ b(n-1) - c\left(\frac{r}{n} + n - 1\right)\left(\frac{1}{n-1} + a + b\right)\right\}\eta(R(X, Y)U) = 0. \tag{38}$$

This motivate us to state the following:

Theorem 2. *Let (M^n, g), $(n > 2)$ be a LP-Sasakian manifold. Then for each of $\bar{C}(\xi, U) \cdot \tilde{R} = 0$, $\bar{L}(\xi, U) \cdot \tilde{R} = 0$, $\bar{P}(\xi, U) \cdot \tilde{R} = 0$ and $\bar{H}(\xi, U) \cdot \tilde{R} = 0$, ξ is semi-torse forming vector field with respect to ∇.*

5 LP-Sasakian manifold with $\tilde{W}(\xi, X) \cdot \tilde{S} = 0$

Let $M^{2n+1}(\phi, \xi, \eta, g)(n > 1)$, be a LP-Sasakian manifold, satisfying the condition

$$\tilde{W}(\xi, X) \cdot \tilde{S} = 0. \tag{39}$$

$$\begin{aligned} i.e.\ \tilde{W}(\xi, X)\tilde{S}(Y, Z) - \tilde{S}(\tilde{W}(\xi, X)Y, Z) - \tilde{S}(Y, \tilde{W}(\xi, X)Z) &= 0 \quad . \\ i.e.\ \tilde{S}(\tilde{W}(\xi, X)Y, Z) + \tilde{S}(Y, \tilde{W}(\xi, X)Z) &= 0 \quad . \end{aligned} \tag{40}$$

As a consequence of (24), we have

$$\begin{aligned}
&\tilde{W}(\xi, X)Y \\
= &\left[a(n-1) - \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right] g(Y,Z)\xi \\
&+ [a(n-1) - b(n-1)]\,\eta(Y)\eta(X)\xi \\
&+ \left[-b(n-1) + \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right]\eta(Y)X. \qquad (41)
\end{aligned}$$

In view of (41), (40) becomes

$$\begin{aligned}
&\left[a(n-1) - \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right] g(Y,Z)\tilde{S}(\xi,Z) \\
&+ [a(n-1) - b(n-1)]\,\eta(Y)\eta(X)\tilde{S}(\xi,Z) \\
&+ \left[-b(n-1) + \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right]\eta(Y)\tilde{S}(X,Z) \\
&+ \left[a(n-1) - \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right] g(X,Z)\tilde{S}(\xi,Y) \\
&+ [a(n-1) - b(n-1)]\,\eta(Z)\eta(X)\tilde{S}(\xi,Y) \\
&+ \left[-b(n-1) + \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right]\eta(Z)\tilde{S}(X,Y) = 0. \qquad (42)
\end{aligned}$$

Using (21), (22) and (23) in the above equation, we obtain

$$\begin{aligned}
&\left[-b(n-1) + \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right] \times \\
&\{\eta(Y)S(X,Z) + \eta(Z)S(Y,X) + 2(n-1)\eta(Y)\eta(X)\eta(Z)\} = 0 \qquad (43)
\end{aligned}$$

which yields

$$\begin{aligned}
&\left[-b(n-1) + \frac{c}{n}\{r+(n-1)\}\left(\frac{1}{n-1}+a+b\right)\right] \times \\
&\{S(X,Z) + (n-1)\eta(X)\eta(Z)\} = 0. \qquad (44)
\end{aligned}$$

for $Y = \xi$. This leads to the following

Theorem 3. *Let (M^n, g), $(n > 2)$ be a LP-Sasakian manifold. Then for each of $\bar{C}(\xi, U)\cdot\tilde{S} = 0$, $\bar{L}(\xi, U)\cdot\tilde{S} = 0$, $\bar{P}(\xi, U)\cdot\tilde{S} = 0$ and $\bar{H}(\xi, U)\cdot\tilde{S} = 0$, the Ricci tensor is of the form $S(X, Z) = -(n-1)\eta(X)\eta(Z)$.*

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