

A PURSUIT-EVASION DIFFERENTIAL GAME ON A COMPACT SUBSET OF \mathbb{R}^2 WITH GEOMETRIC CONSTRAINTS

Akbar ASGHARZADEH-BONAB¹, Hamid BIGDELI^{*,2}
and Javad MOHAMMADKARIMI³

Abstract

In this paper, we study a pursuit differential game with many pursuers and evaders on a compact and not necessarily convex subset M of \mathbb{R}^2 . Their motion's equation is first order differential equations. Players must play in this set M . For control functions of the players we use geometric constraints. Completion of the pursuit means that the state of each of the evader coincides with that of a pursuer in the plane. We derive sufficient conditions for completion of pursuit on this set M . We constructed strategy of the pursuers that guarantees completion of the pursuit. Moreover, we construct winning strategies for pursuers.

2020 Mathematics Subject Classification: 91A24, 49N75.

Key words: differential game, geometric constraint, non-convex.

1 Introduction

Authors in [3]-[21] and some references therein, investigate differential game problems. Many books investigate differential games and systematically examine the topic. The problem of pursuit-evasion has been studied by many authors. For example see, [15], [20], [22], and [11]. Also, many articles have been published on differential games where they used integral constraints for control function. For example see [5], [4], [23], and [8].

In [14] Ibragimov and Salimi studied a differential game for inertial players. They assumed the control resource of the each pursuer is greater than that of evader. Recently, in [24] Salimi et al. studied a differential game that countable objects want to get one evader. All the players must have simple motions and

¹Department of Science and Technology Studies, AJA Command and Staff University, Tehran, I.R. Iran, e-mail: a.asgharzadeh@urmia.ac.ir

^{2*} *Corresponding author*, Department of Science and Technology Studies, AJA Command and Staff University, Tehran, I.R. Iran, e-mail: bigdeli@casu.ac.ir

³Department of Science and Technology Studies, AJA Command and Staff University, Tehran, I.R. Iran, email: javad.mohammadkarimi1369@gmail.com

some pursuers have integral constraints and some other pursuers and the evader have geometric constraints.

Authors in [7] consider the differential game in a compact and not necessarily convex subset.

Leong and Ibragimov in [18] investigate simple motion pursuit-evasion differential game. The game studied on closed convex subset of the Hilbert space l_2 with many pursuers and one evader. They used integral constraints for the control functions of the players. Sufficient conditions for completion of pursuit were obtained. The papers [2], [21] and [16] studied pursuit differential game of many pursuers and one evader in nonempty closed convex set $N \subseteq R^n$. In the papers, the motion of equation of each player is defined by the following $\frac{dH}{dt} = a(t)Q(t)$, $H(0) = H_0$, $\|Q(t)\| \leq 1$ where $H, H_0, Q \in N$, and Q is control parameter of the player that has geometric constraint. In [12] authors investigated a simple motion differential game with many pursuers and one evader in \mathbb{R}^n . For the control functions of the players they used the Gronwall-type constraints.

Pursuit-evasion also has application in biology. In [19] authors analyze the application of pursuit-evasion in artificial systems from three point of view, first, strong pursuer group against weak evader group, weak pursuer group against strong evader group, and equal-ability group. It is crucial to realize the intrinsic mechanisms of biological swarm behaviors in nature, [19] is comprehensive survey of pursuit-evasion for this reason. Authors in [10] investigated a novel method for real-time solutions of the two players pursuit-evasion game. For confirming the Nash equilibrium of the game, they used the min-max principle. They introduced the Lyapunov function and considered the scenario to see when capture accures.

Authors in [17] obtained a solution strategy for deterministic time-optimal pursuit evasion games. The constraint that they used are linear state constraints, convex control, and linear dynamics such they are consistent with linearized relative orbital motion models such as the Clohessy-Wiltshire equations and relative orbital elements. Authors in [27] tried to solve the pursuit evasion game of three pursuers surrounde an evader which is faster. The game is complex because they used the holonomic motion of the agents. In [29], authors studied the process of terminal approach and rendezvous in the mission of spacecraft on-orbit servicing. The cost fuction that they used is terminal zero effort miss distance.

In another line research in the field of pursuit-evasion game, authors in [28] investigated the problem of spacecraft interception game with incomplete-information and proposed switching strategies based on the differential game theory. In the interception process, the target can switch among multiple strategies to evade the interceptor. This leads to the formulation of switching strategies pursuit problem for the interceptor. Also, authors in [30] considered reach-avoid differential game with two evaders and one pursuer in the plane which is divided into a play region and a goal region by a straight line. Two evaders, starting from the play region, aim at reaching the goal region protected by the faster pursuer who tries to capture the evaders. Also very recent result in this field are [26], [13], [25].

In the present paper, we consider a differential game of many pursuers and

many evaders on a compact and not necessarily convex subset M of \mathbb{R}^2 , Figure 1 below, with geometric constraints. Results in [6] is for a convex set. Here we show that same results in [6] still hold for the given non-convex set M .

2 Statement of the problem

We consider a differential game problem with m pursuers and r evaders described by the equations

$$\begin{aligned} \frac{dx_i}{dt} &= \mathcal{P}(t)p_i(t), & x_i(0) &= x_{i0}, & i &= 1, \dots, m, \\ \frac{dy_j}{dt} &= \mathcal{P}(t)e_j(t), & y_j(0) &= y_{j0}, & j &= 1, \dots, r \end{aligned} \quad (1)$$

where $x_i(t)$, $p_i(t)$, $y_i(t)$, $e_i(t) \in \mathbb{R}^2$, $p_i = (p_{i1}, p_{i2})$ is control parameter of the pursuer x_i $i = 1, \dots, m$ and $e_j = (e_{j1}, e_{j2})$ is that of the evader y_j , and the function $\mathcal{P}(t)$ is scalar measurable functions with $0 < \eta \leq \mathcal{P}(t) \leq 1$ over any interval $[0, \mathcal{T}]$, $\mathcal{T} > 0$.

Definition 2.1. [9] Let \mathbb{R}^2 be a real line and $|\cdot|_2$ be its norm, which is defined in term of absolute value. Let M be a subset of \mathbb{R}^2 (for example the set M in Figure 1). We define a set valued mapping $H_M: \mathbb{R}^2 \rightarrow \mathcal{P}(M)$ by $H_M(x) = \{y \in M : \|x - y\| = d(x, M)\}$. M is a Chebyshev set if $H_M(x)$ a singleton for each $x \in \mathbb{R}^2$.

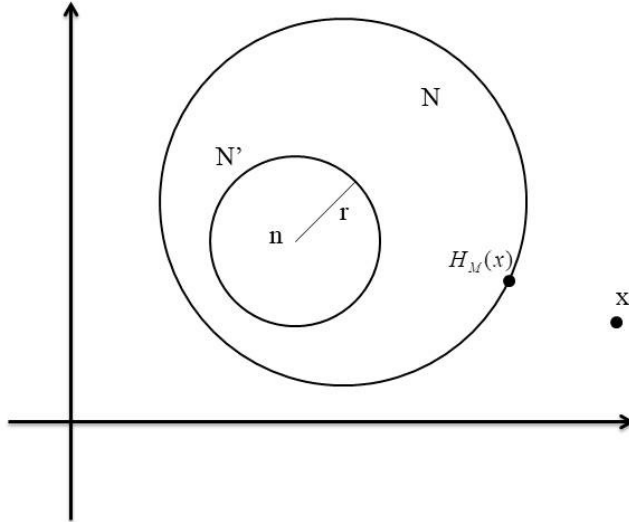


Figure 1: Non-Convex set $M = N - N'$

It is straightforward to check that for any $x \in \mathbb{R}^2 \setminus M$, in Figure 1, $H_M(x) = \left\{ \frac{x}{\|x\|} \right\}$. Therefore, the given set M is a Chebyshev set.

Definition 2.2. [1] A measurable function $p_i(t) = (p_{i1}(t), p_{i2}(t))$ $t \geq 0$, is called an admissible control of the pursuer x_i if $x_i(t) \in M$, $t \geq 0$, and

$$\|p_i(t)\| \leq \mathcal{S}_i, \quad (2)$$

where \mathcal{S}_i , $i = 1, \dots, m$ are given positive numbers.

Definition 2.3. [1] A measurable function $e_j(t) = (e_{j1}(t), e_{j2}(t))$ $t \geq 0$, is called an admissible control of the evader if $y_j(t) \in M$, $t \geq 0$, and

$$\|e_j(s)\| \leq \mathcal{V}_j, \quad (3)$$

where \mathcal{V}_j , $j = 1, 2, \dots, r$ are given positive numbers.

Definition 2.4. [24] Let

$$x_i(t) = x_{i0} + \int_0^t \mathcal{P}(t)p_i(t)dt \in M,$$

$$y_j(t) = y_{j0} + \int_0^t \mathcal{P}(t)e_j(t)dt \in M.$$

Definition 2.5. [1] A Borel measurable function $P_i(x_i, y_j, e_j)$, $P_i: \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is called a strategy of the pursuer x_i if for any control of the evader $e_j(t)$, $j = 1, \dots, r$, $t \geq 0$, the initial value problem

$$\begin{aligned} \frac{dx_i}{dt} &= \mathcal{P}(t)P_i(t), & x_i(0) &= x_{i0}, \\ \frac{dy_j}{dt} &= \mathcal{P}(t)e_j(t), & y_j(0) &= y_{j0}, \quad j = 1, \dots, r \end{aligned} \quad (4)$$

has a unique absolutely continuous solution $(x_i(t), y_j(t))$ with $x_i(t), y_j(t) \in M$ and $t \geq 0$.

Definition 2.6. [6] Pursuit can be completed in the game 1-3, if for all $j = 1, \dots, r$ there exist $i \in \{1, \dots, m\}$ such that $x_i(t) = y_j(t)$ holds for $t \in [0, \mathcal{T}]$.

Given a nonempty closed non-convex set $M \subset \mathbb{R}^2$, according to the rule of the game, all players must not leave the set M .

Problem 1. Find a sufficient condition of completion of pursuit in the game (1)-(3).

3 Results

In this section we give the solutions to the above problem in form of theorems and their proofs. For this goal, we need the following notations:

$$\lambda = \max_{i \in I, j \in J} \frac{\|y_{j0} - x_{i0}\|}{\mathcal{S}_i - \mathcal{V}_j}, \quad \Gamma = \min\{1, \lambda\}.$$

Theorem 3.1. *If for all $i \in I$ there exists $j \in J$ such that $\mathcal{S}_i > \mathcal{V}_j$ then pursuit can be completed in the game (1)-(3) for a finite time $T = \max_{i \in I, j \in J} T_{i,j}$, where*

$$T_{i,j} = \begin{cases} 1 & \lambda \leq \eta \\ \frac{\|y_0 - x_0\|}{\eta(\mathcal{S}_i - \mathcal{V}_j)} & \lambda > \eta. \end{cases}$$

The proof of this theorem is dependent on a lemma which is about a differential game involving one pursuer and one evader ($m = k = 1$). This game has the following motion's equations:

$$\begin{cases} \frac{dx}{dt} = \mathcal{P}(t)p, & x(0) = x_0, \\ \frac{dy}{dt} = \mathcal{P}(t)e, & y(0) = y_0 \end{cases}$$

where $x, y, p, e \in \mathbb{R}^2$, p is the control parameter of the pursuer and e is that of the evader. Pursuit is completed if $x(t) = y(t)$ at some time $t \geq 0$. In this game players move freely in R^2 without any state constraint.

Lemma 3.2. *If $\mathcal{S} > \mathcal{V}$ then we have the completion of the pursuit in the game (1)-(3) for the time*

$$T = \begin{cases} 1 & \lambda \leq \eta \\ \frac{\|y_0 - x_0\|}{\eta(\mathcal{S} - \mathcal{V})} & \lambda > \eta. \end{cases}$$

Proof. We define the following strategy for pursuer:

$$p(t) = \frac{(y_0 - x_0)}{T\mathcal{P}(t)} + e(t), \quad t \geq 0. \quad (5)$$

If the pursuer uses the strategy (5), then we have $x(T) = y(T)$. In fact,

$$\begin{aligned} x(T) &= x_0 + \int_0^T \mathcal{P}(t)p(t)dt \\ &= x_0 + \int_0^T \mathcal{P}(t) \left(\frac{(y_0 - x_0)}{T\mathcal{P}(t)} + e(t) \right) dt \\ &= x_0 + \int_0^T \frac{(y_0 - x_0)}{T} dt + \int_0^T \mathcal{P}(t)e(t)dt \\ &= y_0 + \int_0^T \mathcal{P}(t)e(t)dt = y(T). \end{aligned} \quad (6)$$

Therefore, we have the completion of the pursuit for the time T . We now show that the strategy (5) is admissible. When $\mathcal{S} \leq \mathcal{V}$, we have

$$\begin{aligned}
\|p(t)\| &= \left\| \frac{y_0 - x_0}{\mathcal{P}(t)T} + e(t) \right\| \leq \frac{\|y_0 - x_0\|}{|\mathcal{P}(t)|T} + \|e(t)\| \\
&\leq \frac{\|y_0 - x_0\|}{\eta T} + \mathcal{V} \leq \frac{\|y_0 - x_0\|}{\lambda T} + \mathcal{V} \\
&\leq \mathcal{S} - \mathcal{V} + \mathcal{V} = \mathcal{S}
\end{aligned} \tag{7}$$

On the other hand, if $\mathcal{S} > \mathcal{V}$, we have

$$\begin{aligned}
\|p(t)\| &= \left\| \frac{y_0 - x_0}{\mathcal{P}(t)T} + e(t) \right\| \leq \frac{\|y_0 - x_0\|}{|\mathcal{P}(t)|T} + \|e(t)\| \\
&\leq \frac{\|y_0 - x_0\|}{\eta T} + \mathcal{V} \\
&\leq \mathcal{S} - \mathcal{V} + \mathcal{V} = \mathcal{S}
\end{aligned} \tag{8}$$

Therefore, the strategy (5) is admissible and this completes the proof of the lemma. \square

By using above lemma, we now prove Theorem 3.1.

Proof of Theorem 3.1

Dummy pursuers (DPs): To prove the main theorem, we introduce dummy pursuers D_1, \dots, D_m (if $m = k$) and D_1, \dots, D_r (if $m > k$). whose equations of their motions are described by

$$\frac{dD_i}{dt} = \mathcal{P}(t)\mathcal{K}_i(t), \quad D_i(0) = x_{i0}, \quad \|\mathcal{K}_i(t)\| \leq \mathcal{S}_i, \quad 0 \leq t \leq T \tag{9}$$

where \mathcal{K}_i is control parameter of the pursuer D_i . Dummy pursuers may move out of the nonconvex set $M \subset \mathbb{R}^2$. The aim of the dummy pursuer is for the completion of the pursuit in the finite time.

Let the strategy for the sth pursuer be defined by

$$\mathcal{K}_s(t) = \frac{y_{r0} - x_{m0}}{\mathcal{P}(t)T_{s,r}} + e_r(t), \quad 0 \leq t \leq T_{s,r}, \tag{10}$$

where e_r is the control parameter of the evader whose maximum speed is less than that of dummy pursuer with state D_s . That is, $\mathcal{S}_s > \mathcal{V}_s$. To show the admissibility of this strategy of the dummy pursuer, we have the following:

For $\mathcal{S} \leq \mathcal{V}$, we have

$$\begin{aligned}
\|\mathcal{K}_s(t)\| &= \left\| \frac{y_{r0} - x_{s0}}{\mathcal{P}(t)T_{s,r}} + e_r(t) \right\| \leq \frac{\|y_{r0} - x_{s0}\|}{\mathcal{P}(t)T_{s,r}} + \|e_r(t)\| \\
&\leq \frac{\|y_{r0} - x_{s0}\|}{\eta T_{s,r}} + \mathcal{V}_r \leq \frac{\|y_{r0} - x_{s0}\|}{\lambda T_{s,r}} + \mathcal{V}_r \\
&\leq \frac{\|y_{r0} - x_{s0}\|}{T_{s,r}} \frac{\mathcal{S}_s - \mathcal{V}_r}{\|y_{r0} - x_{s0}\|} + \mathcal{V}_r \\
&\leq \mathcal{S}_s - \mathcal{V}_r + \mathcal{V}_r = \mathcal{S}_s
\end{aligned} \tag{11}$$

For $\mathcal{S} > \mathcal{V}$, we have

$$\begin{aligned}
\|\mathbf{K}_s(t)\| &= \left\| \frac{y_{r0} - x_{s0}}{\mathcal{P}(t)T_{s,r}} + e_r(t) \right\| \\
&\leq \frac{\|y_{r0} - x_{s0}\|}{\mathcal{P}(t)T_{s,r}} + \|e_r(t)\| \\
&\leq \frac{\|y_{r0} - x_{s0}\|}{\eta T_{s,r}} + \mathcal{V}_r \\
&\leq \frac{\|y_{r0} - x_{s0}\|}{T_{s,r}} \frac{\mathcal{S}_s - \mathcal{V}_r}{\|y_{r0} - x_{s0}\|} + \mathcal{V}_r \\
&\leq \mathcal{S}_s - \mathcal{V}_r + \mathcal{V}_r = \mathcal{S}_s
\end{aligned} \tag{12}$$

By Lemma 3.2, if the dummy pursuer with state \mathcal{D}_s uses the strategy (10) then it obtains the equation $\mathcal{D}_s(T_{s,r}) = y_r(T_{s,r})$, for the time $T_{s,r}$. This means that each of the dummy pursuer with state \mathcal{D}_s , $s \in I$, will obtain the equation $\mathcal{D}_s(T_{s,r}) = y_r(T_{s,r})$ for some $r \in J$, for the time $T_{s,r}$. Therefore, in the game (1)-(3), we have the completion of the game for the time $T = \max_{s \in I, r \in J} T_{s,r}$ by dummy pursuers. We now show that completion of the game can be done by the actual pursuers.

We define the controls $p_i(\cdot)$, $i \in I$ of the pursuers x_i by the controls of the dummy pursuers \mathcal{K}_i , $i \in I$.

Let $H_M(x)$ the projection of the point $x \in \mathbb{R}^2$ on the set M , that is

$$\min_{y \in M} |x - y| = |x - H_M(x)|, \tag{13}$$

if $x \in M$, then $H_M(x) = x$. Also, we can see $H_M(x)$ as in Definition 2.1:

$$\begin{aligned}
H_M: \mathbb{R}^2 &\rightarrow \mathbf{P}(M) \\
H_M(x) &:= \{y \in M : |x - y| = d(x, M)\}
\end{aligned} \tag{14}$$

Since $H_M(x)$, based on this given set M , is a singleton for each $x \in \mathbb{R}^2$, so M is a Chebyshev set. Also, for this M , It is easy to see that for any point $x \in \mathbb{R}^2$ there exists a unique point $H_M(x)$. Moreover, for any $x, y \in \mathbb{R}^2$, based on the set M , it is easy to observe that

$$|H_M(x) - H_M(y)| \leq |x - y|, \tag{15}$$

and hence the operator $H_M(\cdot)$, by Corollary 2.20 of [9], is continuous so it relates any absolute continuous function $\mathbf{D}(t)$, $0 \leq t \leq T$ to an absolute continuous function

$$x(t) = H_M(\mathbf{D}(t)), \quad 0 \leq t \leq T, \tag{16}$$

where T is the time that we have the completion of the pursuit by DPs. We construct the strategy for the actual pursuers to satisfy

$$x_i(t) = H_M(\mathbf{D}_i(t)), \quad 0 \leq t \leq T, \quad i = 1, \dots, m. \tag{17}$$

We now prove that (17) is admissible, based on above we have:

$$\begin{aligned}
\mathcal{P}(t)\|p_i(t)\| &= \left\| \frac{dx_i(t)}{dt} \right\| = \lim_{\varsigma \rightarrow 0} \frac{\|x_i(t+\varsigma) - x_i(t)\|}{|\varsigma|} \\
&= \lim_{\varsigma \rightarrow 0} \frac{\|H_M(\mathbf{D}_i(t+\varsigma)) - H_M(\mathbf{D}_i(t))\|}{|\varsigma|} \\
&\leq \lim_{\varsigma \rightarrow 0} \frac{\|\mathbf{D}_i(t+\varsigma) - \mathbf{D}_i(t)\|}{|\varsigma|}
\end{aligned} \tag{18}$$

Also we have:

$$\mathcal{P}(t)\|\mathcal{K}_i(t)\| = \left\| \frac{d\mathcal{D}_i(t)}{dt} \right\| = \lim_{\varsigma \rightarrow 0} \frac{\|\mathbf{D}_i(t+\varsigma) - \mathbf{D}_i(t)\|}{|\varsigma|}.$$

Hence, we have $\|p_i(t)\| \leq \|\mathcal{K}_i(t)\|$ almost every where. Therefore, $\|p_i(t)\| \leq \|\mathcal{K}_i(t)\| \leq S_i$. From the fact that, for any evader y_r , $r \in J$, the equation $\mathcal{D}_s(T_{s,r}) = y_r(T_{s,r})$ is achievable for $T_{s,r}$ and for some $s \in I$ and the evader y_r is in the compact set M throughout the game period then the dummy pursuer with state variable \mathcal{D}_s is also in the compact set M for $t \geq 0$. As a result of this, we have,

$$x_s(T_{s,r}) = H_M(\mathbf{D}_s(T_{s,r})) = \mathbf{D}_s(T_{s,r}) = y_r(T_{s,r}). \tag{19}$$

This means the differential game can be completed. Hence, the proof of the theorem is complete.

4 Conclusion

We have investigated a pursuit differential game of many pursuers and evaders on a compact and not necessarily convex subset M of \mathbb{R}^2 . We obtained a sufficient condition with geometric constraints for this differential game. We showed that results of [6], which is for a closed convex set, still hold on this given non-convex compact set M . The results shows that when the distances between pursuers and evaders are very small then pursuit is completed in a unit time.

References

- [1] Ahmed, I., Kumam, W., Ibragimov, G. and Rilwan, J., *Pursuit differential game problem with multiple players on a closed convex set with more general integral constraints*, Thai J. Math. **18** (2020), no. 2, 551-561.
- [2] Alias, I. A., Ramli, R. N., Ibragimov, G. and Narzullaev, A., *Simple motion pursuit differential game of many pursuers and one evader on convex compact set*, Int. J. Pure Appl. Math. **102** (2015), no. 4, 733-745.
- [3] Azamov, A., *On a problem of escape along a prescribed curve*, J. Appl. Math. Mech. **46** (1982), no. 4, 553-555.

- [4] Azamov, A. and Samatov, B., π -strategy. *An elementary introduction to the theory of differential games*, NUU Press, Tashkent, 2000.
- [5] Azimov, A. Ya., *Linear differential pursuit game with integral constraints on the control*, *Differentsial'nye Uravneniya* **11** (1975), no. 10, 1723-1731.
- [6] Badakaya, A. J., Abdurashheed, A. A. and Iguda, A., *On two pursuit differential game problems with state and geometric constraints in a Hilbert space*, *Uzbek Math. J.* **65** (2021), no. 3, 5-16.
- [7] Bigdeli, H., Mohammadkarimi, J. and Biranvand, N., *Evasion from many pursuers on a compact and not necessarily convex set*, *Caspian J. Math. Sci.* (2022), doi:10.22080/CJMS.2022.23653.1627.
- [8] Ferrara, M., Ibragimov, G., Alias, I. and Salimi, M., *Pursuit differential game of many pursuers with integral constraints on compact convex set*, *Bull. Malays. Math. Sci. Soc.* **43** (2020), 2929-2950.
- [9] Fletcher, J. and Moors, W. B., *Chebyshev sets*, *J. Aust. Math. Soc.* **98** (2015), no. 2, 161-231.
- [10] Gong, Z., He, B., Liu, G., Liu, X. and Zhang, X., *Solution for pursuit-evasion game of agents by adaptive dynamic programming*, *Electronics* **12** (2023), no. 12.
- [11] Hajek, O., *Pursuit games, an introduction to the theory and applications of differential games of pursuit and evasion*, Academic Press, Inc., New York, 1975.
- [12] Ibragimov, G., Egamberganova, O., Arif Alias, I. and Luckraz, S., *On some new results in a pursuit differential game with many pursuers and one evader*, *AIMS Math.* **8** (2023), no. 3, 6581-6589.
- [13] Ibragimov, G., Ferrara, M., Idham, A., Salimi, M. and Ismail, N., *Pursuit and evasion games for an infinite system of differential equations*, *Bull. Malays. Math. Sci. Soc.* **45** (2022), 69-81.
- [14] Ibragimov, G. and Salimi, M., *Pursuit-evasion differential game with many inertial players*, *Math. Probl. Eng.* (2009), Article ID 653723.
- [15] Isaacs, R., *Differential games*, John Wiley & Sons, New York, 1965.
- [16] Ivanov, R. P., *Simple pursuit-evasion on the compact convex set*, *Dokl. Akad. Nauk SSSR* **254** (1980), no. 6, 1318-1321.
- [17] Jansson, O. and Harris, M. V., *A geometrical, reachable set approach for constrained pursuit-evasion games with multiple pursuers and evaders*, *Aerospace* **10** (2023), no. 5.

- [18] Leong, W. J. and Ibragimov, G. I., *A multiperson pursuit problem on a closed convex set in Hilbert space*, Far East J. Appl. Math. **33** (2008), no. 2, 205-214.
- [19] Mu, Z., Pan, J., Zhou, Z., Yu, J. and Cao, L., *A survey of the pursuit–evasion problem in swarm intelligence*, Front. Inform. Technol. Electron. Eng. **24** (2023), no. 8, 1093-1116.
- [20] Petrosyan, L. A., *Differential pursuit games*, Izdat. Leningrad. Univ., Leningrad, 1977.
- [21] Petrov, N. N. and Shchelchkov, K. A., *About the problem of group persecution in linear differential games with a simple matrix and state constraints*, Int. J. Pure Appl. Math. **92** (2014), no. 1, 13-26.
- [22] Rikhsiev, B. B., *The differential games with simple motions*, Fan, Tashkent, 1989.
- [23] Rilwan, J., Kumam, P., Ibragimov, G., Badakaya, A. J. and Ahmed, I., *A differential game problem of many pursuers and one evader in the Hilbert space ℓ_2* , Differ. Equ. Dyn. Syst. **31** (2023), 925-943.
- [24] Salimi, M., Ibragimov, G., Siegmund, S. and Sharifi, S., *On a fixed duration pursuit differential game with geometric and integral constraints*, Dyn. Games Appl. **6** (2016), 409-425.
- [25] Salimi, M., *A pursuit-evasion game with hybrid system of dynamics*, Math. Methods Appl. Sci. (2020), doi:10.1002/mma.7053.
- [26] Sharifi, S., Badakaya, A. J. and Salimi, M., *On game value for a pursuit-evasion differential game with state and integral constraints*, Japan J. Indust. Appl. Math. **39** (2022), 653-668.
- [27] Szots, J. and Harmati, I., *Optimal strategies of a pursuit-evasion game with three pursuers and one superior evader*, Rob. Auton. Syst. **161** (2023).
- [28] Tang, X., Ye, D., Huang, L., Sun, Z. and Sun, J., *Pursuit-evasion game switching strategies for spacecraft with incomplete-information*, Aerosp. Sci. Technol. **112** (2021), 107112.
- [29] Tang, X., Ye, D., Low, K. S., Luo, S. and Sun, Z., *Multi-spacecraft pursuit-evasion-defense strategy based on game theory for on-orbit spacecraft servicing*, Proc. of IEEE Aerospace Conference, 1-9, 2023.
- [30] Yan, R., Shi, Z. and Zhong, Y., *Cooperative strategies for two-evader one-pursuer reach-avoid differential games*, Int. J. Syst. Sci. (2021), 1-19.