

SPATIO-TEMPORAL VIDEO RESTORATION TECHNIQUE USING A 3D ANISOTROPIC DIFFUSION-BASED SCHEME

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Abstract

A spatio-temporal video filtering approach is proposed in this article. The video restoration technique introduced here removes successfully the signal-independent noises, such as the additive white Gaussian noise, that deteriorate the frame sequences. The proposed framework is based on a novel 3D fourth-order nonlinear reaction-diffusion model that reduces the additive white Gaussian noise (AWGN) considerably, overcomes the side-effects and deals properly with the inter-frame correlation problem. A rigorous mathematical treatment is performed on this model, its validity being investigated. A numerical approximation algorithm that solves this nonlinear partial differential equation (PDE)-based model is then provided in this paper and applied successfully in the video denoising tests that are also described here.

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Key words: spatio-temporal video denoising, the additive white Gaussian noise, fourth-order nonlinear 3D diffusion-based model, mathematical validity investigation, numerical approximation algorithm.

1 Introduction

The video noise, which represents a random variation in the brightness of the image, is an unintended side effect of the video capturing process, being a result of the digital gain. The static and video images can be corrupted by multiple noises during the acquisition operation, which deteriorates their quality [1].

The most common noise is the additive white Gaussian noise (AWGN), that has a signal-independent character and comes from the spontaneous thermal generation of electrons. Other types of noises generated by the acquisition instruments include the signal-dependent Poisson (quantum) noise provided by the

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mechanism of quantized photons, multiplicative noise, stripe noise, impulse noise and mixed noises. The AWGN removal represents an essential pre-processing step that facilitates further video analysis processes.

Video filtering is obviously a difficult task, since the adjacent frames are highly correlated. Therefore, the spatial video denoising algorithms that filter each frame individually fail to obtain effective results. Temporal filtering approaches reduce the noise between video frames by using motion compensation [2], but may also generate blurring and dragging effects on the moving zones. Spatio-temporal, or 3D denoising, filters combine the spatial and temporal video denoising solutions [3]. Such a spatio-temporal AWGN filtering technique is proposed in this research article.

The existing video denoising methods represent various extensions of the two-dimension image filters, such as the Gaussian 3D filter [4], Non-local Mean (NLM) 3D filter [5], VBM3D and VBM4D filters [6, 7], deep learning-based video restoration approaches [8] and partial differential equation (PDE) – based denoising schemes [9].

The PDE models have been applied successfully in numerous image processing and analysis fields. We also developed many PDE-based solutions for various image and video processing and analysis tasks [10-13]. Here we consider a new nonlinear fourth-order 3D diffusion-based model for spatio-temporal video denoising and restoration, which is described in the next section. The introduced PDE-based AWGN filter addresses successfully the video frame correlation issue. Due to its fourth-order diffusion component, the proposed filtering scheme overcomes both blurring and staircase effects, preserving and enhancing the essential video details both along and across the frames. A consistent finite difference-based discretization scheme that solves numerically the diffusion model is also proposed in the following section.

The mathematical investigation of the nonlinear PDE model's validity is provided in the third section, while the results of our filtering experiments are presented in the fourth section. The conclusions of this research and the future research plans in this domain are discussed in the final section.

2 A nonlinear fourth-order 3D diffusion-based filtering model

Here we introduce a nonlinear PDE-based video restoration framework that takes into consideration both the spatial and temporal correlation of the movie frames. It is based on a novel 3D fourth-order anisotropic diffusion model.

A video sequence could be represented as a 3D function $v : \Omega \rightarrow R$, where the video image domain $\Omega \subseteq R^3$. Symbols x and y in $v(x, y, z) \in R$ are related to the video spatial domain, and z represents the temporal coordinate. The diffusion-based video restoration is a dynamic process, so a time parameter $t \in (0, T]$ is inserted into this function, that becomes $v : \Omega \times (0, T] \rightarrow R$. The considered PDE-based model has the next form:

$$\begin{cases} \frac{\partial v}{\partial t} + \lambda \delta \left(\frac{\|\nabla^2 v\| + \|\nabla v_\sigma\|}{2} \right) \nabla^2(\psi(\|\nabla^2 v\|)\Delta v) - \alpha(v - v_0) = 0, \\ v(x, y, z, 0) = v_0(x, y, z), \quad \forall (x, y, z) \in \Omega \\ v(x, y, z, t) = 0, \quad \forall (x, y, z) \in \partial\Omega, \quad t \in (0, T) \\ \frac{\partial v}{\partial \mathbf{n}}(x, y, z, t) = 0, \quad \forall (x, y, z) \in \partial\Omega \end{cases} \quad (1)$$

where $\lambda \in [1, 2]$, $\alpha \in (0, 0.5)$, G_σ is the 3D Gaussian kernel, $v_\sigma = v^* G_\sigma$, v_0 is the observed video corrupted by AWGN, the proposed positive and monotonically decreasing diffusivity conductance function is

$$\psi : [0, \infty) \rightarrow [0, \infty) : \psi(s) = \beta \left(\frac{\gamma}{|\xi s^r + \varphi|} \right)^{\frac{1}{r+1}} \quad (2)$$

with $\beta \in [1, 2]$, $\varphi, \gamma \geq 1$, $\xi \in (0, 1)$, and the other positive function used by this model is

$$\delta : [0, \infty) \rightarrow [0, \infty) : \delta(s) = \eta \sqrt[r+1]{\zeta s^r + \varepsilon} \quad (3)$$

where $\zeta, \eta, \varepsilon \in [1, 5]$, $r \geq 2$. The intervals of all these mentioned parameters have been determined empirically, using the trial and error approach.

This nonlinear parabolic PDE-based model is non-variational, since it cannot be derived from any energy functional minimization. The proposed nonlinear diffusion-based filter removes successfully the signal-independent additive white Gaussian noise. Because of its conductance term, the diffusion-based filtering takes places along and not across the edges, within each frame, thus overcoming the blurring effect. And given its fourth-order diffusion term, it avoids the undesired staircase (blocky) effect. The essential video details, like boundaries and corners, are thus preserved during the denoising process.

Temporal correlation between successive frames is also considered. The diffusion operation include pixels having the same location in each video frame. If there is motion between consecutive frames, and so the corresponding pixels are displaced, then the magnitude of 3D Laplacian $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$, increases and the conductance value $\psi(\|\Delta v\|)$ decreases until there is no diffusion occurring. Therefore, the video filtering operation stops whenever it reaches the frontiers of a moving zone. So, this PDE model provides a successful detail preserving smoothing that sharpens the edges of the video objects both along and across adjacent frames, which facilitates video analysis tasks like moving object detection and tracking.

This fourth-order anisotropic diffusion-based model admits also a variational (weak) solution, as demonstrated in the next section. This solution is determined numerically using a finite difference-based approximation algorithm [14]. So, one considers the following time and space coordinate quantization: $x = ih$, $y = jh$, $z = kh$, $t = n\tau$, $n \in \{0, \dots, N\}$, where $i \in \{1, \dots, I\}$, $j \in \{1, \dots, J\}$, $k \in \{1, \dots, K\}$. Its equation can be reformulated as:

$$\frac{\partial v}{\partial t} - \alpha(v - v_0) = -\lambda \delta \left(\frac{\|\nabla^2 v\| + \|\nabla v_\sigma\|}{2} \right) \nabla^2(\psi(\|\nabla^2 v\|)\Delta v) \quad (4)$$

Its left term is then approximated numerically as follows [14]:

$$\frac{v_{i,j,k}^{n+\tau} - v_{i,j,k}^n}{\tau} + \alpha(v_{i,j,k}^0 - v_{i,j,k}^n) = v_{i,j,k}^{n+\tau} \frac{1}{\tau} - v_{i,j,k}^n \left(\alpha + \frac{1}{\tau} \right) + v_{i,j,k}^0 \alpha \quad (5)$$

Next, in the right term of the equation, the term $\delta \left(\frac{\|\nabla^2 v\| + \|\nabla v_\sigma\|}{2} \right)$ is discretized as

$$\delta \left(\frac{\sqrt{(v_{i+h,j,k}^n - v_{i-h,j,k}^n)^2 + (v_{i,j+h,k}^n - v_{i,j-h,k}^n)^2 + (v_{i,j,k+h}^n - v_{i,j,k-h}^n)^2}}{4h} \right) + \frac{(v_\sigma)_{i+h,j,k}^n + (v_\sigma)_{i-h,j,k}^n + (v_\sigma)_{i,j+h,k}^n + (v_\sigma)_{i,j-h,k}^n + (v_\sigma)_{i,j,k+h}^n + (v_\sigma)_{i,j,k-h}^n - 6(v_\sigma)_{i,j,k}^n}{2h^2}$$

and the component $\nabla^2(\psi(\|\nabla^2 v\|)\Delta v)$ is approximated as:

$$\Delta \rho_{i,j,k}^n = \frac{\rho_{i+h,j,k}^n + \rho_{i-h,j,k}^n + \rho_{i,j+h,k}^n + \rho_{i,j-h,k}^n + \rho_{i,j,k+h}^n + \rho_{i,j,k-h}^n - 6\rho_{i,j,k}^n}{h^2} \quad (6)$$

where

$$\rho_{i,j,k}^n = \psi(|\nabla^2 v_{i,j,k}^n|) \Delta v_{i,j,k}^n$$

and

$$\Delta v_{i,j,k}^n = \frac{v_{i+h,j,k}^n + v_{i-h,j,k}^n + v_{i,j+h,k}^n + v_{i,j-h,k}^n + v_{i,j,k+h}^n + v_{i,j,k-h}^n - 6v_{i,j,k}^n}{h^2}.$$

One may consider the parameter values $h = 1$ and $\tau = 1$. Thus, by combining all these approximation results, we obtain the following iterative numerical scheme:

$$\begin{aligned} v_{i,j}^{n+1} = & \alpha v_{i,j}^n - \lambda \delta \left(\left(\sqrt{(v_{i+1,j,k}^n - v_{i-1,j,k}^n)^2 + (v_{i,j+1,k}^n - v_{i,j-1,k}^n)^2 + (v_{i,j,k+1}^n - v_{i,j,k-1}^n)^2} \right. \right. \\ & + 2((v_\sigma)_{i+1,j,k}^n + (v_\sigma)_{i-1,j,k}^n + (v_\sigma)_{i,j+1,k}^n - (v_\sigma)_{i,j-1,k}^n + (v_\sigma)_{i,j,k+1}^n - (v_\sigma)_{i,j,k-1}^n \\ & \left. \left. - 6(v_\sigma)_{i,j,k}^n) / 4 \right) (\rho_{i+1,j,k}^n + \rho_{i-1,j,k}^n + \rho_{i,j+1,k}^n + \rho_{i,j-1,k}^n + \rho_{i,j,k+1}^n + \rho_{i,j,k-1}^n - 6\rho_{i,j,k}^n) \right. \\ & \left. - v_{i,j}^0 \alpha \right) \end{aligned} \quad (7)$$

This numerical solver is stable and consistent to the nonlinear fourth-order diffusion-based model and also converges fast to its numerical solution.

3 Mathematical investigation of the nonlinear diffusion model

A mathematical treatment is then performed on the proposed nonlinear fourth-order PDE model, its well-posedness, or validity, being rigorously investigated.

We intend to determine if the diffusion-based model has a solution and if that solution is unique. So, we set $u = \Delta v$ and rewrite (1) as follows:

$$\begin{cases} \frac{\partial u}{\partial t} + \lambda \Delta \delta \left(\frac{|u| + |\nabla(\Delta^{-1}u)_\sigma|}{2} \right) \Delta(\psi(|u|)u) - \alpha(u - u_0) = 0, \\ u(x, y, z, 0) = u_0(x, y, z), \quad \forall (x, y, z) \in \Omega \\ u(x, y, z, t) = 0, \quad \forall (x, y, z) \in \partial\Omega, \quad t \in (0, T) \\ \frac{\partial u}{\partial \tilde{\mathbf{n}}}(x, y, z, t) = 0, \quad \forall (x, y, z) \in \partial\Omega \end{cases} \quad (8)$$

Now, we set $H(u) \equiv \delta \left(\frac{|u| + |\nabla(\Delta^{-1}u)_\sigma|}{2} \right) \psi(|u|)u$, $u \in L^2(\Omega)$ and rewrite (8) as:

$$\frac{\partial u}{\partial t} + \lambda \Delta^2 H(u) - 2\lambda \Delta \delta \left(\frac{|u| + |\nabla(\Delta^{-1}u)_\sigma|}{2} \right) \Delta(\psi(|u|)u) - \alpha(u - u_0) = 0 \quad (9)$$

with the same boundary conditions.

We set $F(u) = 2\lambda \Delta \left(\delta \left(\frac{|u| + |\nabla(\Delta^{-1}u)_\sigma|}{2} \right) \psi(|u|)u \right)$ and write (9) as

$$\begin{cases} \frac{\partial u}{\partial t} + \lambda \Delta^2 H(u) - \alpha(u - u_0) = F(u), \quad \text{in } \Omega \times (0, T) \\ u(0) = u_0(x, y, z), \quad \text{in } \Omega \\ u = 0, \quad \frac{\partial u}{\partial \tilde{\mathbf{n}}} = 0, \quad \forall (x, y, z) \in \partial\Omega, \quad t \in (0, T) \end{cases} \quad (10)$$

We fix $\bar{u} \in L^\infty(0, T, H^2(\Omega))$ such that $\sup \|\bar{u}(t)\|_{H^2(\Omega)} \leq M < \infty$, where $H^2(\Omega)$ is the Sobolev space of order 2 on Ω . Then, we consider the equation:

$$\begin{cases} \frac{\partial u}{\partial t} + \lambda \Delta^2 H(u) + \alpha(u - u_0) = F(\bar{u}), \quad \text{in } \Omega \times (0, T) \\ u(0) = u_0(x, y, z), \quad \text{in } \Omega \\ u = 0, \quad \frac{\partial u}{\partial \tilde{\mathbf{n}}} = 0, \quad \forall (x, y, z) \in \partial\Omega, \quad t \in (0, T) \end{cases} \quad (11)$$

We consider the space $H_0^2(\Omega) = \{u \in H^2(\Omega); u = 0, \frac{\partial u}{\partial \tilde{\mathbf{n}}} = 0 \text{ on } \partial\Omega\}$ and let $H_0^{-2}(\Omega)$ be the dual space of $H_0^2(\Omega)$. Assume now that $H : L^2(\Omega) \rightarrow L^2(\Omega)$ is a monotone function that is

$$\int_{\Omega} (H(u_1) - H(u_2))(u_1 - u_2) d\Omega \geq 0, \quad \forall u_1, u_2 \in L^2(\Omega) \quad (12)$$

Then we set $V = L^2(\Omega)$ and note that $V \subset H_0^{-2}(\Omega) \subset V'$, where V' is the dual of V in the pairing defining by $H_0^{-2}(\Omega)$. We note that $H_0^{-2}(\Omega)$ is endowed with the scalar product $\langle y_1, y_2 \rangle = - \int_{\Omega} \Delta^{-1} y_1 \Delta^{-1} y_2$, $\forall y_1, y_2 \in H_0^{-2}$ and Δ^2 is the canonical isomorphism from H_0^2 on V to V' on H_0^{-2} . Since the operator $u \rightarrow \lambda \Delta^2 H(u)$ is continuous and monotone from V to V' (because H is monotone in L^2) it follows that for each $u_0 \in H_0^{-2}(\Omega)$, the equation (11) has a unique solution $\bar{u} = \phi(u)$, $u \in C([0, T]; H_0^{-2}(\Omega) \cap L^2(0, T; L^2(\Omega)))$, $\frac{\partial u}{\partial t} \in L^2(0, T; V')$ [15]. Moreover

by multiplying (11) by $H(u)$ and integrating, one gets that for M suitable chosen we have:

$$\sup_{t \in (0, T)} \|\phi(u)(t)\|_{H^2(\Omega)} \leq M \quad (13)$$

Now we consider the closed convex subset $\aleph \subset L^2(0, T; K^2(\Omega))$, where $\aleph = \{u; \sup_{t \in (0, T)} \|u(t)\| \leq M\}$. By (13) we see that $\phi(\aleph) \subset \aleph$ and by (11) we also see that $\sup_{t \in (0, T)} \left\| \frac{d}{dt} \phi(\bar{u}) \right\|_{L^2(0, T; H_0^{-2}(\Omega))} \leq C_M$, $\forall \bar{u} \in \aleph$. Hence $\phi(\aleph)$ is compact in $L^2(0, T; L^2(\Omega))$ and so, by the Schauder fixed point theorem, there is $\bar{u} \in \aleph$, such that $\phi(u) = u$. We have therefore: for each $u_0 \in H_0^{-2}(\Omega)$ there is at least one solution u to equation (8) satisfying (13).

4 Experiments and method comparison

The described nonlinear diffusion-based spatio-temporal video restoration technique was tested successfully on numerous video datasets. The research results disseminated here belong to a computer vision project in the traffic surveillance field [16], so some traffic video databases were used in our video denoising tests. The considered film collections include Kaggle datasets like *Highway Traffic Videos Dataset* [17], *Pedestrian Dataset* and *AAU RainSnow Traffic Surveillance Dataset* [18], that contain traffic sequences. Other traffic movies, which were recorded by us, were also been used in these restoration experiments. The testing videos were contaminated with high levels of AWGN before applying the denoising scheme. The described numerical algorithm removes successfully the signal-independent noises, overcoming also undesired side-effects, such as staircasing, blurring and flickering.

The proposed PDE-based filtering technique addresses properly the inter-frame correlation issue, therefore it preserves properly the essential video features both along and across the frames. This denoising scheme operates fast, given its fast-converging numerical approximation algorithm. Anyway, the needed number of iterations, N , and its running time depend on the video's size and the amount of AWGN which deteriorates it.

The performance of this spatio-temporal video filtering approach was assessed by using a testing dataset based on the mentioned video collections and containing 130 traffic sequences with more than 14000 frames, and applying measures like Peak Signal-to-Noise Ratio (PSNR). This technique achieved good average scores of that performance metric. Method comparison have been performed, too. Our anisotropic diffusion-based 3D denoising scheme outperforms other video denoising approaches, including the spatial video filtering methods using TV-based models [20], the 3D NLM filter, the 3D Gaussian filter and the spatio-temporal VBM3D filter. However it can be outperformed by some deep learning-based methods [8]. Several video restoration method comparison results are displayed in Table 1.

Table 1. Average PSNR values obtained by several video filters

Video Restoration Technique	Average PSNR
Nonlinear diffusion-based video filter	29.3218 (dB)
VBM3D	29.2317 (dB)
3D Non-local Mean (NML) filter	28.4573 (dB)
TV Denoising	27.2648 (dB)
3D Gaussian filter	25.8633 (dB)

A video filtering example is described in Fig. 1. Several frames (#1, #30 and #60) of an original $[1080 \times 1920 \times 100]$ traffic video recorded by us are displayed in a). Those frames of the video affected by a high amount of AWGN, given by the mean $\mu = 0$ and variance $\sigma^2 = 0.12$, are described in b). The video restoration output achieved by our diffusion-based filter after $N = 24$ iterations of (7) is displayed in c).



Fig. 1. Traffic video restoration example

5 Conclusions

A fourth-order anisotropic diffusion based 3D denoising technique that removes efficiently the AWGN was described here. The proposed nonlinear fourth-order PDE model represents the main contribution of this research work. The non-variational diffusion-based filtering model considered here provides an efficient

detail-preserving spatio-temporal restoration of the videos corrupted by additive noise, since it reduces successfully the noise from each frame and deals properly with the inter-frame correlation.

The mathematical investigation of the validity of this 3D nonlinear diffusion-based model constitutes another contribution of our research, as well as the fast converging finite difference-based approximation algorithm which solves numerically the PDE model. It was applied successfully in the video filtering simulations, whose results illustrate the effectiveness of this framework. Since this 3D diffusion-based smoothing approach overcomes the unintended effects and enhances the objects' boundaries both along and across the frames, it facilitates important computer vision processes, like the moving object detection and tracking.

We intend to adapt this nonlinear diffusion-based video filter for other types of noise such as the shot, impulse and mixed Poisson-Gaussian noise, as part of our future work.

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References

- [1] Bi, Z., *Comprehensive Survey on Video Denoise Methods*, 2022.
- [2] Cheng, C., Chen, C.Y., Chen, Y.H. and Chen, L.G., *Analysis and VLSI architecture of update step in motion-compensated temporal filtering*, IEEE International Symposium on Circuits and Systems (ISCAS), 2006
- [3] Szczepanski, M., *Spatio-temporal filters in video stream processing*, Computer Recognition Systems 4. Advances in Intelligent and Soft Computing, vol **95**. Springer, Berlin, Heidelberg, 2011.
- [4] Wirjadi, O. and Breuel, T., *Approximate separable 3D anisotropic Gauss filter*. In IEEE International Conference on Image Processing 2005, **2** (2005), vol. II, 149-152. IEEE.
- [5] Coupé, P., Yger, P. and Barillot, C., *Fast non local means denoising for 3D MR images*. In International conference on medical image computing and computer-assisted intervention, Berlin, Heidelberg, Springer Berlin Heidelberg, 2006, 33-49.
- [6] Ehret, T. and Arias, P., *Implementation of VBM3D and some variants*, Image Processing On Line **11** (2021), 374-395.
- [7] Maggioni, M., Boracchi, G., Foi, A. and Egiazarian, K., *Video denoising, deblocking, and enhancement through separable 4-D nonlocal spatio-temporal transforms*, IEEE Transactions on Image Processing, 2012. <https://doi.org/10.1109/TIP.2012.2199324>.

- [8] Tassano, M., Delon, J. and Veit, T., *Dvdnet: A fast network for deep video denoising*, 2019 IEEE International Conference on Image Processing (ICIP), 2019, 1805-1809. IEEE
- [9] Lee, S. H. and Kang, M.G., (1998, October). *Spatio-temporal video filtering algorithm based on 3-D anisotropic diffusion equation*, Proceedings of 1998 International Conference on Image Processing, ICIP 98 (Cat. No. 98CB36269), Vol. **2** (1998), 447-450. IEEE.
- [10] Barbu, T., *Novel diffusion-based models for image restoration and interpolation*, Book Series: Signals and Communication Technology, Springer International Publishing, 2019.
- [11] Barbu, T., Miranville, A. and Moroşanu, C., *A qualitative analysis and numerical simulations of a nonlinear second-order anisotropic diffusion problem with non-homogeneous Cauchy–Neumann boundary conditions*, Appl. Math. Comput. **350** (2019), 170-180.
- [12] Barbu, T., *Robust contour tracking model using a variational level-set algorithm*, Numer. Funct. Anal. Optim. **35** (2014), no. 3, 263-274.
- [13] Barbu, T., *Deep learning-based multiple moving vehicle detection and tracking using a nonlinear fourth-order reaction-diffusion based multi-scale video object analysis*, Discrete Contin. Dyn. Syst. Ser. S **16** (2023), no. 1, 16-32.
- [14] Boole, G., *Calculus of finite differences*, BoD–Books on Demand, 2022.
- [15] Lions, J. L., *Quelques méthodes de résolution des problèmes aux limites non-linéaires*, Dunod. Gauthier-Villars, Paris, 1969.
- [16] Jain, N. K., Saini, R. and Mittal, P., *A Review on traffic monitoring system techniques*, Soft Computing: Theories and Applications. Advances in Intelligent Systems and Computing, **742**, Springer, 2019.
- [17] Chan, A. B. and Vasconcelos, M., *Probabilistic Kernels for the Classification of Auto-Regressive Visual Processes*, Proc. of IEEE Conf. on Computer Vision and Pattern Recognition, San Diego, 2005.
- [18] Bahnsen C. H. and Moeslund, T. B., *Rain removal in traffic surveillance: Does it matter?*, IEEE Transactions on Intelligent Transportation Systems, **20** (2018), no. 8, 2802-2819.
- [19] Sara, U., Akter. M. and Uddin, M. S., *Image quality assessment through FSIM, SSIM, MSE and PSNR—a comparative study*, Journal of Computer and Communications **7** (2-19), no.3, 8-18.
- [20] Chan, T. F., Osher, S. and Shen, J., *The digital TV filter and nonlinear denoising*, IEEE Trans. Image Process. **10** (2001), no. 2, 231-241.

