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SOME REMARKS RELATED TO THE RAABE-DUHAMEL TEST

Andrei VERNESCU[1]

Abstract

We precise, for some classes of series with positive terms, the expression $\mathcal{R}_n$ which appears in the application of the Raabe-Duhamel test and its limit $\mathcal{R}$, in relationship to the expression of the general term.

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1 Introduction

Let us consider a series with positive terms $\sum_{n=1}^{\infty} a_n$. If the nature of the series is studied by directly applying Cauchy's root test or D'Alembert's ratio test, in the form with the limit ([3], [4]) *without having previously verified a necessary condition for convergence of the series, namely* $a_n \to 0$ and the series is in the inefficiency case of the test (i.e. $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$, respectively $\lim_{n\to\infty} (a_{n+1}/a_n) = 1$) , then it cannot be concluded from this finding *whether the general term of the series tends to* 0 *or not.* Indeed, simple and well-known examples of the inefficiency case show that both situations can occur. So, consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}; \sum_{n=1}^{\infty} \frac{1}{n}; \sum_{n=1}^{\infty} \frac{\lambda(n+1)}{n}, \text{ with } \lambda \neq 0;\ \sum_{n=1}^{\infty} \frac{\mathrm{e}^n n!}{n^n}.$$

All has both previous limits equal to 1. For the first two series, from which the first convergent, the second divergent, the general term tends to 0, and for the last two, the general term does not tend to 0, but to an arbitrary real number $\lambda \neq 0$, respectively to ∞, whence it immediately follows that these last two series are grossly divergent. (Note that obtaining the result $\lim_{n\to\infty} (\mathrm{e}^n n!/n^n) = \infty$ with

[1] *Corresponding author*, Bucharest,Valahia University of Târgovişte (retired), Romania, avernescu@gmail.com .

the help of Stirling's formula for the last series is trivial, but also it exists an elementary obtaining, without Stirling's formula, [10], page 140.)

A nice elementary result [7], due to G. Stoica, shows that

a) If $a > 0$, then $\lim\limits_{n\to\infty} \dfrac{n!}{(1+a)(2+a)(3+a)\cdot\ldots\cdot(n+a)} = 0$.

b) If $(a_n)_{n\geq 1}$ is a sequence of strictly positive numbers and $\lim\limits_{n\to\infty} n\left(\dfrac{a_n}{a_{n+1}} - 1\right) = 1$, then $a_n \to 0$.

Seen through the prism of the theory of series with positive terms, the result from b) brings an interesting clarification, showing that, if we study the nature of a series $\sum\limits_{n=1}^{\infty} a_n$ with positive terms by directly applying the Raabe-Duhamel test, *without having previously checked he necessary condition* $a_n \to 0$ and we find $\lim\limits_{n\to\infty} n\left(\dfrac{a_n}{a_{n+1}} - 1\right) = 1$, which gives the case of inefficiency, then, *even without knowing nature of the series, there is certainty that its general term tends to* 0 (of course, in this case, the use of other convergence tests for the series being necessary.) The very example of the series $\sum\limits_{n=1}^{\infty} a_n$, with $a_n = \dfrac{n!}{(1+a)(2+a)(3+a)\cdot\ldots\cdot(n+a)}$, comes in support of the statement, since although it is obtained (using exactly the Raabe-Duhamel test for which $\mathcal{R}_n = \dfrac{n}{n+1}a$) that for $a > 1$ the series is convergent, and for $a < 1$ is divergent, however, in both these cases its general term tends to 0. [For $a = 1$ the previous series trivially becomes the harmonic one, $\sum\limits_{n=1}^{\infty} 1/(n+1)$, divergent].

In this paper, we propose to make some more remarks on the limit that appears in the Raabe-Duhamel test, for some classical series with positive terms.

2 Some preliminary notational conventions and a short recall of a classic example

In order to facilitate the writing, we will use, according to G. M. Fihtenholţ [1], pp. 250-254, for any series with positive terms $\sum\limits_{n=1}^{\infty} a_n$, the notations (inspired by the initial of Raabe)

$$\mathcal{R}_n = n\left(\frac{a_n}{a_{n+1}} - 1\right), \quad \mathcal{R} = \lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}} - 1\right)$$

The expression $\mathcal{R}_n$ will be called throughout this paper the Raabe-Duhamel expression (in short the R-D expression) of the sequence $a = (a_n)_n$ or of the series $\sum\limits_{n=1}^{\infty} a_n$, and the limit $\mathcal{R}$ will be called the Raabe-Duhamel constant (in short the

R-D constant) of the respective sequence or series. When it will be necessary to specify the sequence $a = (a_n)_n$, to avoid any confusion, we will use the notations $\mathcal{R}_n(a)$, respectively $\mathcal{R}_n(a)$.

Before pass to the main results, remember another classic example of series where the Raabe-Duhamel test leads to the inefficiency result $\mathcal{R} = 1$. It is the series $\sum\limits_{n\geq 2} 1/n(\ln n)^\alpha$ with $a > 0$, for which:

(i) In the case $\alpha > 1$, the series is convergent;

(ii) In the case $\alpha < 1$, the series is convergent.

Indeed,

$$\mathcal{R}_n = n\left(\frac{\dfrac{1}{n(\ln n)^\alpha}}{\dfrac{1}{(n+1)(\ln(n+1))^\alpha}} - 1\right) = \frac{(n+1)(\ln(n+1))^\alpha - n(\ln n)^\alpha}{(\ln n)^\alpha}.$$

Aplying the mean theorem for the function f: $[2,\infty) \to \mathbb{R}$, $f(x) = x(\ln x)^\alpha$ on the compact interval $[n, n+1]$, we obtain

$$\mathcal{R}_n = \frac{(\ln c_n)^\alpha}{(\ln n)^\alpha} + \frac{\alpha(\ln c_n)^{\alpha-1}}{(\ln n)^\alpha},$$

where $c_n \in (n, n+1)$, from which it follows that $\mathcal{R} = \lim\limits_{n\to\infty} \mathcal{R}_n = 1$, i.e. the series is in the inefficiency case of the Raabe-Duhamel test.

In order to investigate the nature of the series, the integral convergence test for series with positive terms can be used. For $\alpha \neq 1$, a primitive of the function $g : [2,\infty) \to \mathbb{R}$, $g(x) = 1/x(\ln x)^\alpha = 1/f(x)$ is the function $G : [2,\infty) \to \mathbb{R}$, $G(x) = -\dfrac{1}{(\alpha-1)(\ln x)^{\alpha-1}}$, then

$$\int_2^\infty \frac{1}{x(\ln x)^\alpha}\mathrm{d}x = \lim_{b\to\infty}\int_2^b \frac{1}{x(\ln x)^\alpha}\mathrm{d}x = \lim_{b\to\infty}\left(-\frac{1}{(\alpha-1)(\ln x)^{\alpha-1}}\right)\Bigg|_{x=2}^{x=b} =$$

$$= \frac{1}{(\alpha-1)}\lim_{b\to\infty}\left(\frac{1}{(\ln 2)^{\alpha-1}} - \frac{1}{(\ln b)^{\alpha-1}}\right).$$

Therefore, if $\alpha > 1$, then

$$\int_2^\infty \frac{1}{x(\ln x)^\alpha}\mathrm{d}x = \frac{1}{(\alpha-1)(\ln 2)^{\alpha-1}},$$

the integral is convergent, so the series is also convergent, and if $\alpha < 1$, then the integral has the value ∞ and the series is divergent.

In the case $\alpha = 1$, a primitive of the function g is $G(x) = \ln(\ln x)$, then

$$\int_2^\infty \frac{1}{x\ln x}\mathrm{d}x = \lim_{b\to\infty}(\ln(\ln b) - \ln(\ln 2)) = \infty,$$

so that the integral and the series are, in this case, both divergent.

3 Main results

Consider applying the Raabe-Duhamel test to Riemann's generalized harmonic series $\sum_{n=1}^{\infty} 1/n^{\alpha} \quad (\alpha > 0)$.

Theorem 1. *For the generalized harmonic series* $\sum_{n=1}^{\infty} 1/n^{\alpha} \quad (\alpha > 0)$, *we have* $\mathcal{R} = \alpha$, *that is the R-D constant of the series is equal to the exponent* α.

Proof. We have

$$\mathcal{R}_n = n\left(\frac{\dfrac{1}{n^{\alpha}}}{\dfrac{1}{(n+1)^{\alpha}}} - 1\right) = n\frac{(n+1)^{\alpha} - n^{\alpha}}{n^{\alpha}} = \frac{\alpha c_n^{\alpha-1}}{n^{\alpha-1}} \xrightarrow[(n\to\infty)]{} \alpha.$$

(We applied the mean theorem to the fonction $x \mapsto x^{\alpha}$ on the interval $[n, n+1]$, $c_n = n + \theta_n$, with $\theta \in (0,1)$, is a point of the interval $(n, n+1)$.) $\square$

So, with theorem 1, based on the Raabe-Duhamel test, we find again, the classical result regarding the nature of the generalized harmonic series, for $\alpha \neq 1$.

Next, we will study the case where the general term of the series for which we apply the Raabe-Duhamel test is *related as order of magnitude to* $1/n^{\alpha}$. To this end, we will intend to first quickly review some simple examples of series with positive terms, all with the general term tending to 0; some of these will contain in the formula of the general term the expression

$$\Omega_n = \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot 2n}, \tag{1}$$

for which it exist the obvious recurrence relation

$$\Omega_{n+1} = \frac{2n+1}{2n+2}\Omega_n. \tag{2}$$

(We introduced this notation in some old works from 1991 and later we used it in [9] and [10].)

We will review, for each of these series, its expression R-D, $\mathcal{R}$ as well as its associated constant R-D, $\mathcal{R}$; in order to simplify the writing, we will write $\sum$ instead of $\sum_{n=1}^{\infty}$.

a) For the series $\sum \dfrac{n^n}{e^n n!}$, we obtain after some calculation

$$\mathcal{R}_n = n\frac{e - \left(1 + \dfrac{1}{n}\right)^n}{\left(1 + \dfrac{1}{n}\right)^n} \xrightarrow[(n\to\infty)]{} \mathcal{R} = \frac{1}{2} < 1 \text{ (then the series is divergent).}$$

[To calculate the limit $\mathcal{R}$, one can switch to the real variable, then applying L'Hospital's rule, or one can use the double estimate from G. Pólya and G. Szegö [5], page 38.

$$\frac{\mathrm{e}}{2n+2} < \mathrm{e} - \left(1+\frac{1}{n}\right)^n < \frac{\mathrm{e}}{2n+1} \tag{3}$$

(for which we gave a very short proof in [8]).]

b) For the series $\sum \Omega_n$, also taking into account that $\Omega_{n+1}/\Omega_n = (2n+1)/(2n+2)$, we have

$$\mathcal{R}_n = \frac{n}{2n+1} \xrightarrow[(n\to\infty)]{} \mathcal{R} = \frac{1}{2} < 1 \text{ (so the series is divergent).}$$

c) For the series $\sum \frac{\Omega_n}{\sqrt{n}}$ we have, after some calculations,

$$\mathcal{R}_n = \frac{8n^2+11n+4}{n(2n+1)\left[\left(2+\frac{2}{n}\right)\sqrt{1+\frac{1}{n}}+\left(2+\frac{1}{n}\right)\right]} \xrightarrow[(n\to\infty)]{} \mathcal{R} = 1 \text{ (so case of inefficiency.)}$$

Indeed, on the other hand, using the left side of the elementary inequality

$$\frac{1}{2\sqrt{n}} \leq \Omega_n < \frac{1}{\sqrt{2n+1}} \tag{4}$$

we have $\frac{\Omega_n}{\sqrt{n}} \geq \frac{1}{2n}$ and the direct comparison with the harmonic series $\sum \frac{1}{n}$ confirms that the series is divergent.

d) For the series $\sum \frac{\Omega_n}{n+p}$ (where $p > 0$), we have

$$\mathcal{R}_n = n\frac{3n+(p+2)}{(2n+1)(n+p)} \xrightarrow[(n\to\infty)]{} \mathcal{R} = \frac{3}{2} > 1 \text{ (so the series is convergent).}$$

d') We mention that, for the particular case $p = 0$, the series was studied by B. Ross in [6], finding the sum ln4, and the general case of d) was treated in [9], where we obtained that

$$\sum \frac{\Omega_n}{n+p} = 2^{p-1}\frac{\Gamma^2(p)}{\Gamma(2p)} - \frac{1}{p},$$

in which $\Gamma(p) = \int\limits_0^\infty t^{p-1}\mathrm{e}^{-t}\mathrm{d}t$, $p > 0$ is, of course, the Gamma function of Euler.

e) For the series $\sum \Omega_n^2$ we have

$$\mathcal{R} = \frac{n(4n+3)}{(2n+1)^2} \xrightarrow[(n\to\infty)]{} \mathcal{R} = 1 \text{ (so, case of inefficiency)}$$

But using once again the left side of the inequality (1), we obtain $\Omega_n^2 \geq 1/4n$ and the comparison with the harmonic series shows that the series is divergent.

f) For the series $\sum \Omega_n^3$ we have

$$\mathcal{R}_n = \frac{12n^2+18n+7}{(2n+1)^2} \xrightarrow[(n\to\infty)]{} \mathcal{R} = \frac{3}{2} > 1 \text{ (so the series is convergent).}$$

g) In the more general case of the series $\sum \Omega_n^p$, with $p>0$, we obtain

$$\mathcal{R}_n = n\frac{(2n+2)^p-(2n+1)^p}{(2n+1)^p}$$

whence, applying the mean theorem of to the function $f(x)=x^p$ on the compact interval $[2n+1, 2n+2]$, and then the squeeze theorem, we obtain

$$\mathcal{R} = \lim_{\to\infty} \mathcal{R}_n = \frac{p}{2}.$$

So, if $p>2$, it results $\mathcal{R}>1$, then the series is convergent, and if $p<2$, it results $\mathcal{R}<1$, then the series is divergent. (The case $p=2$ constituted the example e).)

h) For the series $\sum \dfrac{\Omega_n}{n^q}$ with $q>0$, we obtain after a few slightly longer calculations

$$\mathcal{R}_n = 2\frac{(n+1)^{q+1}-n^{q+1}}{(2n+1)n^{q-1}} - \frac{n^q}{(2n+1)n^{q-1}} = 2\frac{(q+1)(n+\theta_n)^q}{(2n+1)n^{q-1}} - \frac{n}{2n+1},$$

where $n+\theta_n \in (n, n+1)$. Passing to the limit for $n\to\infty$, we find $\mathcal{R}=p+1/2$.

So, for $q>1/2$ the series is convergent, and for $q<1/2$ the series is divergent. (The case $q=0$ was the example b), the case $q=1$, giving $\sum \dfrac{\Omega_n}{n^q}$ is included in the example d), specified in d'), and the case $q=1/2$ constituted the example c)).

We specify that, as we have already done for some of the previous series a) – h), establishing the nature of the other above series (with the Raabe-Duhamel criterion efficient or not) can be also done without its use, but using only an appropriate left or right side of the elementary inequality (4) and the comparison with the generalized harmonic series, of a convenient exponent. Moreover, using both sides of a more refined and concludent two sided estimate, namely

$$\frac{1}{\sqrt{\pi(n+1/2)}} < \Omega_n < \frac{1}{\sqrt{\pi n}}$$

(called in D. S. Mitrinović - P. M. Vasić [3], page 192 the inequality of Wallis), where its coefficient of $\sqrt{n}$ from the denominator is the same in both extreme parts, an asymptotic equivalence can be established between the general term of the previous series and the general term of some generalized harmonic series, slightly modified by multiplying by the coefficient $1/\sqrt{\pi}$. Thus, using for the asymptotic equivalence the notation '$a_n \sim b_n$ in the sense that $\lim\limits_{n\to\infty}(a_n/b_n)=1$ we have, referring to all the previously mentioned series

a) $\dfrac{n^n}{e^n n!} \sim \dfrac{1}{\sqrt{2\pi n}}$; b) $\Omega_n \sim \dfrac{1}{\sqrt{\pi n}}$; c) $\dfrac{\Omega_n}{\sqrt{n}} \sim \dfrac{1}{\sqrt{\pi}}\dfrac{1}{n}$; d) $\dfrac{\Omega_n}{n+p} \sim \dfrac{1}{\sqrt{\pi}} \cdot \dfrac{1}{n^{3/2}}$;
e) $\Omega_n^2 \sim \dfrac{1}{\pi n}$; f) $\Omega_n^3 \sim \dfrac{1}{(\pi n)^{3/2}}$; g) $\Omega_n^p \sim \dfrac{1}{\pi^{p/2} n^{p/2}}$; h) $\dfrac{\Omega_n}{n^p} \sim \dfrac{1}{\sqrt{\pi} n^{n+1/2}}$
therefore all the series a) – h) can be compared to the corresponding generalized harmonic series $\sum 1/n^\alpha$ $(\alpha > 0)$, with an adequate α.

We remark that, every time, the value of the constant R-D, $\mathcal{R}$ *coincides with the exponent of the generalized harmonic series* $\sum 1/n^\alpha$ with which the comparison was made.

All these examples suggest us the formulation of a unitary rule in the case in which the general term of a series with positive terms is asymptotically equivalent (in a slightly more restrictive sense, which will be specified) to $1/n^\alpha$, where α is a certain positive exponent. Before this, we recall a certain asymptotic development of Ω_n ([10], page 148)

$$\Omega_n = \frac{1}{\sqrt{\pi\left(n + \frac{1}{4} + \frac{1}{32n} - \frac{1}{128n} - \frac{5}{2048n} + \frac{23}{4096n} + \dots\right)}},$$

which is of the form

$$\Omega_n = \frac{1}{(n + c + \varepsilon_n)^\alpha},$$

where $\lambda = \dfrac{1}{\sqrt{\pi}}$, $c = \dfrac{1}{4}$, $\varepsilon_n = \dfrac{1}{32n} - \dfrac{1}{128n} - \dfrac{5}{2048n} + \dfrac{23}{4096n} + \dots$, $\alpha = \dfrac{1}{2}$, $(\varepsilon_n)_n$ being a sequence of real numbers that tends to 0.

We can formulate now

Theorem 2. *Given the series with positive terms* $\sum a_n$, *if there exists* $\lambda \in \mathbb{R}$, $c \in \mathbb{R}$, $a > 0$ *and a sequence* $(\varepsilon_n)_n$ *tending to 0, such that*

$$a_n = \frac{\lambda}{(n + c + \varepsilon_n)^2}$$

then $\mathcal{R} = \alpha$, *i.e. the R-D constant of the series* $\sum a_n$ *is the exponent* α *of the generalized harmonic series* $\sum 1/n^\alpha$.

Proof. We have

$$\mathcal{R}_n(\alpha) = n\frac{a_n - a_{n+1}}{a_{n+1}} = n\frac{\lambda\left[(n + c + \varepsilon_n)^{-\alpha} - (n + 1 + c + \varepsilon_{n+1})^{-\alpha}\right]}{\lambda\,(n + c + \varepsilon_{n+1})^{-\alpha}}$$

$$= n\frac{\alpha\,(n + c + \theta_n)^{-\alpha-1}\,(1 + \varepsilon_{n+1} - \varepsilon_n)}{(n + c + \varepsilon_{n+1})^{-\alpha}} = \frac{(n + c + \varepsilon_{n+1})^\alpha\,(1 + \varepsilon_{n+1} - \varepsilon_n)}{\alpha\,(n + c + \theta_n)^{\alpha+1}},$$

where we applied the mean theorem for the function $x \mapsto x^\alpha$ on the compact interval $[n + c + \varepsilon_n, n + 1 + c + \varepsilon_{n+1}]$ and θ_n is a point situed between ε_n and ε_{n+1}. Therefore $\mathcal{R}(a) = \lim\limits_{n\to\infty} \mathcal{R}_n(\alpha) = \alpha$. □

Theorem 3. *If two sequences with positive terms $a = (a_n)_n$ and $b = (b_n)_n$ are asymptotically equivalent in the sense that $a_n = b_n + \varepsilon_n$, in which $(\varepsilon_n)_n$ is a sequence of real numbers tending to spre 0 and if, in addition, the following conditions are satisfied*

α) $\lim\limits_{n\to\infty} \dfrac{\varepsilon_n}{b_n} = 0$, *$\beta$)* $\lim\limits_{n\to\infty} n\dfrac{\varepsilon_n - \varepsilon_{n+1}}{b_{n+1}} = 0$,

then the constants Raabe-Duhamel of the two sequences are equal.

Proof. We have, after some elementary calculations

$$\mathcal{R}_n(a) = \frac{1}{1+\dfrac{\varepsilon_{n+1}}{b_{n+1}}} \cdot \left(n\frac{b_n - b_{n+1}}{b_{n+1}} + n\frac{\varepsilon_n - \varepsilon_{n+1}}{b_{n+1}} \right),$$

therefore $\mathcal{R}_n(a) = \mathcal{R}_n(b)$. This gives $\mathcal{R}(a) = \mathcal{R}(b)$. □

The theorem shows that the series $\sum a_n$ and $\sum b_n$ have the same nature.

Corollary 1. *If the sequence $a = (a_n)_n$ is given by the formula $a_n = \dfrac{\lambda}{n^\alpha} + \dfrac{\mu}{n^{\alpha+2}}$, with $\alpha > 0$, then $\mathcal{R}(\alpha) = \alpha$.*

Indeed, choosing $b_n = \dfrac{\lambda}{n^\alpha}$ and $\varepsilon_n = \dfrac{\mu}{n^{\alpha+2}}$, the conditions of theorem 3 are satisfyied.

An example: For the series $\sum \left(H_n - \ln n - \gamma - \dfrac{1}{2n} \right)$, where $H_n = 1 + \dfrac{1}{2} + \ldots + \dfrac{1}{n}$, and γ is the constant of Euler, $\gamma = \lim\limits_{n\to\infty} (H_n - \ln n) = 0,577\ldots$, by using the formula

$$H_n = \ln n + \gamma + \frac{1}{2n} - \frac{1}{12n^2} + \frac{\theta_n}{120n^4} \ \ (0 < \theta_n < 1),$$

and considering $b_n = \dfrac{1}{12n^2}$ and $\varepsilon_n = \dfrac{\theta_n}{120n^4}$ we deduce that $\mathcal{R}(\alpha) = 2$.

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$$\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{n \cdot 2^n \cdot n!} = \frac{1}{2} + \frac{3}{16} + \frac{15}{144} + \dots = \ln 4n,$$

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$$\Omega_n = \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \dots \cdot 2n},$$

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