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COMMON FIXED POINT VIA NEW TYPE OF IMPLICIT RELATIONS

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Abstract

This paper focuses on three things. Firstly, we define a new type of implicit relations which covers a multitude of contractive conditions in one go. Secondly, we use this implicit relation to prove a new common fixed point theorem for four occasionally weakly biased maps of type (A) in a dislocated metric space. This theorem improves some results existing in the fixed point theory's environment. Thirdly, we will provide an example to illustrate our main theorem and to expound the credibility and generality of our result.

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Key words: dislocated metric space, occasionally weakly biased maps of type (A), unique common fixed points.

1 Introduction

Fixed point theory is of predominant significance in many domains of mathematics, sciences and engineering. It is considered as one of the most active area of research of the last 60 years, or so. It has prolific applications in distinct fields as biology, chemistry, physics, engineering, game theory, economics, image processing, and so on. In the last fifty years or so, several authors have made some alterations to Banach contraction principle sometimes by expanding the contractive condition to a general one and other times by substituting the complete metric space by different generalized metric spaces (see [1], [2], [3], [15], [19], [21], [28], [31], and [32]). In particular, in 1985, Matthews [20] introduced the concept of dislocated metric spaces under the notion of metric domains in order to promote the notion of completeness in domain theory. In 2001, Hitzler and his supervisor Seda [16] used the notion of dislocated metric spaces (d-metric spaces) in order to retain a version of the Banach contraction map theorem. In 2012, Amini-Harandi [6] introduced a new generalization of a partial metric space which is called a

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metric-like space. In fact, dislocated metric spaces and metric-like spaces are the same. Using this definition, several authors studied the existence and uniqueness of common fixed points under diverse conditions (see for example [7], [13], [14], [18], [23], [25], [26], [33], [34]).

On the other hand, in the last few years, in [10] we put in the concepts of occasionally weakly f-biased of type (A) and occasionally weakly g-biased of type (A), and we revealed that the two new definitions coincide with our concepts; occasionally weakly f-biased and occasionally weakly g-biased respectively given in [11]. In this paper, we will demonstrate a unique common fixed point theorem for occasionally weakly biased maps of type (A) on a dislocated metric space through a new type of implicit relations. Our result enlarges and/or ameliorates the one's of [4], [5], [8], [9], [12], [17], [22], [24], [27], [35], [36], and others. Besides, we will enhance our work by giving an example to expound the credibility and generality of our result over the results of the involved articles.

2 Preliminaries

Definition 1. *([20]) A* ***Metric Domain*** *is a pair $< D, d >$ where D is a non-empty set, and d is a function from $D \times D$ to $\mathbb{R}^+$ such that*

1. $\forall\ x,\ y \in D\ d(x, y) = 0 \Rightarrow x = y$
2. $\forall\ x,\ y \in D\ d(x, y) = d(y, x)$
3. $\forall\ x,\ y,\ z \in D\ d(x, y) \leq d(x, z) + d(z, y)$.

Definition 2. *([16, 6]) Let $\mathbb{X}$ be a non-empty set. A function $d : \mathbb{X}\times\mathbb{X} \longrightarrow [0, \infty)$ is said to be a dislocated metric (or a metric-like) (or a d-metric) on $\mathbb{X}$ if for any $x,\ y,\ z \in \mathbb{X}$, the following conditions hold:*

1. $d(x, y) = 0 \Rightarrow x = y$;
2. $d(x, y) = d(y, x)$;
3. $d(x, z) \leq d(x, y) + d(y, z)$.

The pair $(\mathbb{X}, d)$ is then called a ***dislocated metric (metric-like) (d-metric)*** *space.*

Example 1. *If $\mathbb{X} = [0, \infty)$, then $d(x, y) = x + y$ defines a dislocated metric on $\mathbb{X}$.*

Example 2. *Let $\mathbb{X} = [0, \infty)$ define the distance function $d : \mathbb{X} \times \mathbb{X} \longrightarrow \mathbb{X}$ by $d(x, y) = \max\{x, y\}$. Clearly $\mathbb{X}$ is a dislocated metric space.*

Definition 3. *([10]) Let $\aleph$ and $\hbar$ be self-maps of a non-empty set $\mathbb{X}$. The pair $(\aleph, \hbar)$ is said to be* ***occasionally weakly $\aleph$-biased of type*** *(A) and* ***occasionally***

weakly ħ-biased of type (A)*, respectively, if and only if, there exists a point* p *in* $\mathbb{X}$ *such that* $\aleph p = \hbar p$ *implies*

$$d(\aleph\aleph p, \hbar p) \leq d(\hbar\aleph p, \aleph p),$$
$$d(\hbar\hbar p, \aleph p) \leq d(\aleph\hbar p, \hbar p),$$

respectively.

3 Main results

3.1 New type of implicit relations

Popa ([29], [30]) is considered as the first founder of implicit relations because he unified a lot of explicit contractive conditions by inaugurating the implicit contractive condition. Encouraged by this technique, we will instigate a new type of implicit relations.

let Γ be a family of all functions $\gamma : \mathbb{R}_+^6 \to \mathbb{R}$ satisfying the following conditions:

- (γ_1): γ is non-increasing in variables t_2, t_3, t_4, t_5 and t_6,
- (γ_2): $\gamma(t, t, 2t, 2t, t, t) > 0 \ \forall t > 0$.

Example 3. $\gamma(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \alpha t_2 - \beta t_5 - \delta t_6$, *where* $\alpha, \beta, \delta > 0$ *and* $\alpha + \beta + \delta < 1$.

- (γ_1)*: It is obvious to see that* γ *is non-increasing in variables* t_2, t_3, t_4, t_5 *and* t_6,
- (γ_2)*:* $\gamma(t, t, 2t, 2t, t, t) = t - \alpha t - \beta t - \delta t = t(1 - \alpha - \beta - \delta) > 0 \ \forall t > 0$.

Example 4. $\gamma(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \nu(t_2 + t_3 + t_4 + t_5 + t_6)$, *where* $\nu \in \left(0, \frac{1}{7}\right)$.

- (γ_1)*: It is clear to see that* γ *is non-increasing in variables* t_2, t_3, t_4, t_5 *and* t_6,
- (γ_2)*:* $\gamma(t, t, 2t, 2t, t, t) = t(1 - 7\nu) > 0 \ \forall t > 0$.

Example 5. $\gamma(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \mu \max\{t_2, t_3, t_4, t_5, t_6\}$, *where* $\mu \in \left(0, \frac{1}{2}\right)$.

- (γ_1)*: Obviously,*
- (γ_2)*:* $\gamma(t, t, 2t, 2t, t, t) = t(1 - 2\mu) > 0 \ \forall t > 0$.

Example 6. $\gamma(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \alpha t_2 - \beta t_3 - \eta t_4 - \delta t_5 - \lambda t_6$, *where* $\alpha, \beta, \eta, \delta, \lambda > 0$ *and* $\alpha + 2\beta + 2\eta + \delta + \lambda < 1$.

- (γ_1)*: Clearly,*

- (γ_2): $\gamma(t,t,2t,2t,t,t) = t(1-\alpha-2\beta-2\eta-\delta-\lambda) > 0$ $\forall t > 0$.

Example 7. *$\gamma(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - \omega t_2 - \upsilon \max\{t_3,t_4\} - \tau t_5$, where $\omega, \upsilon, \tau > 0$ and $\omega + 2\upsilon + \tau < 1$.*

- (γ_1): *Trivial,*
- (γ_2): $\gamma(t,t,2t,2t,t,t) = t(1-\omega-2\upsilon-\tau) > 0$ $\forall t > 0$.

Example 8. *$\gamma(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - at_2 - b\max\{t_3,t_4\} - ct_5 - dt_6$, where $a,b,c,d > 0$ and $a + 2b + c + d < 1$.*

- (γ_1): *Evident,*
- (γ_2): $\gamma(t,t,2t,2t,t,t) = t(1-a-2b-c-d) > 0$ $\forall t > 0$.

Example 9. *$\gamma(t_1,t_2,t_3,t_4,t_5,t_6) = t_1^\kappa - \rho t_2^\kappa - \varrho \max\{t_3^\kappa, t_4^\kappa\} - \sigma t_5^\kappa$, where κ is a positive integer, $\rho, \varrho, \sigma > 0$ and $\rho + 2^\kappa \varrho + \sigma < 1$.*

- (γ_1): *Clear,*
- (γ_2): $\gamma(t,t,2t,2t,t,t) = t^\kappa(1-\rho-2^\kappa\varrho-\sigma) > 0$ $\forall t > 0$.

Example 10. *$\gamma(t_1,t_2,t_3,t_4,t_5,t_6) = t_1^\iota - \zeta t_2^\iota - \xi \max\{t_3^\iota, t_4^\iota\} - \varsigma t_5^\iota - \chi t_6^\iota$, where ι is a positive integer, $\zeta, \xi, \varsigma, \chi > 0$ and $\zeta + 2^\iota \xi + \tau + \chi < 1$.*

- (γ_1): *Obvious,*
- (γ_2): $\gamma(t,t,2t,2t,t,t) = t^\iota(1-\zeta-2^\iota\xi-\varsigma-\chi) > 0$ $\forall t > 0$.

3.2 A unique common fixed point theorem for quadruple maps

Theorem 1. *Let $\aleph$, $\hbar$, $\imath$ and $\jmath$ be self-maps of a d-metric space $\mathbb{X}$ satisfying the following condition*

$$\gamma\left(d(\aleph x, \hbar y), d(\imath x, \jmath y), d(\aleph x, \imath x), d(\hbar y, \jmath y), d(\imath x, \hbar y), d(\aleph x, \jmath y)\right) \leq 0 \qquad (1)$$

for all x, $y \in \mathbb{X}$, where $\gamma \in \Gamma$. If the pair $(\aleph, \imath)$ as well as $(\hbar, \jmath)$ is occasionally weakly $\imath$-biased of type (A) and occasionally weakly $\jmath$-biased of type (A), respectively, then $\aleph$, $\hbar$, $\imath$ and $\jmath$ have a unique common fixed point.

Proof. By hypotheses, there are two points u and v in $\mathbb{X}$ such that $\aleph u = \imath u$ implies $d(\imath\imath u, \aleph u) \leq d(\aleph\imath u, \imath u)$ and $\hbar v = \jmath v$ implies $d(\jmath\jmath v, \hbar v) \leq d(\hbar\jmath v, \jmath v)$.

First, we are going to prove that $\aleph u = \hbar v$. Suppose that $\aleph u \neq \hbar v$, from inequality (1) we have

$$\begin{aligned} & \gamma(d(\aleph u, \hbar v), d(\imath u, \jmath v), d(\aleph u, \imath u), d(\hbar v, \jmath v), d(\imath u, \hbar v), d(\aleph u, \jmath v)) \\ = \; & \gamma(d(\aleph u, \hbar v), d(\aleph u, \hbar v), d(\aleph u, \aleph u), d(\hbar v, \hbar v), d(\aleph u, \hbar v), d(\aleph u, \hbar v)) \leq 0. \end{aligned}$$

Since γ is non-increasing in t_3 and t_4, using the triangle inequality we get

$$\begin{aligned} 0 &\geq \gamma(d(\aleph u, \hbar v), d(\aleph u, \hbar v), d(\aleph u, \aleph u), d(\hbar v, \hbar v), d(\aleph u, \hbar v), d(\aleph u, \hbar v)) \\ &\geq \gamma(d(\aleph u, \hbar v), d(\aleph u, \hbar v), 2d(\aleph u, \hbar v), 2d(\aleph u, \hbar v), d(\aleph u, \hbar v), d(\aleph u, \hbar v)), \end{aligned}$$

contradicts (γ_2) thus $\aleph u = \hbar v$.

Now, we assert that $\aleph\aleph u = \aleph u$. If not, then the use of condition (1) gives

$$\begin{aligned} 0 &\geq \gamma(d(\aleph\aleph u, \hbar v), d(\imath\aleph u, \jmath v), d(\aleph\aleph u, \imath\aleph u), d(\hbar v, \jmath v), d(\imath\aleph u, \hbar v), d(\aleph\aleph u, \jmath v)) \\ &= \gamma(d(\aleph\aleph u, \aleph u), d(\imath\aleph u, \aleph u), d(\aleph\aleph u, \imath\aleph u), d(\aleph u, \aleph u), d(\imath\aleph u, \aleph u), d(\aleph\aleph u, \aleph u)). \end{aligned}$$

Since the pair $(\aleph, \imath)$ is occasionally weakly $\imath$-biased of type (A) and γ is non-increasing in t_2, t_3, t_4 and t_5, using the triangle inequality we get

$$\gamma(d(\aleph\aleph u, \aleph u), d(\aleph\aleph u, \aleph u), 2d(\aleph\aleph u, \aleph u), 2d(\aleph\aleph u, \aleph u), d(\aleph\aleph u, \aleph u), d(\aleph\aleph u, \aleph u)) \leq 0,$$

contradicts (γ_2) hence $\aleph\aleph u = \aleph u$ and so $\imath\aleph u = \aleph u$.

Suppose that $\hbar\hbar v \neq \hbar v$. Using inequality (1) we obtain

$$\begin{aligned} &\gamma(d(\aleph u, \hbar\hbar v), d(\imath u, \jmath\hbar v), d(\aleph u, \imath u), d(\hbar\hbar v, \jmath\hbar v), d(\imath u, \hbar\hbar v), d(\aleph u, \jmath\hbar v)) \\ =\ &\gamma(d(\hbar v, \hbar\hbar v), d(\hbar v, \jmath\hbar v), d(\hbar v, \hbar v), d(\hbar\hbar v, \jmath\hbar v), d(\hbar v, \hbar\hbar v), d(\hbar v, \jmath\hbar v)) \leq 0. \end{aligned}$$

As γ is non-increasing in t_2, t_3, t_4 and t_6, and the pair $(\hbar, \jmath)$ is occasionally weakly $\jmath$-biased of type (A), the use of the triangle inequality gives

$$\gamma(d(\hbar v, \hbar\hbar v), d(\hbar v, \hbar\hbar v), 2d(\hbar v, \hbar\hbar v), 2d(\hbar v, \hbar\hbar v), d(\hbar v, \hbar\hbar v), d(\hbar v, \hbar\hbar v)) \leq 0$$

contradicts (γ_2) thus $\hbar\hbar v = \hbar v$ and so $\jmath\hbar v = \hbar v$; i.e., $\hbar\aleph u = \aleph u$ and $\jmath\aleph u = \aleph u$. Put $\aleph u = \imath u = \hbar v = \jmath v = w$, therefore w is a common fixed point of maps $\aleph$, $\hbar$, $\imath$ and $\jmath$.

Finally, let w and t be two distinct common fixed points of maps $\aleph$, $\hbar$, $\imath$ and $\jmath$. Then, $w = \aleph w = \hbar w = \imath w = \jmath w$ and $t = \aleph t = \hbar t = \imath t = \jmath t$. By condition (1) we have

$$\begin{aligned} &\gamma(d(\aleph w, \hbar t), d(\imath w, \jmath t), d(\aleph w, \imath w), d(\hbar t, \jmath t), d(\imath w, \hbar t), d(\aleph w, \jmath t)) \\ =\ &\gamma(d(w, t), d(w, t), d(w, w), d(t, t), d(w, t), d(w, t)) \leq 0. \end{aligned}$$

Since γ is non-increasing in t_3 and t_4, by the triangle inequality we obtain

$$\begin{aligned} 0 &\geq \gamma(d(w, t), d(w, t), d(w, w), d(t, t), d(w, t), d(w, t)) \\ &\geq \gamma(d(w, t), d(w, t), 2d(w, t), 2d(w, t), d(w, t), d(w, t)) \end{aligned}$$

which implies that $t = w$. □

3.3 Illustrative example

Example 11. *Let* $\mathbb{X} = [0,31)$ *with the d-metric* $d(x,y) = \max\{x,y\}$*. Define*

$$\aleph x = \begin{cases} \frac{x}{2} \text{ if } x \in [0,1] \\ \frac{1}{5} \text{ if } x \in (1,31), \end{cases} \quad \hbar x = \begin{cases} 0 \text{ if } x \in [0,1] \\ \frac{1}{4} \text{ if } x \in (1,31), \end{cases}$$

and

$$\imath x = \begin{cases} 30x \text{ if } x \in [0,1] \\ 3 \text{ if } x \in (1,31), \end{cases} \quad \jmath x = \begin{cases} 0 \text{ if } x \in [0,1] \\ 30 \text{ if } x \in (1,31). \end{cases}$$

First it is clear to see that $\aleph$ *and* $\imath$ *are occasionally weakly* $\imath$*-biased of type* (A) *and* $\hbar$ *and* $\jmath$ *are occasionally weakly* $\jmath$*-biased of type* (A)*. Take* $\gamma(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - \frac{1}{3}\max\{t_2,t_3,t_4,t_5,t_6\}$*, we get*

1. *for* $x,\ y \in [0,1]$ *we have* $\aleph x = \frac{x}{2}$, $\hbar y = 0$, $\imath x = 30x$ *and* $\jmath y = 0$ *and*

$$\begin{aligned} & \gamma\left(d(\aleph x,\hbar y), d(\imath x,\jmath y), d(\aleph x,\imath x), d(\hbar y,\jmath y), d(\imath x,\hbar y), d(\aleph x,\jmath y)\right) \\ = & \frac{x}{2} - \frac{1}{3}\max\left\{30x, 30x, 0, 30x, \frac{x}{2}\right\} \\ = & -\frac{19x}{2} \\ \leq & \ 0. \end{aligned}$$

2. *For* $x,\ y \in (1,31)$*, we have* $\aleph x = \frac{1}{5}$, $\hbar y = \frac{1}{4}$, $\imath x = 3$, $\jmath y = 30$ *and*

$$\begin{aligned} & \gamma\left(d(\aleph x,\hbar y), d(\imath x,\jmath y), d(\aleph x,\imath x), d(\hbar y,\jmath y), d(\imath x,\hbar y), d(\aleph x,\jmath y)\right) \\ = & \frac{1}{4} - \frac{1}{3}\max\{30,3,30,3,30\} \\ = & -\frac{39}{4} \\ \leq & \ 0. \end{aligned}$$

3. *For* $x \in [0,1]$, $y \in (1,31)$*, we have* $\aleph x = \frac{x}{2}$, $\hbar y = \frac{1}{4}$, $\imath x = 30x$, $\jmath y = 30$ *and*

$$\begin{aligned} & \gamma\left(d(\aleph x,\hbar y), d(\imath x,\jmath y), d(\aleph x,\imath x), d(\hbar y,\jmath y), d(\imath x,\hbar y), d(\aleph x,\jmath y)\right) \\ = & \max\left\{\frac{x}{2},\frac{1}{4}\right\} - \frac{1}{3}\max\left\{30, 30x, 30, \max\left\{30x,\frac{1}{4}\right\}, 30\right\} \\ = & \max\left\{\frac{x}{2},\frac{1}{4}\right\} - 10 \\ \leq & \ 0. \end{aligned}$$

4. *Finally, for* $x \in (1,31)$, $y \in [0,1]$, *we have* $\aleph x = \dfrac{1}{5}$, $\hbar y = 0$, $\imath x = 3$, $\jmath y = 0$ *and*

$$\begin{aligned}&\gamma\left(d(\aleph x,\hbar y),d(\imath x,\jmath y),d(\aleph x,\imath x),d(\hbar y,\jmath y),d(\imath x,\hbar y),d(\aleph x,\jmath y)\right)\\=&\ \frac{1}{5}-\frac{1}{3}\max\left\{3,3,0,3,\frac{1}{5}\right\}\\=&\ -\frac{4}{5}\\\leq&\ 0,\end{aligned}$$

so, all hypotheses of the above theorem are satisfied and 0 *is the unique common fixed point of maps* $\aleph$, $\hbar$, $\imath$ *and* $\jmath$.

Remark 1. *Note that the main results of [9], [17], [22], [24], [27] and [35] are not applicable because the four maps are not continuous and* $\aleph(\mathbb{X}) = \left[0,\dfrac{1}{2}\right] \nsubseteq \jmath(\mathbb{X}) = \{0,30\}$.

3.4 Some results

Corollary 1. *Let* $\aleph$, $\hbar$, $\imath$ *and* $\jmath$ *be self-maps of a d-metric space* $\mathbb{X}$ *satisfying the following condition*

$$d(\aleph x,\hbar y) \leq \alpha d(\imath x,\jmath y)+\beta d(\imath x,\hbar y)+\gamma d(\aleph x,\jmath y)$$

for all x, $y \in \mathbb{X}$, *where* $\alpha,\beta,\gamma > 0$ *and* $\alpha+\beta+\gamma < 1$. *If the pair* $(\aleph,\imath)$ *as well as* $(\hbar,\jmath)$ *is occasionally weakly* $\imath$*-biased of type* (A) *and occasionally weakly* $\jmath$*-biased of type* (A), *respectively, then* $\aleph$, $\hbar$, $\imath$ *and* $\jmath$ *have a unique common fixed point.*

Proof. Use Theorem 1 and Example 3. □

Remark 2. *The above corollary improves Theorem 3.1 of [17] and Theorems 2.1 and 2.6 of [9].*

Corollary 2. *Let* $\aleph$, $\hbar$, $\imath$ *and* $\jmath$ *be self-maps of a d-metric space* $\mathbb{X}$ *satisfying the following condition*

$$d(\aleph x,\hbar y) \leq \jmath(d(\imath x,\jmath y)+d(\aleph x,\imath x)+d(\hbar y,\jmath y)+d(\imath x,\hbar y)+d(\aleph x,\jmath y))$$

for all x, $y \in \mathbb{X}$, $\jmath \in \left(0,\dfrac{1}{7}\right)$. *If the pair* $(\aleph,\imath)$ *as well as* $(\hbar,\jmath)$ *is occasionally weakly* $\imath$*-biased of type* (A) *and occasionally weakly* $\jmath$*-biased of type* (A), *respectively, then* $\aleph$, $\hbar$, $\imath$ *and* $\jmath$ *have a unique common fixed point.*

Proof. Use Theorem 1 and Example 4. □

Remark 3. *The above corollary improves and extends Theorems 2 and 4 of [24] and improves Theorems 1 and 3 of [27].*

Corollary 3. *Let $\aleph$, $\hbar$, $\imath$ and $\jmath$ be self-maps of a d-metric space $\mathbb{X}$ satisfying the following condition*

$$d(\aleph x, \hbar y) \leq \varsigma \max\{d(\imath x, \jmath y), d(\aleph x, \imath x), d(\hbar y, \jmath y), d(\imath x, \hbar y), d(\aleph x, \jmath y)\}$$

for all x, $y \in \mathbb{X}$, where $\varsigma \in \left(0, \frac{1}{2}\right)$. If the pair $(\aleph, \imath)$ as well as $(\hbar, \jmath)$ is occasionally weakly $\imath$-biased of type (A) and occasionally weakly $\jmath$-biased of type (A), respectively, then $\aleph$, $\hbar$, $\imath$ and $\jmath$ have a unique common fixed point.

Proof. Use Theorem 1 and Example 5. □

Remark 4. *The above corollary improves and extends Theorems 1 and 3 of [24] and improves Theorems 2 and 4 of [27].*

Corollary 4. *Let $\aleph$, $\hbar$, $\imath$ and $\jmath$ be self-maps of a d-metric space $\mathbb{X}$ satisfying the following condition*

$$d(\aleph x, \hbar y) \leq \alpha d(\imath x, \jmath y) + \beta d(\aleph x, \imath x) + \gamma d(\hbar y, \jmath y) + \delta d(\imath x, \hbar y) + \lambda d(\aleph x, \jmath y)$$

for all x, $y \in \mathbb{X}$, where $\alpha, \beta, \gamma, \delta, \lambda > 0$ and $\alpha + 2\beta + 2\gamma + \delta + \lambda < 1$. If the pair $(\aleph, \imath)$ as well as $(\hbar, \jmath)$ is occasionally weakly $\imath$-biased of type (A) and occasionally weakly $\jmath$-biased of type (A), respectively, then $\aleph$, $\hbar$, $\imath$ and $\jmath$ have a unique common fixed point.

Proof. Use Theorem 1 and Example 6. □

Remark 5. *The above corollary improves Theorem 3.1 of [22] and Theorem 3.1 of [35].*

Corollary 5. *Let $\aleph$, $\hbar$, $\imath$ and $\jmath$ be self-maps of a d-metric space $\mathbb{X}$ satisfying the following condition*

$$d^p(\aleph x, \hbar y) \leq a d^p(\imath x, \jmath y) + b \max\{d^p(\aleph x, \imath x), d^p(\hbar y, \jmath y)\} + c d^p(\imath x, \hbar y)$$

for all x, $y \in \mathbb{X}$, where p is a positive integer, $a, b, c > 0$ and $a + 2^p b + c < 1$. If the pair $(\aleph, \imath)$ as well as $(\hbar, \jmath)$ is occasionally weakly $\imath$-biased of type (A) and occasionally weakly $\jmath$-biased of type (A), respectively, then $\aleph$, $\hbar$, $\imath$ and $\jmath$ have a unique common fixed point.

Proof. Use Theorem 1 and Example 9. □

Remark 6. *The above corollary extends and improves Theorem 1 and Corollary 1 of [12].*

Corollary 6. *Let $\aleph$, $\hbar$, $\imath$ and $\jmath$ be self-maps of a d-metric space $\mathbb{X}$ satisfying the following condition*

$$\begin{aligned} d^p(\aleph x, \hbar y) \leq\ & a d^p(\imath x, \jmath y) + b \max\{d^p(\aleph x, \imath x), d^p(\hbar y, \jmath y)\} + c d^p(\imath x, \hbar y) \\ & + d d^p(\aleph x, \jmath y) \end{aligned}$$

for all x, $y \in \mathbb{X}$, where p is a positive integer, $a, b, c, d > 0$ and $a + 2^p b + c + d < 1$. If the pair $(\aleph, \imath)$ as well as $(\hbar, \jmath)$ is occasionally weakly $\imath$-biased of type (A) and occasionally weakly $\jmath$-biased of type (A), respectively, then $\aleph$, $\hbar$, $\imath$ and $\jmath$ have a unique common fixed point.

Proof. Use Theorem 1 and Example 10. □

3.5 A unique common fixed point theorem for a sequence of maps

Theorem 2. *Let $\imath$, $\jmath$ and $\{\aleph_n\}_{n=1,2,...}$ be self-maps of a d-metric space $\mathbb{X}$ satisfying the following condition*

$$\begin{aligned}&\gamma(d(\aleph_n x, \aleph_{n+1} y), d(\imath x, \jmath y), d(\aleph_n x, \imath x), d(\aleph_{n+1} y, \jmath y),\\ &d(\imath x, \aleph_{n+1} y), d(\aleph_n x, \jmath y)) \leq 0\end{aligned}$$

for all x, $y \in \mathbb{X}$, where $\gamma \in \Gamma$. If the pair $(\aleph_n, \imath)$ as well as $(\aleph_{n+1}, \jmath)$ is occasionally weakly $\imath$-biased of type (A) and occasionally weakly $\jmath$-biased of type (A), respectively, then $\imath$, $\jmath$ and $\{\aleph_n\}_{n=1,2,...}$ have a unique common fixed point.

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