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A THEORETHICAL MATHEMATICAL MODEL FOR ENERGY BALANCE IN GREENHOUSES

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Abstract: *A mathematical model for greenhouse climate which consists of a set of algebraic equations was developed. The equations were written for four parts of a greenhouse: outdoor air, indoor air, surface and ground surface not covered by crop. It was assumed that the air is well mixed in the greenhouse, the thermal properties of building materials does not change in time and solar radiation passing through the transparent material without absorption. Model input parameters are ambient air temperature, solar radiation values on the surface, normal solar radiation on the surface, soil temperature inside the greenhouse and soil temperature at a depth of 5 cm. A computer program was written in C ++ language.*

Keywords: *energy balance, mathematical model, thermal properties.*

1. Introduction

A plant greenhouse is a site where the environment can be controlled or modified to make it suitable for growing different types of plants. Environment emissions are represented by a group of spatial mean values of climatic factors, such as radiation, temperature, humidity and carbon dioxide concentration, that have effects on plant growth and development. Architectural parameters of plants and climate variables are not homogeneous in greenhouses. Considering environmental factors around live plants in a greenhouse is called greenhouse microclimate. This greenhouse microclimate regulates the direct exchange of energy and mass and affects the metabolic activities of plants. Greenhouse microclimate is influenced by the macro-climate, the external environment and the physical state (geometrical parameters, thermal conditions, etc.) of plants in the greenhouse [1].

The difference in emissions and the climate outside is caused mainly by two mechanisms. The first is containing air, which is stagnant due to the built and covered enclosure, in the greenhouse. So, air exchange with the outside of the greenhouse is significantly lower than when there is no coverage. Moreover, local air velocities are small compared to the outdoor greenhouse ones. Reducing air

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exchange or ventilation, directly affects the energy exchange and the mass of air emissions, while the speeds of the local air affects to a lesser extent the exchange of energy, water vapor and carbon dioxide from air emissions and balances crop, soil surface, inside air and heating system emissions [2].

Second, there is the mechanism for the absorption of radiation by activating radiation shortwave, from direct sunlight and scattered in the sky and clouds, and they are captured by the opaque and transparent components of the greenhouse, while the radiation wavelength exchange between the indoor and the outdoor environment is affected due to the radiative properties of the roofing materials.

The microclimate of the greenhouse quantitatively describes the processes of energy and mass transfer in the canopy, the exchange processes between air and plants and other surfaces, and how plants respond to environmental factors. The greenhouse microclimate is affected by the orientation of the greenhouse, latitude, greenhouse area, the canopy inner surface of the greenhouse, uncovered soil surface inside the greenhouse, structural design (shape and size) of the greenhouse, properties of the material used for building greenhouses. The microclimate depends also on the ventilation inside the greenhouse, if it is naturally ventilated or is equipped with fans.

2. The Greenhouse Model

The present work is a mathematical model which was developed and based on the energy balance of a greenhouse for plants, consisting of a set of algebraic equations that describe a system with lumped parameters [8]. The equations were written for the four components of the greenhouse respectively, outside surface coatings, indoor air, indoor surface of canopy and soil surface, uncovered by crops. The system of algebraic equations was solved by using the Gauss-Seidel iterative method [3], [4].

The following simplifying assumptions have been made for the plant greenhouse model:
- The air inside the greenhouse is well mixed and homogeneous at any time, so there are gradients of temperature or humidity in the air,
- Condensation and evaporation on the greenhouse floor is negligible,
- Indoor air is permeable by infrared wavelength radiation,
- Each component of the greenhouse temperature: exterior roof, indoor air, indoor roof is the same throughout the greenhouse [5].

2.1. Energy Balance of the Cover Roof of the Greenhouse

The energy balance of the cover roof of the greenhouse includes solar radiation absorbed by the layer of coating, solar radiation reflected from crest culture of plants and soil uncovered plants that are absorbed by the coating, heat transfer by convection between the roof and the air outside the greenhouse, heat transfer by convection between the roof and indoor air, heat radiation exchange from the roof and outdoor air, thermal radiation emitted by the inner surface of the coating material of the greenhouse thermal radiation emitted by the bare soil surface and absorbed coating (Fig. 1) [6].

Energy balance of the cover surface of the greenhouse is presented below:

$$\begin{aligned} & I_g \times A_{ci} \times \alpha_c + I_g \times A_{cii} \times \alpha_c (1+\tau_c) - h_{co} \times A_c \times (T_c - T_0) - (\mathrm{E}_c \times \sigma \times T_c^4 \times A_c - \\ & - E_{sky} \times \sigma \times T_{sky}^4 \times A_c \times \alpha_{ct}) + I_{gh} \times \tau_c \times A_{ca} \times \alpha_{ct} + E_{sb} \times \sigma \times T_{sb}^4 \times A_{sb} \times \alpha_{ct} - \\ & - h_{ci} \times A_c (T_c - T_i) - E_c \times \sigma \times T_c^4 \times (1 - F_c) \times A_c = 0 \end{aligned} \tag{1}$$

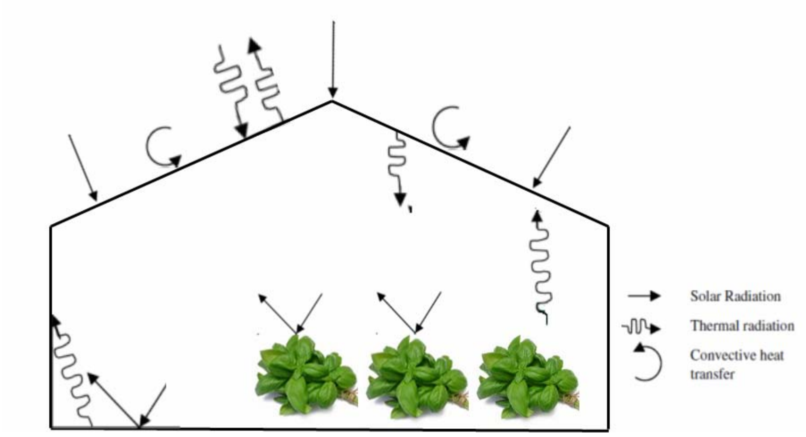

Fig. 1. *Energy balance of greenhouse cover*

where:

$I_g \times A_{ci} \times \alpha_c + I_g \times A_{cii} \times \alpha_c (1+\tau_c)$ – solar radiation absorbed by the cover;

$h_{co} \times A_c (T_c - T_0)$ – convective heat transfer from cover to ambient air;

$(E_c \times \sigma \times T_c^4 \times A_c - E_{sky} \times \sigma \times T_{sky}^4 \times A_c \times \alpha_{ct})$ – net thermal radiation exchange between cover and sky;

$I_{gh} \times \tau_c \times A_{ca} \times \alpha_{ct}$ – solar radiation reflected by the canopy and absorbed by the cover;

$I_{gh} \times \tau_c \times A_{sb} \times \rho_{sbca} \times \alpha_c$ – solar radiation reflected by bare soil and absorbed by the cover;

$E_{ca} \times \sigma \times T_{ca}^4 \times A_{ca} \times \alpha_{ct}$ – thermal radiation emitted by the canopy and absorbed by the cover;

$E_{sb} \times \sigma \times T_{sb}^4 \times A_{sb} \times \alpha_{ct}$ – thermal radiations emitted by bare soil and absorbed by the cover;

Convective heat transfer from cover to inside air: $h_{ci} \times A_c (T_c - T_i)$;

$E_c \times \sigma \times T_c^4 \times (1 - F_c) \times A_c$ – radiation emitted by the inside cover.

Values of properties of different materials/components used in greenhouses Table 1

Material/component	Property	Symbol	Value
Polyethylene cover	Transmissivity	τ_c	0.65
	Absorptivity of solar radiations	α_c	0.20
	Reflectivity of solar radiations	ρ_c	0.15
	Emissivity	E_c	0.9
	Absorptivity of thermal radiations	α_{ct}	0.9
Tomato canopy	Transmissivity	τ_{ca}	0
	Absorptivity of solar radiations	α_{ca}	0.77
	Reflectivity of thermal radiations	ρ_{ca}	0.23
	Emissivity	E_{ca}	0.98
	Absorptivity of thermal radiations	α_{cat}	0.98

Soil	Transmissivity	τ_{sb}	0
	Absorptivity of solar radiations	α_{sb}	0.8
	Reflectivity of solar radiations	ρ_{sb}	0.2
	Emissivity	E_{sb}	0.9
	Thermal conductivity	K	0.7
Sky	Emissivity	E_{sky}	0.9

Values of different constants used in the model Table 2

Name of constant	**Symbol**
Area of canopy inside the greenhouse	A_{ca}
Area of bare soil	A_{sb}
Area of greenhouse cover	A_c
Area of roof of the greenhouse	A_r
Area of side wall above canopy	A_{s1}
Thickness of soil for first layer	Z_1
Area of side walls	A_{sw}
Wind velocity inside the greenhouse	V_i
Wind velocity outside the greenhouse	V
Canopy coverage area factor	S_{hf}
Shape factor for roof to canopy	S_{fr}
Shape factor for first side wall to canopy	S_{f1}
Shape form for second side wall to canopy	S_{f2}
Shape factor for roof to bare soil	S_{frs}
Shape factor for side wall to bare soil	S_{fsws}

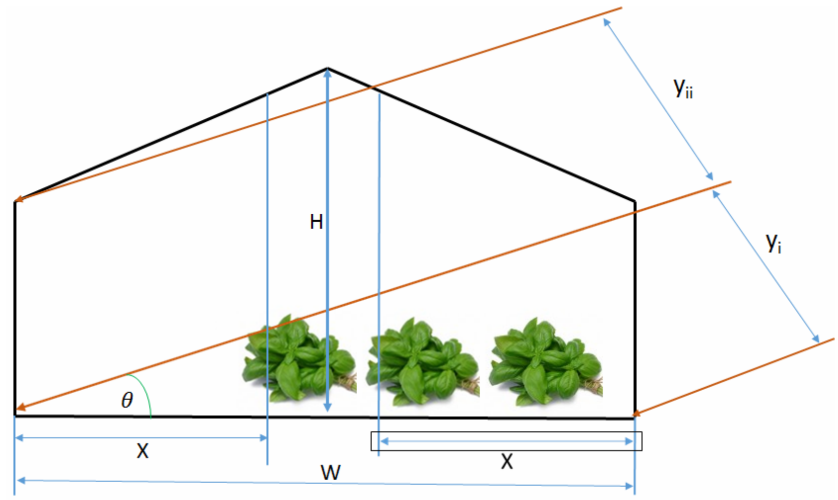

Fig. 2. *Line diagram of the greenhouse to calculate A_{ci} and A_{cii}*

The sunlight vertical angle East-West varies in time. It depends on the latitude of the location of the greenhouse, the angle of elevation, azimuth, declination and hour angle. To calculate the East-West vertical angle the following equation is used [7]:

$$\theta = \tan^{-1}\left(-\frac{\cos(90-\beta)}{\cos(\gamma)}\right) \tag{2}$$

where:

$$\beta = \sin^{-1}(\cos(l)\cos(h)\cos(d) + \\ + \sin(l)\sin(d)) \tag{3}$$

$$\gamma = \cos^{-1}(\sec\beta(\cos(l)\sin(d) - \\ - \cos(d)\sin(l)\cos(h)) \tag{4}$$

$$d = 23.5\sin\frac{2n\pi}{365} \tag{5}$$

All heat transfer convective coefficients are calculated with the relation:

$$h = 2.8 + 3.0 \cdot V \tag{6}$$

Sunlight temperature is calculated with the relation:

$$T_{sky} = T_{amb} - 6. \tag{7}$$

2.2. Energy Balance of the Inside Air

The energy balance of inside air of the greenhouse is considered only from the perspective of convective heat transfer. It is assumed that solar radiation is absorbed by heat or air. For indoor heat transfer convection, heat transfer by convection from plants and air heat transfer by convection at the surface of the canopy but also indoor air convection and the bare surface of the soil are taken into consideration.

Heat losses due to ventilation or air infiltration exchanges are assumed to be equal to zero for this phase model. This may be considered valid if it involves stopping the fans in winter. The heat absorbed by the water spray is also assumed to be zero at this stage. All convective heat transfer coefficients are calculated using the equation (8).

$$h_{ica}(T_i - T_{ca})A_{ca} - h_{isb}(T_i - T_{sb})A_{sb} - \\ - h_{ic}(T_i - T_c) = 0 \tag{8}$$

where:

- $h_{ica}(T_i - T_{ca})A_{ca}$ convective heat transfer from inside air to canopy;
- $h_{isb}(T_i - T_{sb})A_{sb}$ – convective heat transfer from inside air to bare soil surface;
- $h_{ic}(T_i - T_c)$ – convective heat transfer from inside air to cover.

2.3. Energy Balance of the Inside Cover

The heat exchange between cover and air occurs with convective heat transfer, radiation heat exchange with the ground surface between the canopy and the soil surface under the canopy. The canopy absorbs heat and emits the thermal radiation emitted by the greenhouse covering material. Solar radiation is also absorbed by the canopy. The heat loss from the canopy generated by the transfer of heat through convection and by ventilation is shown in (Fig. 4).

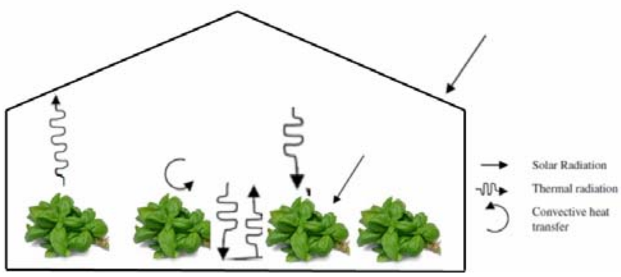

Fig. 4. *Energy balance for canopy surface of greenhouse*

Energy balance for the canopy surface of the greenhouse is given below:

$$\begin{aligned}&\tau_c I_{gh} S_{hf} \alpha_{ca} A_{ca} + E_c \sigma T_c^4 (A_\gamma S_{fr} + A_{s1} S_{f1} + A_{s2} S_{f2}) \alpha_{cat} - E_{ca} \sigma T_{ca}^4 A_{ca} \\ &- h_{cai} A_{ca} (T_{ca} - T_i) - h_{ca} A_{ca} (T_{ca} - T_i) + (E_s \sigma T_{sc}^4 A_{sc} \alpha_{cat} - E_{ca} \sigma T_{ca}^4 A_{ca}) = 0\end{aligned} \quad (9)$$

where:

$\tau_c I_{gh} S_{hf} \alpha_{ca} A_{ca}$ – shortwave radiations absorbed by the canopy;

$E_c \sigma T_c^4 (A_\gamma S_{f\gamma} + A_{s1} S_{f1} + A_{s2} S_{f2}) \alpha_{cat}$ – thermal radiations emitted by the cover and absorbed by the canopy;

$E_{ca} \sigma T_{ca}^4 A_{ca}$ – thermal radiation emitted by the canopy;

$h_{cai} A_{ca} (T_{ca} - T_i)$ – convective heat transfer from canopy to inside air;

$h_{ca} A_{ca} (T_{ca} - T_i)$ – latent heat flux due to transpiration;

$E_s \sigma T_{sc}^4 A_{sc} \alpha_{cat} - E_{ca} \sigma T_{ca}^4 A_{ca}$ – net thermal radiation exchange between canopy and soil.

2.4. Energy Balance of the Soil

Heat flows at the soil surface are due to thermal radiation emitted by the greenhouse roof and which are absorbed by the soil's uncovered surface, and direct solar radiation absorbed by the soil surface. The soil heat is transferred by radiation emitted from the uncovered soil surface, by the conduction heat transfer in the soil and by the convective heat transfer between the soil surface and the air in the greenhouse (Fig. 5).

$$I_{gh} A_{sb} \tau_c \alpha_{sb} - h_{si} (T_{sb} - T_i) A_{sb} - \left(\frac{K}{Z_1} \right) (T_{sb} - T_{g1}) A_{sb} - E_{sb} \sigma T_{sb}^4 A_{sb} + E_c \sigma T_c^4 (A_\gamma S_{f\gamma s} + A_{sw} S_{fsws}) \alpha = 0 \quad (10)$$

where:

- $h_{si} (T_{sb} - T_i) A_{sb}$ – conduction from soil surface to first soil layer;
- $\left(\frac{K}{Z_1} \right) (T_{sb} - T_{g1}) A_{sb}$ – conduction from soil surface to first soil layer;
- $E_{sb} \sigma T_{sb}^4 A_{sb}$ – thermal radiation emitted by soil;
- $I_{gh} A_{sb} \tau_c \alpha_{sb}$ – solar radiation absorbed by soil;
- $E_c \sigma T_c^4 (A_\gamma S_{f\gamma s} + A_{sw} S_{fsws}) \alpha$ –
- thermal radiations emitted by the greenhouse cover and absorbed by the soil

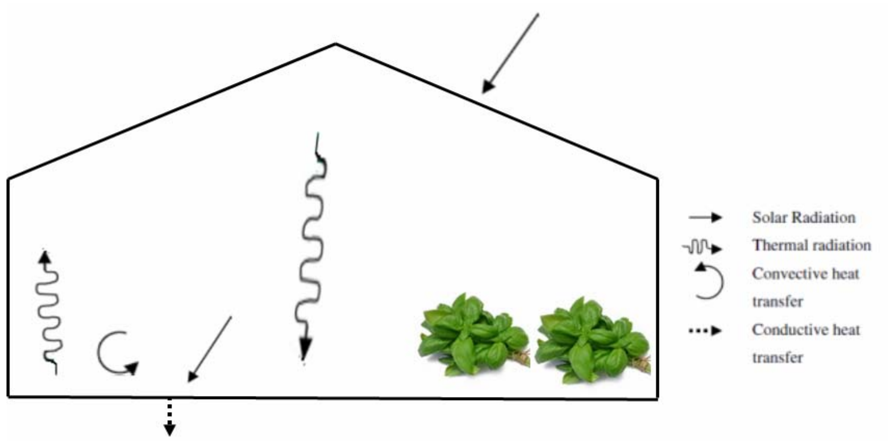

Fig. 5. *Energy balance of the bare soil surface of the greenhouse*

The material properties, such as absorptivity, reflectivity, emissivity, conductivity, etc. are selected from the literature on the materials used in this type of greenhouse. Some of the values used are measured, for example the ones for polyethylene cover transmissivity. Some properties are determined by heat transfer relations. The values of material properties used in the model are shown in Table 1 and the values of wind speed and the shape factor are given in Table 2 [9].

Rearranging Eq. (1), (11)-(13) the following system of equations results:

$$(h_{ica}A_{ca} + h_{isb}A_{sb} + h_{ic}A_c)T_i = h_{ica}A_{ca}T_{ca} + h_{isb}A_{sb}T_{sb} + h_{ic}A_cT_c \tag{11}$$

$$\begin{aligned}(h_{si} + \frac{K}{Z_1})A_{sb}T_{sb} + E_{sb}\sigma T_{sb}^4 A_{sb} &= I_{gh}A_{sb}\tau_c\alpha_{sb} + \frac{K}{Z_1}A_{sb}T_{g1} + \\ &+ h_{si}A_{sb}T_i + E_c\sigma T_c^4(A_\gamma S_{f\gamma s} + A_{sw}S_{fsws})\alpha_{sbt}\end{aligned} \tag{12}$$

$$\begin{aligned}(h_{co}A_c + h_{ci}A_c)T_c + E_c\sigma A_c(2 - F_c)T_c^4 &= I_g A_{ci}\alpha_c + I_g A_{cii}\alpha_c + I_g A_{cii}\tau_c\alpha_c + h_{co}T_oA_c + \\ &+ E_{sky}\sigma A_cT_{sky}^4\alpha_{ct} + I_{gh}\tau_cA_{ca}\rho_{ca}\alpha_c + I_{gh}\tau_cA_{sb}\rho_{sb}\alpha_c + E_{ca}\sigma T_{ca}^4A_{ca}\alpha_{ct} + \\ &+ E_{sb}\sigma T_{sb}^4A_{sb}\alpha_{ct} + h_{ci}A_cT_i\end{aligned} \tag{13}$$

$$\begin{aligned}2E_{ca}\sigma A_{ca}T_{ca}^4 + (h_{cai} + h_{ep})A_{ca}T_{ca} &= \tau_cI_{gh}S_{hf}\alpha_{ca}A_{ca} + E_s\sigma T_{sc}^4A_{ca}\alpha_{cat} + \\ &+ E_c\sigma T_c^4(A_\gamma S_{fr} + A_{s1}S_{f1} + A_{s2}S_{f2})\alpha_{cat} + (h_{cai} + H_{ca})A_{ca}T_i\end{aligned} \tag{14}$$

The equations were solved using the Gauss-Seidel iterative method, to determine cover temperature (T_C), indoor air temperature (T_i), the surface temperature of the canopy (T_{CA}) and the temperature of the soil surface (T_{SB}). The initial values of temperatures are supposed to be 0°C.

Acknowledgments

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