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DEVELOPMENT OF MATHEMATICAL MODEL FOR PREDICTING THE CUPPING OF LUMBER

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Abstract: *Deformations are a significant disadvantage of wood as a structural material for products made of it. Therefore, the purpose of this work was to determine the magnitude and nature of the deformations of lumber used for the manufacturing of laminated panels in construction. The main causes of the possible changes in the shape of the surfaces of laminated panels in construction are characterized. In this paper, a model has been developed for the theoretical study of shape change in lumber, which is proposed to be determined through the warping force, which causes the deformation of lumber. The engineering formulas for calculating the warping force that leads to lumber shape change were obtained. This force mainly depends on size, changes in moisture content and the location of the lumber in the cross-section of the log.*

Key words: *lumber, laminated panel constructions, shape change, shrinkage, swelling, wood moisture content, warping force.*

1. Introduction

Laminated panel constructions (LPC) are common among the structural elements for manufacturing various wood products [12, 17]. Mostly, the indicators of physical-and-mechanical characteristics, in particular strength and dimensional stability, are decisive when determining the quality of LPC; in the case of using LPC for facade surfaces, indicators of its aesthetics also become important [6, 11, 17, 23].

An important factor affecting the quality of wood products is the warping of the structural elements from which the products are made. There are many causes of this phenomenon: anisotropy of the physical and mechanical properties of wood, nonuniform shrinkage in different structural directions, residual stresses arising during the drying and mechanical processing of work pieces, etc. It is clear that it is practically impossible to take into account the influence of all these factors, and especially their interaction, on the magnitude of the warping deformation of

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lumber. To determine the magnitude of cupping (cupping is a warp across the width of the face, in which the edges are higher or lower than the center of the wood), it is advisable to use the warping force, the value of which can be predicted depending on the geometric and physical-mechanical parameters of the lumber and the changes in its moisture content. Therefore, the creation of a mathematical model for predicting the warping force, which would be quite simple in computational terms and, at the same time, would make it possible to adequately describe the process of lumber cupping, is an urgent task.

One of the disadvantages of wood products is that during operation, they are exposed to the influence of the external environment and, as a result, can change their shape and size. Wood moisture has the greatest influence on the physical and mechanical properties of wood. With a change in its moisture content, wood also changes its volume in proportion to the amount of hygroscopic moisture removed or introduced. In this case, a change in the shape of the surfaces (deformation) of lumber is observed (Figure 1) [1, 24], and most often such warping is transverse to its width [2, 13, 14, 23, 24]. A similar situation is observed for two-layer materials (glued), but the elasticity of the glue and the gluing of compacted wood make it possible to reduce cupping [9, 10]. In lumber with rectangular cross-sections (boards, beams, bars), warping is characterized by the movement of the edges of the lumber by a certain amount towards the convexity of the annual rings. This phenomenon is mostly caused by the nonuniformity of wood drying in different directions, in particular in the radial and tangential directions, and depends on the location of the lumber in the log [7, 8], which can be determined by the direction of the annual rings on the lumber surfaces [14, 24].

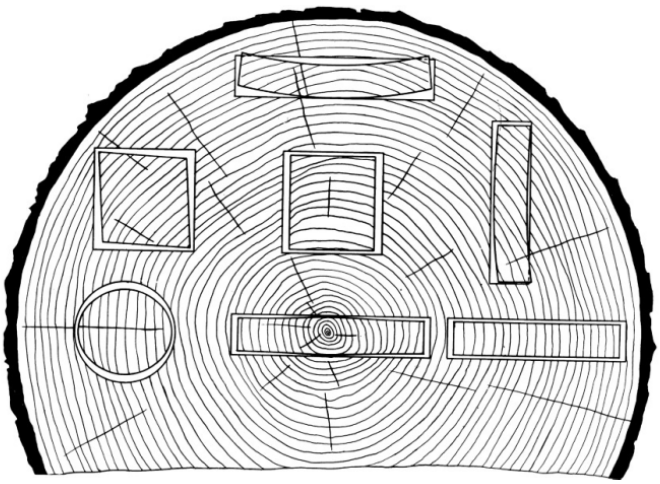


Fig. 1. *Deformation of lumberdepending on the place of its sawing out [6]*

It should be noted that neglecting the basic recommendations for selecting sections of wood before gluing them into a LPC is one of the main causes of obtaining low-quality products in terms of shape stability [11].

The problem of deformation of sawn timber in the process of its drying as such, which leads to its unsuitability for use in the constructions of various purposes, is considered in the paper [14]. The authors consider the main causes of this phenomenon to be the high sensitivity of wood to changes in moisture content, the unequal properties of wood in the direction from pith to bark, as well as the structure and direction of the wood fibers. Wood is considered to be a material with a large difference in properties between the longitudinal, radial, and tangential directions in terms of rigidity parameters; also, the influence of moisture content and temperature on wood, as well as the place in the log from where the piece of lumber is sawn out should be taken into account.

The results of experimental studies on the dimensional stability of laminated panels [4] and numerical-and-experimental studies of the dimensional stability of laminated columns [5] indicate that the dimensional stability of these elements strongly depends on the internal orientation of the individual sections of the wood of these elements, and the results of experimental and numerical studies are in good agreement. In another work [15], wood is considered an orthotropic material with great differences in properties between the longitudinal, radial, and tangential directions, and the directional numerical modeling of deformations and stresses in wood is carried out for variable moisture content.

Interesting approaches to reducing the cupping of lumber were studied by Peterson Silva Do Carmo et al. [16] and Xiang et al. [25]. The essence of the first approach [25] is to apply an impermeable coating to the inside (closer to the pith) and/or outside (closer to the peripheral part of the log) face of the lumber. The results of experimental studies showed that the application of an impermeable coating, especially on the inside face of the lumber, can reduce its cupping. In addition, the results of an experimental study on lumber cupping without coating its surfaces are in good agreement with the results of a theoretical study for the directional numerical model presented in [25].

The essence of the second approach [16] is to apply pressure to level curved lumber, in particular cupped lumber which can be used for LPC. The results of an experimental study of levelling curved lumber are in good agreement with the results of a theoretical study using the mechanical (numerical) model presented in [16]. The difference between the experimental and theoretical values is less than 5 kPa for most of the analyzed samples.

To study the stability of the shape of lumber during the drying process and in order to obtain information about unfavorable deformations and stresses that can develop during the drying process, Mitchell [13] simulated the drying process of lumber using known formulas and data [24]. In particular, the equation below was used to calculate the expected dimensional changes as the moisture content of the wood decreases or increases, according with the Equation (1):

$$\Delta D = D \cdot S\left(\frac{\Delta MC}{FSP}\right) \qquad (1)$$

where:

ΔD is the change in size (dimension);

D – the initial size;

S – the total drying from FSP to 0%;

ΔMC – the change in moisture content (below *FSP*);
FSP – fiber saturation point (average value is 30%).

The calculation using the above formula assumes that radial drying and tangential drying occur parallel to the surfaces of the lumber, and therefore does not take into account the curvilinear nature of the annual rings. The formula takes into account the following input data: initial width and thickness of the board, coordinates of the center of the board, initial and final moisture content (%), wood species, log diameter.

Scientists mainly consider the warping of lumber as the influence of residual and partially regenerated plastic-elastic strains that arise during the drying process [3, 19, 22, 26]. The issue of determining the magnitude and nature of such deformations is still relevant but extremely complex, because wood, in addition to its generally known properties, is also characterized by such properties as the complex nature of spatial correlations, the self-organization effects, and the presence of an ultra-slow diffusion-type process [18, 20, 21].

The search for alternative approaches, other than those described above, to determine the warping of lumber, which are suitable for effectively modelling such warping, including cupping, is timely and relevant. To study the deformation of lumber warping, relevant imulation studies were performed using the obtained mathematical dependencies to determine the warping force, which should be considered a conditional force that counteracts lumber cupping. The developed model and the obtained engineering formulas for the theoretical study of the shape change in lumber will make it possible to implement in practice design and technological solutions for the manufacturing of LPC in compliance with the requirements of their strength and form stability, and will contribute to both solving the tasks of manufacturing high-quality products and the rational use of wood.

2. Materials and Methods

The cross-sectional shape of a warped board resembles the elastic line of a hinged beam (propped cantilever beam). Then, during the shrinkage (swelling) process, a conditional force (let us call it the warping force) arises, which corresponds to the load of the beam with a concentrated force. To determine the force of warping, as well as the amount of deformation of the lumber along the vertical axis (deflection) at an arbitrary point in the cross-section, we will use the methods of material resistance. If the same force is applied to the cross-section of the board, but directed in the opposite direction, then there will be no warping.

Consider an arbitrary cross-section of a board ABCD, which will be presented as a two-support beam, loaded with force P_W (Figure 2). In the first approximation, the modulus of elasticity (Young's modulus) of the first kind *E* and the shrinkage factor S_F change according to a linear law in thickness. Thus, for the calculation diagram of the element shown in Figure 2, could be obtained by Equations (2) and (3):

$$E = E(y) = E_0 + A_y \quad (2)$$

$$S_F = S_F(y) = S_{F0} + C_y \quad (3)$$

where:

$$E_0 = \frac{E_1 \cdot b_2 - E_2 \cdot b_1}{b_2 - b_1} \quad (4)$$

$$S_{F0} = \frac{S_{F1} \cdot b_2 - S_{F2} \cdot b_1}{b_2 - b_1} \quad (5)$$

$$A = \frac{E_2 - E_1}{b_2 - b_1} \quad (6)$$

$$C = \frac{S_{F2} - S_{F1}}{b_2 - b_1} \quad (7)$$

In Equations (2) – (7) – E_1, S_{F1} and E_2, S_{F2}, respectively, represent the elasticity modulus and the shrinkage factor on the inside ($y=b_1$) and outside ($y=b_2$) faces of the board.

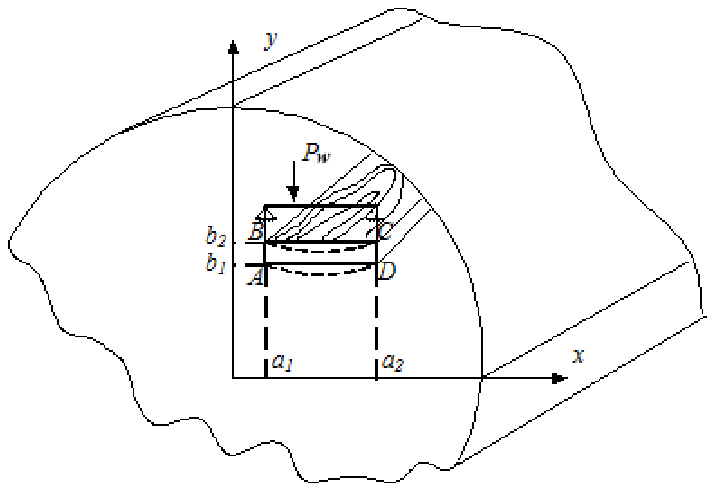


Fig. 2. *Calculation diagram for determining the warping force*

If, in addition, we take into account that the physical-and-mechanical characteristics of wood depend on moisture content, then Young's modulus can be represented as follows:

$$E = \left(E_0 + A_y\right) \cdot \left(1 + \beta \cdot \Delta \cdot W\right) \quad (8)$$

where:

$$\Delta W = W_{sp} - W_{fmc} \quad (9)$$

W_{sp} is the moisture content corresponding to the saturation point (the limit of hygroscopicity of a given wood species);

W_{fmc} – the final moisture content (moisture content at a given point at a given time);

β – the coefficient that takes into account the effect of the changes in moisture content on the modulus of elasticity.

According to Gayda and Kiydo [8], the stress in an arbitrary layer of lumber will be considered proportional to the difference between free shrinkage and contraction (Eq. (10)).

$$\sigma = E \cdot (\delta - \gamma) \quad (10)$$

where:

δ is the relative deformation at free shrinkage:

$$\delta = \frac{l_i - l}{l_i} = S_F \cdot \Delta W \quad (11)$$

l_i – the initial length of the longitudinal layer of lumber at a moisture content above (or equal to) the saturation point;

$l = f(y)$ – the length of the longitudinal layer with coordinate *y*;

S_F – the shrinkage factor;

γ – the contraction which occurs when the selected element is freed from fasteners (supports):

$$\gamma = \frac{l_i - l_{act}}{l_i} \quad (12)$$

l_{act} – the actual length.

In this case, an equilibrium state arises, leading to the same deformation for all fibers (Eq. (13)):

$$\gamma = const \quad (13)$$

Since a linear stress state is considered, two of the six integral equations of equilibrium describing the equilibrium state of the body in space are not identical (Equations (14) and (15)):

$$\int_{b_1}^{b_2} \sigma dy = 0 \quad (14)$$

$$\int_{b_1}^{b_2} \sigma y dy = M \quad (15)$$

where: *M* is the bending moment leading to the curvature of the element axis and associated with the presence of the warping force.

The practice of wood drying shows that, depending on the position of the lumber in the sawing scheme, the resistance of the boards to cupping is different. The warping force is the greatest for the central boards and decreases as the boards move away in the sawing scheme from the center (the pith of a log) to the outer portion of the log. In this regard, we introduce one more assumption. Let us assume (Figure 3) that the distance from the point of application of the warping force to the middle of the cross-section of the board is proportional to the distance between the center of the cross-section of the log and the middle of the cross-section of the board (Equations (16) and (17)):

$$\frac{x_0}{\frac{B}{2}} = \frac{a_0}{R} \quad (16)$$

or

$$x_0 = \frac{B \cdot a_0}{2R} \quad (17)$$

where:

B is the width of the board:

$$B = a_2 - a_1 \quad (18)$$

a – the abscissa of the center of the board:

$$a_0 = \frac{a_1 + a_2}{2} \tag{19}$$

R – the radius of the log from which the board is sawn out.

Thus, for the bending moment included in the second Equations (14) and (15), based on the well-known formula for the resistance of materials to determine the bending moment from the force in the span of the beam, we obtain:

$$M = \frac{P_w \cdot \left(\frac{B}{2} - x_0\right) \cdot \left(\frac{B}{2} + X_0\right)}{B} \tag{20}$$

where P_w is the warping force.

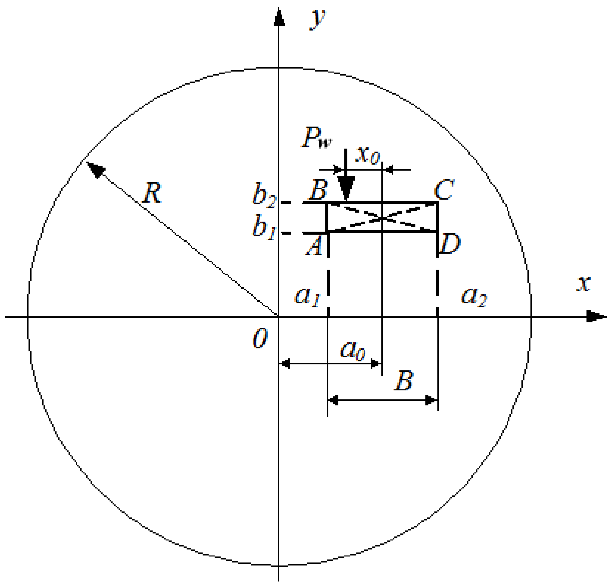


Fig. 3. *Scheme for determining the point of application of the warping force*

We determine the amount of contraction γ using the first equilibrium Equations (14) and (15). Taking into account the Equations (2), (3), (8)-(13), we obtain:

$$\int_{b1}^{b2} (E_0 + A_Y)(1 + \beta \cdot \Delta W)((S_{F0} + C_Y)\Delta W - \gamma)dy = 0 \tag{21}$$

or

$$\int_{b1}^{b2} (E_0 + A_Y)(S_{F0} + C_Y)\Delta W dy = \int_{b1}^{b2} \gamma (E_0 + A_Y) dy \tag{22}$$

or

$$\Delta W\left(E_o \cdot S_{F0} \cdot y + \left(A \cdot S_{F0} + CE_0\right) \cdot \frac{y^2}{2} + AC \cdot \frac{y^3}{3}\right)\Bigg|_{b1}^{b2} = \gamma\left(E_0 \cdot y + A\frac{y^2}{2}\right)\Bigg|_{b1}^{b2} \tag{23}$$

Hence we obtain:

$$\gamma = \frac{E_0 \cdot S_{F0} \cdot \left(b_2 - b_1\right) + \left(A \cdot S_{F0} + CE_0\right) \cdot \dfrac{b_2^2 - b_1^2}{2} + A \cdot C \cdot \dfrac{b_2^3 - b_1^3}{3}}{E_0 \cdot \left(b_2 - b_1\right) + A \cdot \dfrac{b_2^2 - b_1^2}{2}} \Delta W \tag{23}$$

Now we use the second equilibrium Equations (14) and (15):

$$\int_{b1}^{b2} \left(E_0 + A_y\right)\left(1 + \beta\Delta W\right)\left(\left(S_{F0} + Cy\right)\Delta W - \gamma\right) y dy = M \tag{24}$$

or

$$\int_{b1}^{b2} \left(E_0 + A_y\right)\left(S_{F0} + Cy\right) y \left(1 + \beta\Delta W\right)\Delta W dy - \gamma \int_{b1}^{b2} \left(E_0 + A_y\right)\left(1 + \beta\Delta W\right) y dy = M \tag{25}$$

or

$$\left(1 + \beta\Delta W\right)\Delta W \int_{b1}^{b2} \left(E_0 \cdot S_{F0} \cdot y + \left(E_0 \cdot C + S_{F0} \cdot A\right) y^2 + A \cdot C \cdot y^3\right) dy -$$

$$- \gamma\left(1 + \beta\Delta W\right)\int_{b1}^{b2} \left(E_0 \cdot y + A \cdot y^2\right) dy = M \tag{26}$$

or

$$\left(1 + \beta\Delta W\right)\Delta W\left(\frac{E_0 \cdot S_{F0} \cdot \left(b_2^2 - b_1^2\right)}{2} + \frac{\left(E_0 \cdot C + S_{F0} \cdot A\right) \cdot \left(b_2^3 - b_1^3\right)}{3} + \frac{A \cdot C \cdot \left(b_2^4 - b_1^4\right)}{4}\right) -$$

$$- \gamma\left(1 + \beta\Delta W\right) \cdot \left(\frac{E_0 \cdot \left(b_2^2 - b_1^2\right)}{2} + \frac{A \cdot \left(b_2^3 - b_1^3\right)}{3}\right) = \frac{Pw\left(\dfrac{B}{2} - x_0\right) \cdot \left(\dfrac{B}{2} + x_0\right)}{B} \tag{27}$$

.

Then we finally obtain for the warping force *Pw*:

$$Pw = \frac{B\cdot(1+\beta\Delta W)\Delta W}{\left(\frac{B}{2}-x_0\right)\cdot\left(\frac{B}{2}+x_0\right)}\cdot\Bigg[\left(\frac{E_0\cdot S_{F0}\cdot(b_2^2-b_1^2)}{2}\right)+\frac{(E_0\cdot C+S_{F0}\cdot A)\cdot(b_2^3-b_1^3)}{3}+\frac{A\cdot C\cdot(b_2^4-b_1^4)}{4}-$$

$$-\frac{E_0\cdot S_{F0}\cdot(b_2-b_1)+(A\cdot S_{F0}+C\cdot E_0)\cdot\frac{(b_2^2-b_1^2)}{2}+\frac{A\cdot C\cdot(b_2^3-b_1^3)}{3}}{E_0\cdot(b_2-b_1)+A\cdot\frac{b_2^2-b_1^2}{2}}\cdot\left(\frac{E_0\cdot(b_2^2-b_1^2)}{2}+\frac{A\cdot(b_2^3-b_1^3)}{3}\right)^2\Bigg] \quad (24)$$

To study the deflection of a certain cross-section of lumber, taking into account the obtained values of the warping force, we write the differential equation of the curved axis of the beam (Eq. (25)):

$$EIy''(z) = M(z) \quad (25)$$

where:

- *E* is the Young's modulus of the material (wood of a certain species);
- *I* – the moment of inertia of the cross-section relative to the neutral line;
- *M(z)* – the bending moment in the section with coordinate z.

For a two-support beam loaded asymmetrically with a concentrated force *Pw* applied in the beam span, it is advisable to consider two characteristic sections (Figure 3): to the left of the force *Pw*(length *a*) and to the right of the force *Pw*(length *b, (a+b=B)).* Therefore, Equation (25) must be written and integrated twice within each characteristic section.

As a result of the mathematical transformations, we will obtain the following Equations for the deflections and angles of rotation within each characteristic section:

1. for the first section of length *a:*

$$EIy(z) = \frac{Pw\cdot b}{l}\cdot\frac{z^3}{6}-\frac{Pw\cdot b}{6}\cdot\left(l^2-b^2\right)\cdot z = \frac{Pw\cdot b\cdot z}{6\cdot l}\cdot\left(z^2+b^2-l^2\right) = -\frac{Pw\cdot b\cdot z}{6\cdot l}\cdot\left(a^2+2\cdot a\cdot b^2-z^2\right) \quad (26)$$

$$EI\Theta(z) = \frac{Pw\cdot b}{2\cdot l}\cdot\left(z^2+\frac{b^2-l^2}{3}\right) = -\frac{Pw\cdot b}{6}\cdot\left(a^2+2\cdot a\cdot b^2-3\cdot z^2\right) \quad (27)$$

2. for the second section of length *b:*

$$EIy(z) = \frac{Pw \cdot b}{l} \cdot \frac{z^3}{6} - \frac{Pw \cdot (z-a)^3}{6} - \frac{Pw \cdot b}{l} \cdot \left(l^2 - b^2\right) \cdot z =$$

$$= \frac{Pw \cdot b \cdot z}{6 \cdot l} \cdot \left(z^2 + b^2 - l^2\right) - \frac{Pw \cdot (z-a)^3}{6} = -\frac{Pw \cdot a}{6 \cdot l} \cdot \left(z^3 - 3 \cdot l \cdot z^2 + \left(a^2 + 2 \cdot l^2 \cdot z - a^2 \cdot l\right)\right) \tag{28}$$

$$EI\Theta(z) = -\frac{Pw \cdot a}{6 \cdot l} \cdot \left(3 \cdot z^3 - 6 \cdot l \cdot z + a^2 + 2 \cdot l^2\right) \tag{29}$$

Let us determine the abscissa of the section with the largest deflection Z_f and the magnitude of the largest deflection y_{max}.To do this, we will solve the resulting equations, understanding that the largest deflection will occur within the larger of the two sections (if a >b, then the largest deflection will occur on the left section of length *a,* and vice versa).

$$EIy'(Z_f) = EI\Theta(Z_f) \geq 0 \tag{30}$$

or

$$Zf = \sqrt{\frac{a^2 + 2 \cdot a \cdot b}{3}} = \sqrt{\frac{a \cdot (a + 2 \cdot b)}{3}} = \sqrt{\frac{(l-b) \cdot (l+b)}{3}} = \sqrt{\frac{l^2 - b^2}{3}} \tag{31}$$

Then the maximum deflection (Equation (32)):

$$ymax = -\frac{Pw \cdot b}{6 \cdot l} \cdot \left(a^2 + 2 \cdot a \cdot b^2 - \frac{a^2 + 2 \cdot a \cdot b}{3}\right) \cdot \sqrt{\frac{a \cdot (a + 2 \cdot b)}{3}} =$$

$$= \frac{Pw}{9 \cdot \sqrt{3}} \cdot \frac{b \cdot (a^2 + 2 \cdot a \cdot b)^{\frac{3}{2}}}{a+b} = \frac{Pw \cdot b}{9 \cdot \sqrt{3}} \cdot \frac{(l^2 - b^2)^{\frac{3}{2}}}{l} \tag{32}$$

Experimental studies of the resistance to cupping of 100 pine boards confirmed the above pattern. The difference between the results obtained by theoretical and experimental methods did not exceed 5 %.

3. Results and Discussion

To study the change patterns in the warping force which causes the warping deformation of lumber depending on its size, moisture content (initial and final), and location in the cross-section of the log, appropriate simulation studies were conducted using the obtained mathematical dependencies.

The subject of the study is the change in shape and the magnitude of the deformation of lumber in the cross-section. The primary cause of deformations is internal stresses that appear in the wood when the content of hygroscopic moisture changes. The characteristics of wood shrinkage (swelling) are the corresponding shrinkage factors, which characterize the change in the geometric dimensions of lumber. Therefore, we will consider two shrinkage factors: in the radial direction – S_{Fr} and the tangential direction – S_{Ft}. As an example, we present the results of calculating the warping force that causes a change in the shape of pine lumber, which occurred in variable temperature and moisture fields with a change (decrease) in moisture content by 22%, in particular the surface of the upper and lower faces (Figure 2). In the calculations, it was assumed that: the cross-sectional dimension of the lumber is 20×70mm, the lumber is cut asymmetrically relative to the vertical Y axis (Figure 4), the shrinkage (swelling) factors are taken as follows: S_{Fr} = 0.17, S_{Ft} = 0.28.

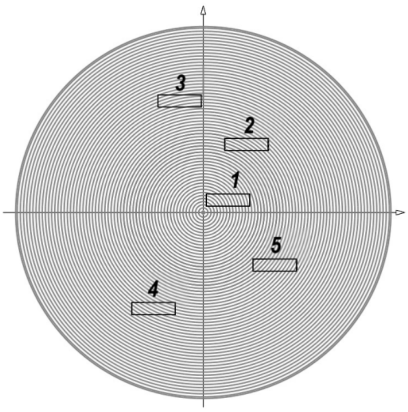


Fig. 4. *The location of the lumber in the log*

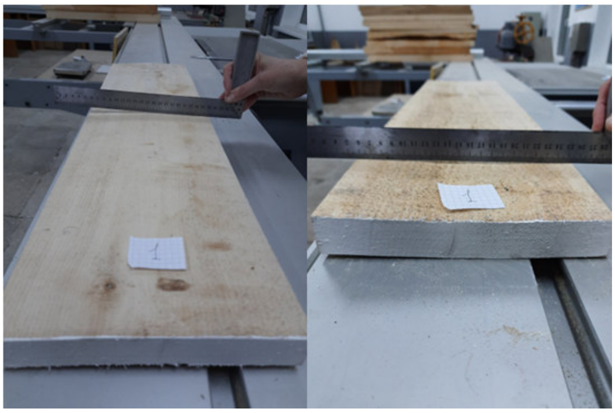
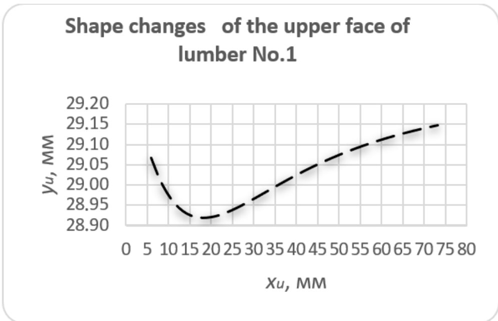


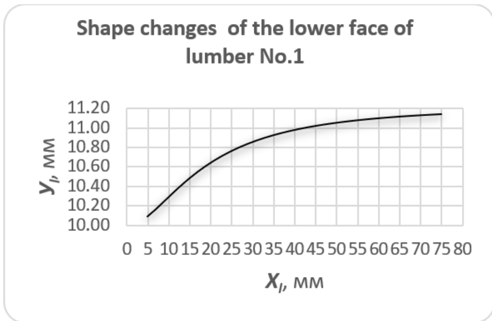
Fig. 5. *General appearance of the sawn lumber*

Fig. 6. *Predicted appearance of the panel glued sequentiallyedge-to-edge from pieces of lumber No_s 1-5*

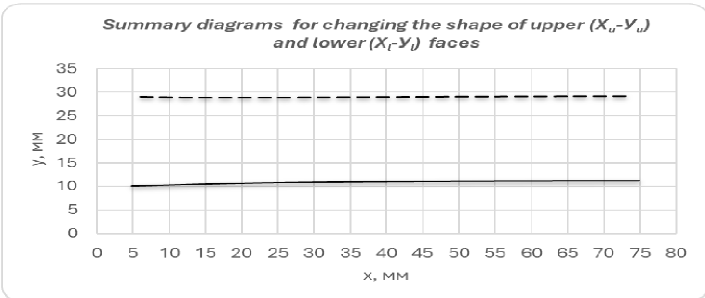
The places in the log from where five pieces of lumber were sawn out, and which compose the laminated panel under study, are shown in Figure 4. The general appearance of the sawn lumber is shown in Figure 5, and the predicted appearance of the panel glued from the five pieces of lumber is shown Figure 6.



a.



b.



c.

Fig. 7. *Shape changes of the faces of piece of lumber No. 1: a. upper face; b. lower face; c. summary diagram for changing the shape of the faces of lumber No. 1*

By studying the anisotropy of the deformability of the wood across the fibers, it is possible to determine the dimensional parameters of the lumber for which, other conditions being equal, the shrinkage (swelling) stresses will be minimal. In this way, the quality of the lumber can be planned already at the stage of drawing up the scheme for cutting logs into the lumber, and by the deformability of the individual components of the LPC, the quality of the LPC can be predicted even before gluing, taking into account the textural features of the future LPC.

The study of the shape changes of lumber indicates that the shape of the surfaces of the lumber mainly depends on its size, moisture content (initial and final) and location in the cross-section of the log. The maximum reduction in the thickness of the lumber is 1.997 mm, which is a significant value and requires consideration when using lumber in the further manufacture of products.

Depending on the location in the sawing scheme, lumber has different resistance to cupping. In fact, this is an external manifestation of the action of internal stresses that arise in lumber when moisture content changes, which is described by the warping force parameter. As an example, the distribution of the warping force along the length of lumber No. 1 is shown in Figure 8, the shape changing of the faces of lumber No_s 1-5, sequentially joined into a panel – in Figure 9, and a summary diagram of the maximum values of the shape changing in the faces of the lumber and the warping force of lumber No_s 1-5, sequentially joined into a panel, is shown in Figure 10.

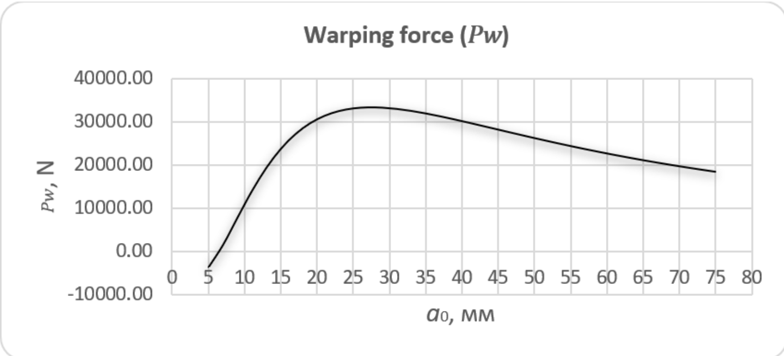


Fig. 8. *Distribution of the warping force along the length of lumber No. 1*

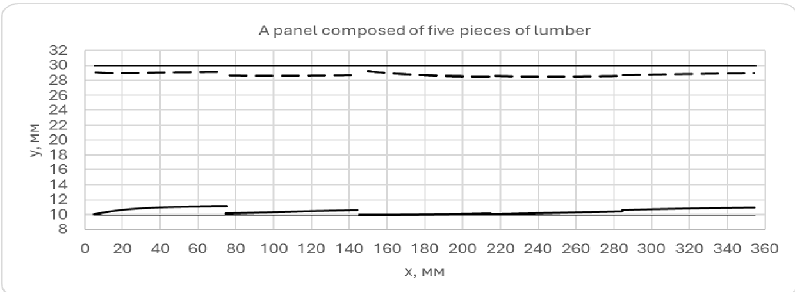


Fig. 9. *Shape change of the faces of lumber No_s 1-5, joined sequentially edge-to-edge into a panel*

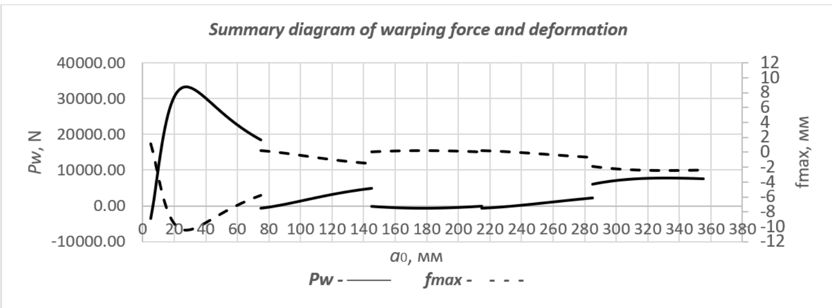


Fig. 10. *Summary diagram of shape change in the faces of the lumber and the warping force of lumber No_s 1-5, joined sequentially edge-to-edge into a panel*

The results of the obtained calculations (Figure 10) indicate a number of change patterns in the warping force, which causes the warping deformation of the lumber. The warping force itself varies depending on the location of the lumber in the cross- section of the log and is proportional to the difference in the shrinkage factors of the lumber faces. This force acquires a different value for each piece of the lumber depending on their location in the sawing scheme, and therefore the lumber has different resistance to cupping.

It should be noted that the nature and magnitude of the shape change of the lumber is consistent with the results of Mitchell [13], in particular, if we assume that $b_1 = -R_1$, $b_2 = R$, we obtain the result presented in Mitchell [13].

The obtained results indicate that a change in the moisture content of lumber affects its shape change. In particular, the maximum deviation of the upper and lower faces compared to the initial parameters (before shrinkage) is about 1.081 mm for the upper face and 0.851 mm for the lower face, which in total is 1.997 mm. According to the calculations of Mitchell [13], for similar lumber, the maximum deflection is 1.866 mm. Thus, the nature and magnitude of the shape change of the lumber we studied is consistent with the research results by Mitchell [13].

4. Conclusions

A mathematical apparatus has been developed for the study of the shape change of lumber and to determine the cupping force. This force depends on the size of the lumber, its moisture content (initial and final), the location in the cross-section of the log, and it is proportional to the difference in the shrinkage factors of the lumber faces. The magnitude of the warping force of pine lumber varies from -3500 Nto 33400 N. This mathematical apparatus makes it possible to implement consistent engineering calculations of the warping force of lumber, taking into account its operation in variable temperature and moisture fields.

The results of simulation modeling of the shape change of lumber indicate that within the cross-section of a log there are lumbers with different resistance to cupping, and therefore such lumber can be sorted into those suitable or unsuitable for use in LPC.

The developed model will make it possible to implement in practice design and technological solutions for the manufacturing of LPC in compliance with the requirements for their strength and shape stability, and will help to improve product quality and the rational use of wood.

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