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IMPACT MODELLING OF THE COEFFICIENT OF RESTITUTION OF POTATOES BASED ON THE KELVIN-VOIGHT PAIR

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Abstract: *This paper aims at presenting a model to study the impact between a potato tuber and contact surfaces, for the purpose of facilitating the projection of devices separating hard objects from agricultural products. The impact modelling of the coefficient of restitution is important because it is a measure of the energy value absorbed by the viscoelastic object, and the absorbed energy is correlated with the mechanical damages incurred by the tubers.*

Key words: *coefficient of restitution, energy, impact, potato.*

1. Introduction

During the process of handling, starting with taking the tubers out of the soil and until their use, tubers are subject to certain mechanical shocks resulting from their contact with the surfaces of organs and constituents of the equipment used. Impact or collision are dynamic stresses, characterized by high speed stress applied in a very short time (regularly, the impact lasts for several milliseconds). During the short impact duration, stress and deformation waves are formed, acting on the tuber structure. These waves fade as the distance from the place of impact increases.

Potato tubers are made of viscoelastic materials and, for this reason, the impact caused on them is also a complicated phenomenon, which involves both the sample and material reaction, as well as the formation (initiation) and propagation of damages [1]. To understand the causes and to prevent damages, the dynamic reaction of the sample and the influence of various parameters on the reaction should be determined. For most cases of low speed impact, the damages brought are small and their surface, located around the point of

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impact, is insignificant, so that damages do not affect the dynamic properties of samples. Therefore, generally, for dynamic impact analysis, damages developed during the impact are not considered. The analytical treatment of impact causes substantial difficulties due to contact (nonlinear) between the sample and the impact object and to coupling the transitional movement of the sample with the forced contact phenomenon.

According to the classical theory of elasticity in the area of small deformations, the relations between tensions and deformations shall be considered linear, and are time independent [2]. Experimental studies have shown that most of the real objects which, in theory, are considered elastic in order to apply the elasticity theory only have stress areas, sometimes quite short, where elastic properties are more striking [3]. The viscoelastic behaviour is achieved by adding the time factor, which represents the real properties of objects more accurately. This differentiation between the two models: elastic and viscoelastic is done precisely by the method of writing the relations between stress and deformation.

The viscoelastic behaviour of fruits and vegetables to mechanical stress means that the force, distance and time, as length and duration of the stress, shall determine the value of measurements.

In the present paper the theoretical research of the tuber research process under dynamic stresses occurring under exterior forces is done based on the study of the tuber-impact surface impact dynamics, using dynamic models equivalent to real physical systems, actually applied for the working conditions. Also the mathematical models set by dynamic modelling of the real mechanical system simulate the behaviour of the real system, on a computer, for impact modelling the coefficient of restitution using a Kelvin-Voight model (referred to as impact pair).

The use of the mathematical model allows simulating the effect of changing various contact parameters on the force-time and force-displacement variation curve values, of the various values for the coefficients of restitution and for the Poisson coefficients of the impact object material. The effect of changing one of the impact parameters can be noted on the global reaction of the sample (tuber) subject to impact.

The dynamic stresses of impact-type tubers are characterized by a high stress application speed, in a very short interval, (regularly, the impact lasts for several milliseconds). During the short impact duration, stress and deformation waves are formed, acting on the tuber structure. These waves fade as the distance from the place of impact increases. Given how dynamic stresses manifest, they can be considered collisions [5].

2. Impact modelling of the coefficient of restitution

The separation of potatoes from other objects, after harvesting, is necessary for the removal of foreign objects or for product sorting. Removal of foreign objects is initially done using a simple grill, after which, subsequent separations are based on other physical properties. One of these properties is the coefficient of restitution K, which can be used as a basis for sorting. A first step in modelling the coefficient of restitution consists in determining the speeds before and after the collision. For low up to moderate impact speeds, the collision between the objects is elastic-plastic, thus causing permanent imperceptible penetrations on the contact surface, in case of hard objects. The force - penetration relation is irreversible from the moment when the work given by the contact force during compression is partially consumed by elastic waves, plastic deformations in the contact area and friction. These modes of initial kinetic energy dispersion during collision depend on the relative speed of the bodies at the point of impact, the material properties and inertial properties (which depend on impact configuration and on the support conditions of the deformable sample).

The coefficient of restitution for the one-dimensional impact between the two bodies, in pure translation movement, is defined as the relative velocity ratio (with changed sign) between the two bodies, at the beginning and end of collision. This is a measure of impact energy lost due to internal sources: elastic waves, plastic deformations and frictions in the contact area [4]. Considering the case of transversal impact (frictionless), perfectly elastic, the coefficient of restitution will be in this case heat dissipation in the contact area. For a limited range of initial impact speeds Vi, the coefficient of restitution can be written as:

$$K = 1 - \alpha V_i \tag{1}$$

where α is a material coefficient ranging from 0.08 to 0.32 s/m for steel, bronze and ivory. The linear equation (1) cannot be applied over certain limits of the impact initiated speed V_i, which depends on the material and on the geometry of surfaces found in contact.

To consider the irreversibly lost energy upon impact, Hunt and Crossley [x] modelled the coefficient of restitution with a Kelvin-Voight model (called an impact pair Figure 1) where the viscous damping force *Fd* is given by the relation:

$$F_d = \lambda_1 \delta^{\frac{3}{2}} \dot{\delta} \tag{2}$$

where:

$$\lambda_1 = \frac{3}{2} \alpha \kappa_c \tag{3}$$

δ is the deformation in the contact area;
k_c – contact stiffness.

The α parameter can be determined experimentally by measuring the coefficient of restitution for the impact on the sample supported on a rigid support across the entire surface. In this case, the contact law shall be given by the equation:

$$F(t) = \kappa_c \delta^{\frac{3}{2}} + \frac{3}{2} \alpha \kappa_c \delta^{\frac{3}{2}} \dot{\delta} \tag{4}$$

By replacing *F(t)* with $M_1 \ddot{\delta}$ in the relation (4) we obtain the equation of motion:

$$M_1 \ddot{\delta} + \frac{3}{2} \alpha \kappa_c \delta^{\frac{3}{2}} \dot{\delta} + \kappa_c \delta^{\frac{3}{2}} = 0 \tag{5}$$

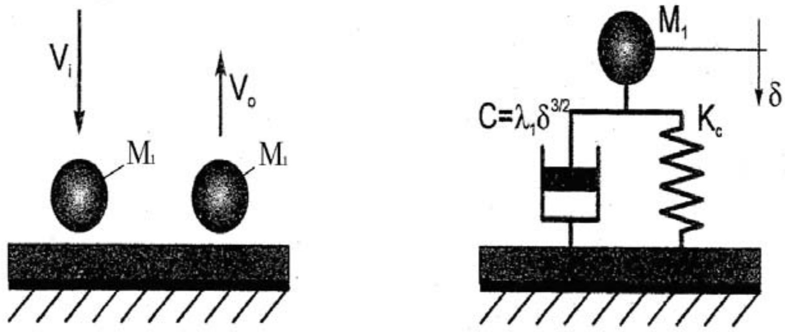

Fig. 1. *The impact pair model [4]:*
a – impact model; b – elastic equivalent model

The equation (5) was numerically solved using the Runge-Kutta with the MATLAB software. To validate the software, two impact examples done by Hunt and Crossley were analysed. They studied the case of the impact of a spherical steel object, with a 25.4 mm radius and a mass m = 0.453 kg, on a rigid steel plate with initial speeds of 0.203 m/s and 0.406 m/s. The contact coefficient of rigidity was $\kappa_c = 3.4 \cdot 10^{10}\ \text{N}/\text{m}^{3/2}$. The results obtained match the ones given by Hunt and Crossley (Figure 2).

The model developed by Hunt and Crossley can be used to simulate the impact energy loss phenomenon in the case of the impact between a rigid impact object with an elastic semi-space (represented by the tuber). For this reason, the model can be adapted to simulate the impact behaviour of the potato tubers which are viscoelastic materials and, in low impact speeds, the contact surface acts as an elastic semi-space. Figure 2 presents the force variation upon impact.

Additionally, Figure 3 presented below illustrated force variation upon impact (total and elastic component) in time. The impact behaviour of potato tubers was analysed for the case of impact between a potato tuber and an impact object using the equation of motion (5) numerically solved with the prior-referred to software, namely Matlab software. The contact stiffness coefficient *k* was calculated using the formula:

$$\kappa_c = \frac{4}{3} E R^{1/2} \tag{6}$$

resulting values comprised within the limits: k_c = 0.66·10^6 ... 1.20·10^6 N/m$^{3/2}$. The formula (1) was used to calculate α, the coefficient of restitution K being experimentally determined by measuring the initial and final impact speed for potato samples with different contact radii. The parameter R necessary to determine the contact stiffness coefficient was calculated using the formula (7) considering the impact object radius R2 = ∞, and the tested tuber contact radii comprised within the limits R1 = 0.009 m...0.05 m.

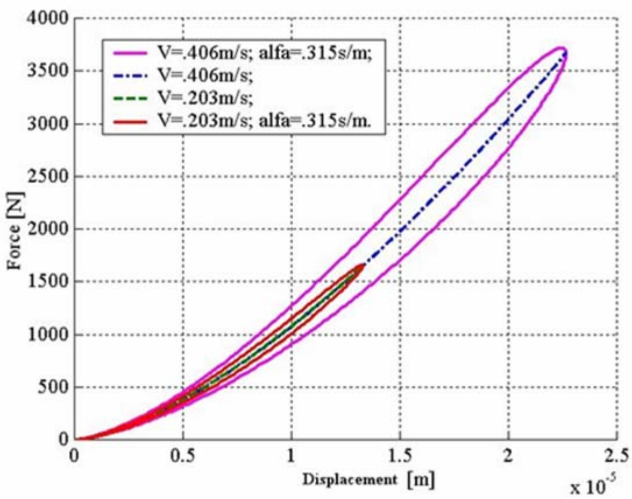

Fig. 2. *Force variation (elastic and total component) upon impact depending on penetration in the contact area (Hunt and Crossley) [4]*

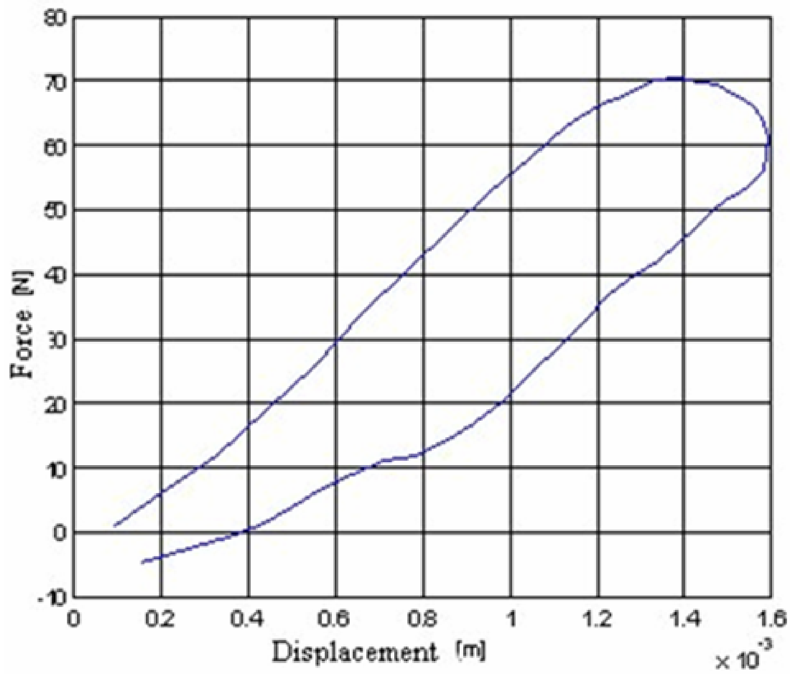

Fig. 3. *Theoretical curves of the force-deformation variation in the impact area for potato tubers*

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{7}$$

Another parameter necessary to obtain the contact stiffness, marked with E, was determined using the relation (8). Using the Poisson coefficient of the tuber, the value stipulated in the specialised literature ν_1 = 0.49 and for the impact object material ν_2 = 0.3 [5, 7].

$$\frac{1}{E} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \tag{8}$$

All parameters characterising the impact between tubers and the flat surface were inserted in the equation of motion (5) and by running the application developed in Matlab, resulting the variation curves of the contact force F depending on the deformation δ (presented in Figure 3).

3. Conclusions

To assess dynamic response of samples and the influence of various parameters on the response, the displacement of the impact object and local deformations in the contact area were considered. The mathematical model developed to study the impact of the energy conservation law is based on the equivalent elastic and dynamic models of mass-spring-type. Choosing a particular method for modelling should be based on careful consideration of the effects of various factors occurring during impact.

The measure of tuber absorbed energy upon impact is characterized by the coefficient of restitution upon impact and, therefore, shaping this coefficient is important to correlate with the mechanical damages suffered by potato tubers. The higher the absorbed energy upon impact, the higher the likelihood of mechanical damages by impact becomes.

The modelling (irreversibly) of lost energy upon contact in the tuber was done by using a Kelvin-Voight model (referred to as impact pair), for which the equations of motion can be solved using the software MATLAB with the Runge Kutta method.

Using the mathematical model presented in paragraph above, the influence of various contact parameters can be simulated on the force-time and force-displacement variation curves. To solve the equation in MATLAB, different contact radii as tuber coefficients of restitution can be introduced, as well as different Poisson coefficients for the impact object material. The analysis of resulting curves after running the program can show to what extent changing one of the parameters affects the overall response of the sample to impact.

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