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MEASUREMENTS OF WAVE VELOCITY IN WOOD SAMPLES USING THE INTRINSIC TRANSFER MATRIX

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Abstract: *This study presents experimental data of wave velocity measurements in wood samples, obtained by using the intrinsic transfer matrix method. The method is based on the properties of the behavior of the eigenvalues of the transfer matrix in resonance cases, and this means the method is a particular modal approach of a resonance method. To find the wave velocity in a sample, respective sample is builtin an embedded system containing gauge material and the sample of interest. A numerical analysis applied to the analytical expression of the eigenvalue permits the wave velocity estimation.*

Key words: *transfer matrix, eigenmodes, elastic wave velocity in wood.*

1. Introduction

This work approaches the determination of the phase velocity in solid elastic materials starting from a method used in computer simulations of the wave propagation, especially inhomogeneous media and multilayered media [3], [6], [10], [12], [13]. For semi-infinite and continuous inhomogeneous media, the most convenient methods are the finite difference method, the finite element method and the method of series expansion of the solution and of the function that describes the inhomogeneity [1], [5], [9].

The matrix method can be used for semi-infinite or finite media, in particular for multilayered media, provided that we know the kind of interface between the layers, which is required to writing the necessary boundary conditions. In that case, we can obtain the solution to problems of elastic wave propagation in multilayered media, in which each layer is homogeneous, by solving a product of matrices [8], [11].

2. The Intrinsic Transfer Matrix

If we consider a simple solid homogeneous elastic rod with the length l, much larger than its diameter, and the characteristic impedance $Z = \rho c$, placed between two semi-infinite media with the characteristic impedances Z_{in} and Z_{out}, which propagates a longitudinal plan wave, the transfer matrix (TM) which connects the amplitudes of the Fourier components of the incident and the reflected wave

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displacements in the sample, $A(\omega)$ and $B(\omega)$, has the expression[4]:

$$\begin{pmatrix} A_{out}(\omega,l) \\ B_{out}(\omega,l)\end{pmatrix} = \frac{1}{4}\begin{pmatrix}1+\frac{Z}{Z_{out}} & 1-\frac{Z}{Z_{out}} \\ 1-\frac{Z}{Z_{out}} & 1+\frac{Z}{Z_{out}}\end{pmatrix}\begin{pmatrix} e^{-\beta l} e^{i\frac{\omega}{c}l} & 0 \\ 0 & e^{\beta l}e^{-i\frac{\omega}{c}l}\end{pmatrix}\begin{pmatrix}1+\frac{Z_{in}}{Z} & 1-\frac{Z_{in}}{Z}\\ 1-\frac{Z_{in}}{Z} & 1+\frac{Z_{in}}{Z}\end{pmatrix}\begin{pmatrix}A_{in}(\omega,0)\\B_{in}(\omega,0)\end{pmatrix} \quad (1)$$

where:

β is the attenuation factor related to amplitude;

ω – the angular frequency;

c – is the speed of the wave.

In the stationary case, when the wave is confined inside the sample, only the intrinsic part of the TM is involved, i.e.

$$TM(\beta,\omega) = \begin{pmatrix} e^{-\beta l} e^{i\frac{\omega}{c}l} & 0 \\ 0 & e^{\beta l} e^{-i\frac{\omega}{c}l}\end{pmatrix} \quad (2)$$

The intrinsic part of the TM has the eigenvalues given by Eq. (3):

$$\lambda_{1,2} = \left[\cos\left(\frac{\omega l}{c}\right)\cdot\cosh(\beta l) - i\cdot\sin\left(\frac{\omega l}{c}\right)\cdot\sinh(\beta l)\right] \pm \sqrt{\left[\cos\left(\frac{\omega l}{c}\right)\cdot\cosh(\beta l) - i\cdot\sin\left(\frac{\omega l}{c}\right)\cdot\sinh(\beta l)\right]^2 - 1} \quad (3)$$

The eigenmodes of the rod correspond to real values of the eigenvalues, and satisfy the condition (4) and, implicitly, (5).

$$\sin\left(\frac{\omega l}{c}\right) = 0, \quad (4)$$

and,

$$l = \frac{n\lambda_{wave}}{2}, \quad n = 1,2,... \quad (5)$$

3. The Transfer Matrix of a Ternary System

Consider a ternary system which contains three layers having the thicknesses l_1, l_2, l_3, the characteristic impedances Z_1, Z_2 and Z_3, which propagates longitudinally waves with the wavenumbers k_1, k_2, k_3, placed between two semi-infinite elastic media, with the characteristic impedances Z_{in} and Z_{out} (Figure 1).

Fig. 1. *A ternary system*

The spectral amplitudes of the waves at input and at output are connected by a TM given below:

$$\begin{pmatrix} A_{out}(\omega,l) \\ B_{out}(\omega,l) \end{pmatrix} =$$

$$\frac{1}{16}\begin{pmatrix} 1+\frac{Z}{Z_{out}} & 1-\frac{Z}{Z_{out}} \\ 1-\frac{Z}{Z_{out}} & 1+\frac{Z}{Z_{out}} \end{pmatrix}\begin{pmatrix} e^{ik_3l_3} & 0 \\ 0 & e^{-ik_3l_3} \end{pmatrix}\cdot\begin{pmatrix} 1+\frac{Z_2}{Z_3} & 1-\frac{Z_2}{Z_3} \\ 1-\frac{Z_2}{Z_3} & 1+\frac{Z_2}{Z_3} \end{pmatrix}\cdot\begin{pmatrix} e^{ik_2l_2} & 0 \\ 0 & e^{-ik_2l_2} \end{pmatrix}\cdot \quad (6)$$

$$\cdot\begin{pmatrix} 1+\frac{Z_1}{Z_2} & 1-\frac{Z_1}{Z_2} \\ 1-\frac{Z_1}{Z_2} & 1+\frac{Z_1}{Z_2} \end{pmatrix}\cdot\begin{pmatrix} e^{ik_1l_1} & 0 \\ 0 & e^{-ik_1l_1} \end{pmatrix}\begin{pmatrix} 1+\frac{Z_{in}}{Z} & 1-\frac{Z_{in}}{Z} \\ 1-\frac{Z_{in}}{Z} & 1+\frac{Z_{in}}{Z} \end{pmatrix}\begin{pmatrix} A_{in}(\omega,0) \\ B_{in}(\omega,0) \end{pmatrix}$$

The intrinsic part of the TM, taking into consideration the attenuation is:

$$TM(\omega)=\frac{1}{4}\cdot\begin{pmatrix} e^{i\frac{\omega}{c_3}l_3-\beta_3l_3} & 0 \\ 0 & e^{-i\frac{\omega}{c_3}l_3+\beta_3l_3} \end{pmatrix}\cdot\begin{pmatrix} 1+\frac{Z_2}{Z_3} & 1-\frac{Z_2}{Z_3} \\ 1-\frac{Z_2}{Z_3} & 1+\frac{Z_2}{Z_3} \end{pmatrix}\cdot\begin{pmatrix} e^{i\frac{\omega}{c_2}l_2-\beta_2l_2} & 0 \\ 0 & e^{-i\frac{\omega}{c_2}l_2+\beta_2l_2} \end{pmatrix}\cdot$$

$$\cdot\begin{pmatrix} 1+\frac{Z_1}{Z_2} & 1-\frac{Z_1}{Z_2} \\ 1-\frac{Z_1}{Z_2} & 1+\frac{Z_1}{Z_2} \end{pmatrix}\cdot\begin{pmatrix} e^{i\frac{\omega}{c_1}l_1-\beta_1l_1} & 0 \\ 0 & e^{-i\frac{\omega}{c_1}l_1+\beta_1l_1} \end{pmatrix} \quad (7)$$

The eigenvalues of $TM(\omega)$ for a configuration with identical materials of the layers 1 and 3 are:

$$\lambda_{1,2}(\omega)=$$

$$\left[\left(\frac{Z_1+Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\cos\left(\sum_{m=1}^{3}\frac{\omega l_m}{c_m}\right)\cdot\cosh\left(\sum_{m=1}^{3}\beta_m l_m\right)-i\cdot\sin\left(\sum_{m=1}^{3}\frac{\omega l_m}{c_m}\right)\cdot\sinh\left(\sum_{m=1}^{3}\beta_m l_m\right)\right]\right]-$$

$$\left[\left(\frac{Z_1-Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\begin{matrix}\cos\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\frac{\omega l_m}{c_m}\right)osh\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\beta_m l_m\right)- \\ i\cdot\sin\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\frac{\omega l_m}{c_m}\right)\sinh\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\beta_m l_m\right)\end{matrix}\right]\right]\pm\sqrt{F(\omega)} \quad (8)$$

where:

$$F(\omega)=\left\{\left[\left(\frac{Z_1+Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\begin{array}{l}\cos\left(\sum_{m=1}^{3}\frac{\omega l_m}{c_m}\right)\cdot\\ \cosh\left(\sum_{m=1}^{3}\beta_m l_m\right)-\\ i\cdot\sin\left(\sum_{m=1}^{3}\frac{\omega l_m}{c_m}\right)\cdot\\ \sinh\left(\sum_{m=1}^{3}\beta_m l_m\right)\end{array}\right]\right]-\left[\left(\frac{Z_1-Z_2}{2\sqrt{Z_1Z_2}}\right)^2\left[\begin{array}{l}\cos\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\frac{\omega l_m}{c_m}\right)\cdot\\ \cosh\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\beta_m l_m\right)-\\ i\cdot\sin\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\frac{\omega l_m}{c_m}\right)\cdot\\ \sinh\left(\sum_{m=1}^{3}(-1)^{m+1}\cdot\beta_m l_m\right)\end{array}\right]\right]\right\}^{2}-1 \quad (9)$$

A modal analysis combined with a numerical method able to study the behavior of the eigenvalues can be proposed to find the elastic constants of solid materials. Such a method suggests the possibility to study by modal analysis the simple embedded systems, containing gauge materials and the sample of interest.

The simple systems are characterized in the case of 1D propagation by a simple distribution of the eigenmodes and also simple and convenient analytical expression of the TM. Examples of such simple systems are binary or ternary built-in systems, which contain gauge materials and materials for investigation.

4. Application of the Method for Some Wood Species

A suitable application of the method is to characterize the elastic properties of wood samples [7]. Because the elastic properties of wood (elastic module, mass density, and Poisson's ratios) are random variables that vary significantly for the same wood species [14], the intrinsic transfer method offers a fast and convenient method to characterize such materials. Instead of huge poles with the ends connected to emitters and receivers [2], the samples used in the transfer matrix method are much smaller. The wood samples consisting of small cylinders were built-in ternary systems brass-wood-brass, taking the brass as gauge material. Moreover, the small cylinders can be cut so as to comply the cylindrical geometry used in the characterization of the orthotropic behavior of wood (Figure 2).

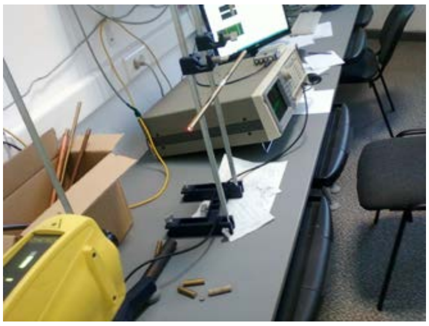

Fig. 2. *Experimental setup*

Tables 1 and 2 express the configuration of the experimental setup, the sample sizes and the obtained values of sound velocity along the fiber c_l and perpendicular to it c_r. ν represents the value of the frequency of the eigenmodes obtained by Fourier analysis which was taken into consideration for the numerical analysis.

Table 1

Elastic wave velocity along the fiber, estimated by the intrinsic transfer method

No.	Species of wood	l_2 [mm]	Diameter [mm]	ρ [Kg/m^3]	l_1 (brass) [mm]	l_3 (brass) [mm]	ν [Hz]	c_l along [m/s]
1	Fir tree	29.19	10	473.14	160.2	93.57	3300.78	4563
2	Oak	29.98	9.95	724.85	199	129.16	3164	4161
3	Beech	29.90	9.98	703.77	123.4	123.4	4355	4905
4	Spruce	29.45	9.47	450.73	160	93.39	3808	5437
5	Ash	29.5	9.52	801.11	198.4	129.15	3398.44	4253

Table 2

Elastic wave velocity transversal to the fiber,
estimated by the method of the intrinsic transfer matrix

No.	Species of wood	l_2 [mm]	Diameter [mm]	ρ [Kg/m^3]	l_1 [mm]	l_3 [mm]	ν [Hz]	c_r Radial [m/s]
1	Fir tree	31.37	10.02	473.14	160.2	93.57	1269	1512
2	Oak	31.07	10.0	724.85	199	129.16	1562.5	1707
3	Beech	31.15	10.1	703.77	123.4	123.4	1738.28	1692
4	Spruce	16.03	9.8	450.73	159.64	92.72	1171.88	1019
5	Ash	22.14	9.93	801.11	198.07	129.42	1972.66	1850

5. Conclusions

This paper proposes a resonance method based on the properties of the eigenvalues of the wave transfer matrix combined with a numerical method, in order to find the velocity of elastic waves in solid elastic samples. The study also considers the attenuation and shows that the attenuation affects the frequency of the eigenmodes, for an embedded system. The ternary system is preferred because such a system preserves the longitudinally plan wave much better, a special case for which the transfer matrix has a simple mathematical form. It is difficult to introduce dispersion, and according to experiments, even the attenuation factor has a dispersive behavior.

A theoretical and experimental analysis of the accuracy, involving the study of the influence of the geometrical setup, of attenuation and the influence of the gauge properties reveals that good estimations of wave velocity can be obtained by a precise Fourier analysis. Usually, the experimental data are obtained by data acquisition, hence such a technique is recommended to obtain a fine frequency resolution, that will lead to accuracy in estimation of the phase velocity of wave.

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