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CALCULATION OF OPTIMAL LENGTH FOR SKYLINE

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Abstract: *As important for the design of cable logging systems both theoretically and practically, the optimization problem regarding the optimal length of the skyline was determined. In the process of modelling the shape of the sagging curve of the holding cable, the objective function is constructed in the form of a transcendental equation, whose roots are determined by iterative and graphical-analytical methods. A numerical analysis of the problem is carried out. To calculate the optimal length of the skyline, a software algorithm, an engineering formula, and nomograms are proposed.*

Key words: *single span skyline, minimal cable tension, optimal length, catenary, deflection.*

1. Introduction

Technical means of skyline system are widely used in many branches of engineering [1], [15, 16], in particular in the logging and timber industries. At present, in the process of forestry, especially in difficult terrain, soil, and hydrological conditions, cable logging systems and snubbing have been widely used [2], [7], [11], [13, 14]. Their advantages are economy, environmental friendliness, and technological compatibility. However, there are a number of shortcomings that hinder the widespread introduction of such systems,

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including the high cost of technological equipment, the complexity of its installation and maintenance, and the fixed cost associated with its installation.

The main element that determines the performance of skyline installations is the cable rigging. Cable logging systems for timber transportation are usually made with single-cable or multi-cable single-span with pendulum carriage movements. As a guide for the cargo carriage uses a tension skyline stretched between supports (with relatively rigidly fixed ends). The skyline is the main element of the cable logging systems, the operating conditions of which largely determine the nature of the operation of the machine as a whole.

2. Objectives

The results of the calculation of the skylines are given in many works [3], [5, 6], [9], [11]. However, it should be noted that some results were obtained for systems with small arrows of sagging (deflection) of the skyline ($f_{max} \leq 1/20\ell$) [8], [12], and the study mainly concerned specific tasks of designing cable rigging [8], [10], [13, 14].

At present, there are tendencies in forest engineering practice to access such forested areas, where it is necessary to use skidding units, in which skyline deflection f_{max} is greater than 5% of the span length ℓ [9], [12]. Since such systems violate the fundamental assumptions of the existing methods for calculating skylines with small deflection arrows (e.g., uniform load distribution along the span; modelling the sagging curve of the holding cable with a parabola, etc.), there is a need to refine and improve them. The first step in this direction is to determine the geometric characteristics and strength parameters of fixed anchored skyline at both ends under the assumption that its weight is evenly distributed along the length. In case of the uniform distribution of the vertical load (skyline self-weight) along the length of the skyline and neglection of deformations at the first stage (its longitudinal and transverse stiffness, temperature deformation, pliability of supports, etc.) [3, 4], [7], [12], the shape of the sagging curve is described by a catenary [1], [9], [15] and Equations (1) and (2), for the system of flat rectangular coordinates x_0 – y_0, which starts at point C (Figure 1):

$$y_0 = C_1 \cdot \cosh\left(\frac{x_0}{C_1} - 1\right) \tag{1}$$

Where: C_1 is the catenary parameter [m].

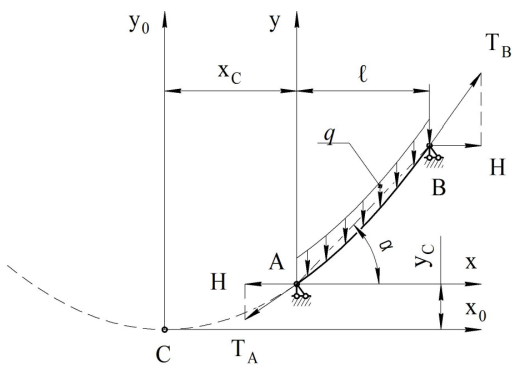


Fig. 1. *Schematic diagram placement of skyline in the systems of flat rectangular coordinates*

$$C_1 = \frac{H}{q} \tag{2}$$

where: H is the projection of skyline tension on the horizontal axis x_0 (x) [N], and q is the vertical load, which is evenly distributed along the length of the skyline (running weight, etc.) [N/m].

It is well known that in this case the tension of the skyline at any point is expressed in Equation (3):

$$T_0 = H \cdot \cosh\left(\frac{x_0}{C_1}\right) \tag{3}$$

The algebraic expression (taking into account that x_c could have negative values) to calculate the horizontal projection of the distance from the top of the catenary (point C) to the lower support (point A) has the form

$$x_C = \frac{\ell}{2} - C_1 \cdot \mathrm{Arsinh}\frac{\ell \cdot \tan\alpha}{2 \cdot C_1 \cdot \sinh\left(\frac{\ell}{2 \cdot C_1}\right)} \tag{4}$$

where: ℓ is the span length (horizontal projection) [m] and α is the slope of the span chord to the horizon [degrees].

For the convenience of engineering calculations, we will carry out plane-parallel movement of a system of rectangular coordinates $x_0 - y_0$ with shift of its beginning from point C towards point A (Figure 2).

In this case, Equation (1) is transformed as follows:

$$y = C_1 \cdot \cosh\frac{x - C_2}{C_1} - C_3 \tag{5}$$

Where: C_2 and C_3 are coefficients of the catenary [m], described in Equations (6) and (7).

$$C_2 = x_C \tag{6}$$

$$C_3 = C_1 - y_C \tag{7}$$

where: x_c and y_c are the abscissa and the ordinate of the vertex of the catenary in the coordinate system $x - y$ [m], respectively.

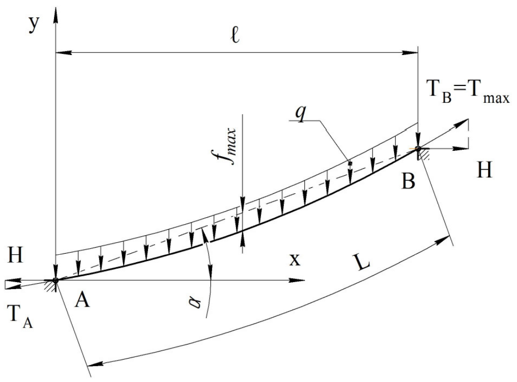


Fig. 2. *Calculation scheme of the fixed skyline in the system of flat rectangular coordinates x – y*

In the shape modelling process of the deflection curve of the skyline, the parameter C_1, and coefficients C_2, C_3 of the catenary are determined, based on the geometric conditions of the fixing ends of the cable (points A and B) and the fact that its length L is a known value.

The length of the skyline [1], [4], [9], [12] is calculated in Equation (8):

$$L = \int_{-x_c}^{\ell - x_c} \sqrt{1 + \left(y'\right)^2}\, dx = \sqrt{4 \cdot C_1^2 \cdot \sinh^2\left(\frac{\ell}{2 \cdot C_1}\right) + (\ell \cdot \tan\alpha)^2} \tag{8}$$

Based on Equation (3), the tension of the skyline at any point, in this case, will be:

$$T = H \cdot \cosh \frac{x - C_2}{C_1} \tag{9}$$

From Equation (9), it follows that when loading the fixed skyline vertically, its maximum tension occurs in the section with the highest ordinate [3, 4], [16], i.e., for the scheme shown in Figure 2, $T_B = T(\ell) = T_{max}$.

After analyzing the general properties of hyperbolic function *cosh(x),* the possible ranges of components, and the structure of Equation (9) to calculate the maximum tension of the cable T_{max} depending on its length *L* and the slope of the chord to the horizon *α*, we can assume that for single-span cable systems with span length ℓ and arbitrary deflection arrows f_{max}, which are loaded vertically evenly with distributed load *q*, there is such a length L_{opt} [15, 16], in which the maximum tension T_{max} takes the lowest value (if $L \rightarrow L_{opt}$, then $T_{max} \rightarrow min$ and $T_{max} = T_{opt}$) (Figure 3).

The goal of this study is to substantiate the optimal length L_{opt} of a non-stretchable asymmetrical skyline with rigidly fixed ends and loaded with a vertically evenly distributed load.

The definition for cable logging systems of this length is an important optimization task of both theoretical and practical purposes. In particular, the solution of this problem allows for the system with specific parameters to set the characteristic intervals, where one value of tension for the skylines corresponds to its two lengths (if $T_{max} > T_{opt}$), one length (if $T_{max} = T_{opt}$) or area where it is impossible to provide such tension in production conditions ($T_{max} < T_{opt}$). This aspect is important to consider when improving theories of calculation of cable logging systems [3], [6], [9], as well as when developing systems of automated engineering design of cable equipment for forestry and timber production [3], [6], [8], [11], [13, 14].

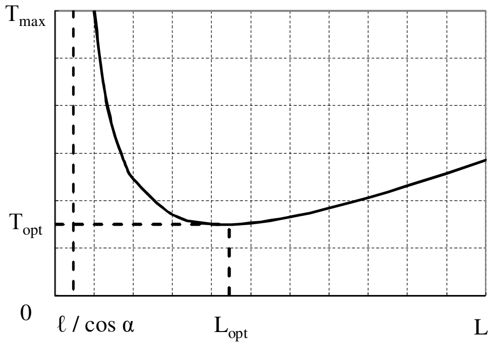


Fig. 3. *Dependence graph $T_{max} = f(L)$ for ℓ = const, α = const and q = const*

3. Material and Methods

To calculate the maximum tension that occurs in the skyline, we substitute in Equation (9) the values $x = \ell$, and Equations (2), (4), (6):

$$T_{max} = C_1 \cdot q \cdot \cosh\left(\frac{\ell}{2 \cdot C_1} + \operatorname{Arsinh} \frac{\ell \cdot \tan\alpha}{2 \cdot C_1 \cdot \sinh\left(\frac{\ell}{2 \cdot C_1}\right)} \right) \tag{10}$$

Let us perform a transformation of the obtained Equation (11):

$$\frac{2\cdot T_{max}}{q\cdot \ell}=\frac{2\cdot C_1}{\ell}\cdot\cosh\left(\frac{\ell}{2\cdot C_1}+\mathrm{Arsinh}\frac{\ell\cdot\tan\alpha}{2\cdot C_1\cdot\sinh\left(\frac{\ell}{2\cdot C_1}\right)}\right) \tag{11}$$

If we enter the notation $X = \ell/(2C_1)$ and $Y = 2T_{max}/\ell$, and take into account the accepted assumption that in operation $q = const$, then in mathematical terms, the optimization problem is to determine an extremum point of the objective function:

$$Y=\frac{1}{X}\cdot\cosh\left(X+\mathrm{Arsinh}\frac{X\cdot\tan\alpha}{\sinh X}\right) \tag{12}$$

To determine the extremum point (taking into account the rules of differentiation of the sum, product and fraction of functions, including hyperbolic and inverse hyperbolic, as well as the theorem on the derivative of a composite function) we differentiate the function Y and equate its derivative to zero. As a result, we obtain the following transcendental equation:

$$Y'=\frac{1}{X}\cdot\sinh\left(X+\mathrm{Arsinh}\frac{X\cdot\tan\alpha}{\sinh X}\right)\cdot\left(1+\frac{1}{\sqrt{1+\frac{X^2\cdot\tan^2\alpha}{\sinh^2 X}}}\cdot\frac{\tan\alpha\cdot\sinh X-X\cdot\tan\alpha\cdot\cosh X}{\sinh^2 X}\right)- \tag{13}$$

$$-\cosh\left(X+Ar\sinh\frac{X\cdot\tan\alpha}{\sinh X}\right)\Big/X^2=0$$

We perform the transformation of the transcendental Equation (13), given that the value X cannot be zero (because when $X = 0$ the optimization problem will not have technical meaning):

$$\mathrm{cotanh}\left(X+\mathrm{Arsinh}\frac{X\cdot\tan\alpha}{\sinh X}\right)=X\cdot\left(1+\frac{\tan\alpha\cdot(\sinh X-X\cdot\cosh X)}{\sinh X\cdot\sqrt{\sinh^2 X+X^2\cdot\tan^2\alpha}}\right) \tag{14}$$

At a fixed value α ($\alpha \neq \pm 90°$ according to design considerations), the solution of the transcendental Equation (14) will be the desired value X, which corresponds to the optimal length of the cable L_{opt}.

Given the value of Equation (8) and the fact that $X = \ell/(2C_1)$, we obtain:

$$L_{opt} = \ell \cdot \sqrt{\left(\frac{\sinh X}{X}\right)^2 + \tan^2 \alpha} \qquad (15)$$

4. Results and Discussions

Since for design reasons $\ell > 0$, as well $C_1 > 0$ as ($H > 0$, $q > 0$), the desired solution of Equation (14) is a positive value of *X*, negative and zero values of *X* will not be considered in the future.

Considering the structure of composite functions and the performance numerical analysis of Equation (14), we can conclude that for different inclination angles of chords to the horizon α in the range of values $\alpha \in$ (-90°; 90°), the function $X = f(\alpha)$ is even i.e., for any numerical value of αi this set of values is equal $f(-\alpha i) = f(\alpha i)$ (Figure 4).

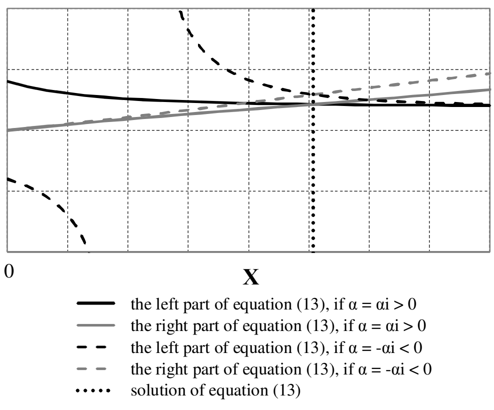


Fig. 4. *Finding the solutions of Equation (14) for cases α = ± αi using the graphical method*

Since the function $X = f(\alpha)$ is even, its graph is symmetric with the vertical *X*-axis. Therefore, for the convenience of presenting the results of the numerical analysis of Equations (14) and (15) in the graphs (Figures 5 and 6), the range of values $\alpha \in$ (-90°; 90°) is presented as a function of the module $|\alpha|$ with only the absolute value of the number.

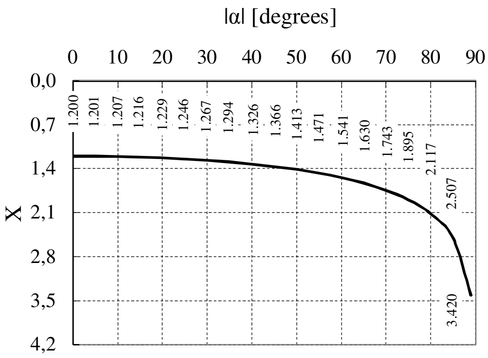


Fig. 5. *Dependency chart* $X = f(|\alpha|)$

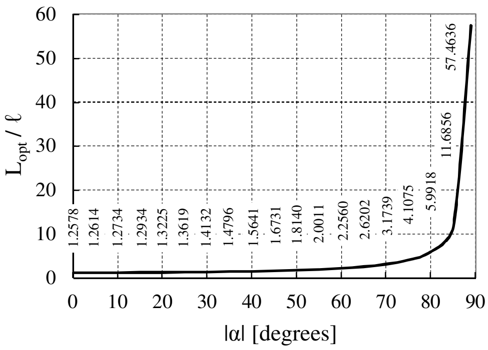


Fig. 6. *Dependency chart* $L_{opt}/\ell = f(|\alpha|)$

Having analyzed the nature of the dependence $L_{opt}/\ell = f(|\alpha|)$ (Figure 6), in order to ensure the possibility of convenient use of the obtained results in engineering practice, we approximate it by a polynomial of the third degree in the interval $|\alpha| \in [0°; 45°]$ (Figure 7).

After evaluating the results of approximation and taking into account the obtained coefficients of the approximating function, an engineering formula is proposed, which with a slight error in the parameter of the length of the skyline (δ_{max} < 0.1% in range of $|\alpha| \in [0°;\ 45°]$ allows to determine the value L_{opt} depending on the characteristics of the cable logging systems.

$$L_{opt} = \left(1.2578 - 0.0055 \cdot \tan|\alpha| + 0.5511 \cdot \tan^2|\alpha| - 0.1304 \cdot \tan^3|\alpha|\right) \cdot \ell \tag{16}$$

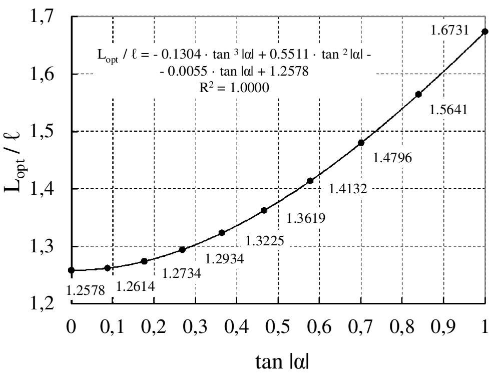


Fig. 7. *Approximation of the dependence $L_{opt} / \ell = f(\tan(|\alpha|))$ with graphical method on the interval $|\alpha|$ = 0-45°: R^2 - characteristic of the reliability of the approximation (square of mixed correlation)*

For symmetric systems in which the ends of the skyline are located at the same level (α = 0°), from the algebraic Equation (16) we can obtain the known results [1, 15, 16], where for such cases it is proposed to determine the value L_{opt} by the formula $L_{opt} = 1.258\,\ell$. This confirms the correctness of the results of the study, and at the same time significantly expands the scope of their application, as symmetrical schemes of cable logging systems in engineering practice, in particular in forest engineering, are used very rarely.

If the slope of the span chord to the horizon of the cable logging system exceeds the modulus 45° (but not more than 90°), to determine the approximate value L_{opt} nomograms can be used (Figure 8), which are constructed as a result of the numerical analysis of Equations (14) and (15). In particular Figure 8a shows the dependence of the optimal length of the cable on the slope L_{opt}, of the slope of the span chord to the horizon α at different values of the length of the span ℓ, and in Figure 8b - dependence L_{opt} on the length of the span ℓ at different values of the slope of the chord of the span to the horizon α.

In forest engineering practice, cable logging systems mostly work with the following installation (preliminary) T_{inst} and operational (working) T_{expl} tensions:

$$T_{inst} \approx (0.4 - 0.8) \cdot T_{expl} \tag{17}$$

$$T_{expl} \le \frac{T_{br}}{n_{sk}} \tag{18}$$

where:

T_{br} is the tensile force (breaking load) of the skyline as a whole, which is determined according to the standard [N];

n_{sk} – the safety factor of the skyline; depending on the type of cable logging systems and in accordance with most professional recommendations (best practices) n_{sk} ≈ 2-3 [10].

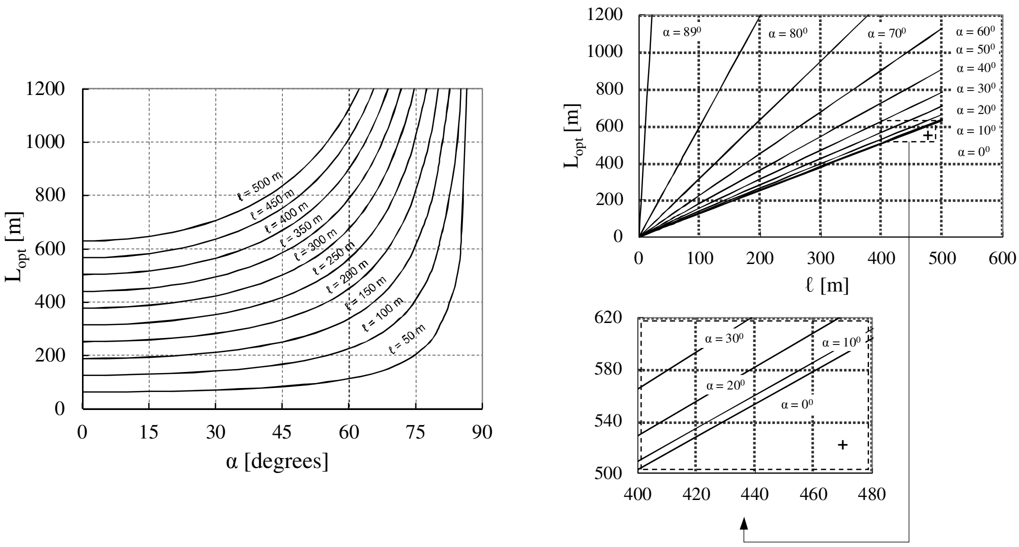
Therefore, the recommended ratio range for breaking and mounting tensions of the skyline is:

$$\frac{T_{br}}{T_{inst}\ (T_{opt})} \geq 2.5-7.5 \qquad (19)$$

On the other hand, in forest engineering practice, cable logging systems mostly work with such sagging arrows f_{max} that do not exceed 1/20 of the span length, sometimes a little more, but generally not more than 1/2 of the span length i.e.,

$$\frac{f_{max}}{\ell} \leq 0.05-0.5 \qquad (20)$$

For purposes of skyline construction in Ukraine, the most widely used cables are GOST 2688-80* with a diameter of d_r = 17-27 mm, the estimated cross-sectional area of wires A_r = 74.4-274.31 mm^2, the weight of a running meter of oiled cable m_r = 0.728-2.685 kg/m, tensile strength of cable T_{br} = 110250-408750 N (similar constructions: DIN 3059 ISO 434; BS 302 6 × 19 (12/6/1) FC; NFA47-200 group 4 class 6 × 19 organic core (12 + 6 + 1); JIS G3525 type 36 × 19).



a) *b*)

Fig. 8. *Nomograms of dependence $L_{opt} = f\,(\ell, \alpha)$*

Given the magnitude of the tensions and especially the maximum skyline deflection, which are mass-produced and widely used in the timber industry (Figures 9 and 10), it can be concluded that they cover the spans of considerable length (ℓ > 500 m) and the use of skyline cables of considerable length (L_{opt} > 1200 m) in industrial practice is not rational.

Based on these principles, it can be concluded that for the purpose of practical application to indicate numerical values on nomograms (Figure 8) for cases L_{opt} > 1200 m, as well as ℓ > 500 m, there is no practical need.

In the case of increased requirements for the accuracy of calculations and the possible application of the results obtained in forestry-related engineering to find the optimal length of the holding element (cable, chain, string, cord, etc.) L_{opt}, including when 45° < $|\alpha|$ < 90°, computational algorithms of many specialized software products can be used.

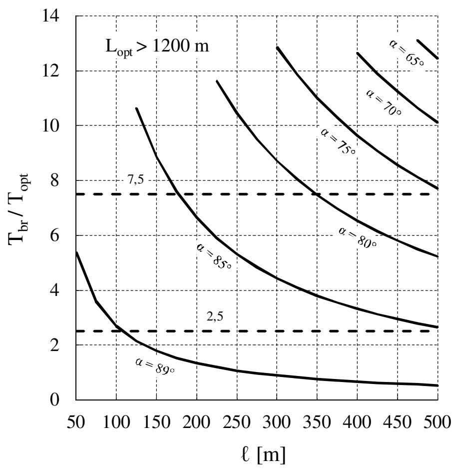


Fig. 9. *Dependency chart* $T_{br} / T_{opt} = f(\ell, \alpha)$ *for a case when* $L_{opt} > 1200$ *m*

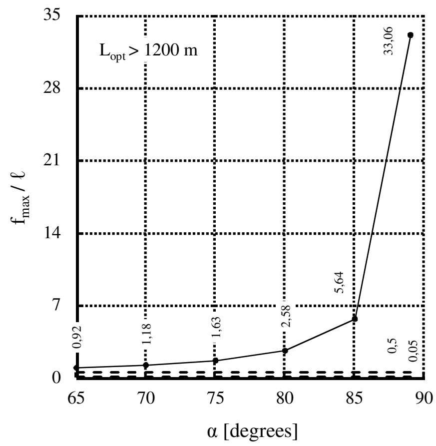


Fig. 10. *Dependency chart* $f_{max} / \ell = f(\alpha)$ *for a case when* ℓ *= 50-500 m and* $L_{opt} > 1200$ *m*

For example, in the computational knowledge engine environment WolframAlpha (wolframalpha.com) the calculation can be performed in the following order:

1. record the input data, for example: ℓ = 300 m, α = -30°;
2. to form Equation (14) in functions understandable to the operating environment, for example:

$$\operatorname{cotanh}\left\{\mathrm{X}+\operatorname{Arsinh}\left(\frac{\mathrm{X}\cdot\tan\left(\operatorname{radians}(-30)\right)}{\sinh(\mathrm{X})}\right)\right\}=$$

$$=\mathrm{X}\cdot\left\{1+\frac{\tan\left(\operatorname{radians}(-30)\right)\cdot\left(\sinh(\mathrm{X})-\mathrm{X}\cdot\cosh(\mathrm{X})\right)}{\sinh(\mathrm{X})\cdot\operatorname{sqrt}\left(\left(\sinh(\mathrm{X})\right)^{2}+\mathrm{X}^{2}\cdot\left(\tan\left(\operatorname{radians}(-30)\right)\right)^{2}\right)}\right\}$$

3. to obtain numerical solutions:
   $X \approx$ -0.980983556134288...
   $X \approx$ 1.26725202408530...,
   and do not consider the negative value of X in the future, because in this case the problem has no technical meaning;
4. to form Equation (15) in functions understandable to the operating environment, for example:

$$\mathrm{L} = 300 \cdot \mathrm{sqrt}\left\{\left[\frac{\sinh(1.26725202408530)}{1.26725202408530}\right]^2 + \left[\tan(\mathrm{radians}(-30))\right]^2\right\}$$

5. obtain the numerical value of the optimal length L_{opt} of the holding element: L ≈ 423.988097136952 m;
6. for these inputs, the result of the numerical analysis in Equation (16) will be:

$$\mathrm{L} = \{1.2578 - 0.0055 \cdot \tan(\mathrm{radians}(\mathrm{ABS}(-30))) +$$
$$+0.5511 \cdot \left[\tan(\mathrm{radians}(\mathrm{ABS}(-30)))\right]^2 - 0.1304 \cdot \left[\tan(\mathrm{radians}(\mathrm{ABS}(-30)))\right]^3\} \times$$
$$\times 300 = 423.968724545604\ldots \text{ m}$$

7. the relative error of calculations on the length of the holding element is equal to δ = -0.0046% and does not exceed the modulus 0.1%.

In the process of developing your own applications for calculating the optimal length of the holding element L_{opt}, it should be borne in mind that Equation type (14) in closed form is not solved by analytical methods. The roots of this equation with a given degree of accuracy can be found using a number of methods based on the known physical content of input parameters and belong mainly to iterative methods (dichotomy, simple iterations, tangent lines, secant lines, chords, etc.) [12] or graphical-analytical methods [16].

5. Conclusions

The proposed method of determining optimal length of the unstretchable skyline, loaded evenly distributed along the vertical load is universal and the use of special software allows to perform automatic calculations with high accuracy.

The results of the numerical analysis of the obtained dependences (14) and (15), as well as the proposed formula (16), show that the value L_{opt} is uniquely determined by the geometric characteristics of the cable systems: span length ℓ and slope of the chord to the horizon α [1], and force parameter q (distributed load) affects only the absolute value of the maximum tension T_{max}.

The article shows how to determine the characteristic intervals of possible skyline tension for specific input data, where one or two cable lengths can correspond to one of its values, or to establish a zone in which it is impossible to provide the required cable tension in production conditions. This aspect, as well as other proposed recommendations, is important to consider in improving calculation theories and the development of automated engineering design tools of cable logging systems.

In addition to the field of technological and transport equipment design for forest harvesting, the results of the study can be used in other related fields of mechanical engineering, which study the operating conditions of flexible retaining elements (e.g., cables, chains, strings, cords, etc.), that are evenly loaded on length of the vertical load (own weight or another load). The proposed approach also allows, if necessary, to take into account such operational factors as elastic and temperature deformation of the holding element, the flexibility of the end supports, and so on.

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