MATHEMATICAL EXPRESSION OF INTERNODE CHARACTERISTICS OF YELLOW AMPEL BAMBOO 
(\textit{Bambusa vulgaris var. striata})

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\textbf{Abstract:} Most bamboo research has focused on growth performance, physico-chemical attributes, and biomass estimation. There are few studies on elaborating the biometric of bamboo, especially in internodes mathematical expression. In this study, we revealed the mathematical expression of bamboo internode along the culm. We used 30 samples of culms and cut each culm by its internodes. A total of 928 sample internodes were measured for the outer diameter, wall thickness, and length of internodes. The findings showed that the mathematical expression for two-dimensional culm morphometric was a two and three-order function. In contrast, the three and four-dimensional culm morphometric were a four-order function. Moreover, the relationship between relative internode number (RIN) and relative internode length (RIL), and relative cumulated internode length (RCIL) showed a third-order function.

\textbf{Key words:} Non-linear regression, biomass, volume, basal area, hollow area.

1. Introduction

Bamboo is an environmentally friendly biomaterial that can be applied in construction, industry, and traditional usage. The potency of bamboo is around 37 million hectares globally, spread out on almost all continents [26]. Ethnobotanically, bamboo has been recognized and utilized by ethnic groups in Indonesia [21], [40], India [22], [31], [34], and Benin [11], for food sources (shoots),

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fuels, construction materials (fences and housing), and traditional usage (handicraft and music instruments).

In terms of climate change mitigation, bamboo should be considered carbon stock and rural livelihood poverty alleviation [10], [20], [27], [30]. Moreover, bamboo has also been recommended for ecological restoration in degraded lands [28], [29], water conservation, soil control [41], and forest healing [8].

Internodes play a vital role in bamboo culm growth. Bamboo culm morphology is unique because multiple hollow internodes along the bamboo culm are separated by nodes [3]. Generally, most bamboo species can find cylindrical hollows [24]. The cell orientation in the internodes is axial and formed by parenchyma and vascular bundles [23]. In the early growth phase, a full-height 3-30 m is reached in a few months, and the culm elongation results from the expansion of the internodes along the culm [25]. Furthermore, bamboo culm growth will stop and enter the maturation period with minor anatomical changes.

The morphology of bamboo internodes has been less elaborated on because researchers have focused on bamboo culm as a sample rather than on the internodes [3]. Some examples of such research include bamboo culm form factor [12], [13], [18], bamboo culm volume [14], [16], [17], [33], [36], and culm bamboo biomass and carbon stock [7], [14], [30], [38], [39].

Mathematically, Cheng et al. [3] divided culm morphometrics into four groups, namely one dimension (diameter, length, and wall thickness), two dimensions (cross-sectional area, hollow area, and ring area), three dimensions (hollow cylinder volume and total cylinder volume), and four-dimensions (weight and density). However, the internodes’ non-dimensional characteristics have been elaborated on, such as culm form or volume reduction factor [12], [13], [18]. On the other hand, to eliminate culm size, Higuchi [9], Inoue et al. [15], and Inoue et al. [19] analyzed the culm morphometric using the relativized approach. Unfortunately, the mathematical expressions on bamboo biometrics still lack information [33].

This study aimed to elaborate on the mathematical expression of culm morphometrics. The mathematical expression has been applied to elaborate the characteristics of basal area (two-dimensional), volume (three-dimensional), biomass (four-dimensional), and form factor as non-dimensional morphometric. We chose yellow ampel bamboo (Bambusa vulgaris var. Striata) for several reasons. Firstly, Bambusa vulgaris (Schrad. ex J.C. Wendl.) is widely distributed in Indonesia [5] and is well utilized by communities. Moreover, the cultivation of Bambusa vulgaris (Schrad. ex J.C. Wendl.) in marginal lands has also been economically feasible [6].

2. Materials and Methods

The study was conducted in the private forest and home gardens in Kapanewon Pakem, Sleman Regency, Yogyakarta, Indonesia. The total sample of yellow ampel bamboo was 30 culms. The samples were collected from 5 villages in Kapanewon Pakem, where the bamboo culm was mature, and the owner was also permitted to harvest the bamboo culm. The selection of samples was based on the owner’s permission and the structure of vegetation in the private forests or
home gardens, also conducted by Roshetko et al. [32] in Sumatra, Indonesia. The bamboo culm was felled and divided into many sections based on the number of internodes for each culm. The diameter at breast height and the total height were measured for the felled bamboo culm. Referring to Inoue et al. [19], the internode number (IN) was started from the base to the tip. Each internode was measured for the diameter of the internode (D), the length of the internode (l), and the wall thickness of the internode (WT) (Figure 1).

The data analysis for internode morphometrics followed Cheng et al. [3], who proposed the morphological parameters for bamboo internode as the basal area of the internode (cross-sectional area, hollow area, and ring area) and the volume of the internode (hollow cylinder volume, total cylinder volume, and ring cylinder volume). Furthermore, the calculation of internode morphology can be employed as follows in (Equations (1) to (7)).

\[ HR = \left( \frac{D}{2} \right)^2 - WT \]  

\[ CSA = \pi \left( \frac{D}{2} \right)^2 \]  

\[ HA = \pi \cdot HR^2 \]  

\[ RA = CSA - HA \]  

\[ HCV = \ell \cdot HA \]  

\[ TCV = \ell \cdot CSA \]  

\[ RCV = TCV - HCV \]  

\[ B = RCV \cdot WD \]
where:

HR is hollow radius [cm];
D – diameter of internode [cm];
WT – wall thickness [cm];
CSA – cross-section area [cm$^2$];
HA – hollow area [cm$^2$];
RA – ring area [cm$^2$];
HCV – hollow cylinder volume [cm$^3$];
l – length of internode [cm];
TCV – total cylinder volume [cm$^3$];
RCV – ring cylinder volume [cm$^3$];
B – biomass [gr.];
WD – wood density [gr./cm$^3$]; the wood density for *Bambusa vulgaris* was 0.69 gr/cm$^3$ [1].

The relationship between internode number and internode length is referred to in Inoue et al. [15]. Furthermore, Inoue et al. [15] have explained how to eliminate the culm size that has affected the variation among the culm on internode number and internode length, the analysis using relative internode length (RIL), relative internode number (RIN), and relative cumulated internode length (RCIL). The schematic of RIL and RIN is displayed in Figure 2.

The form factor analysis has also been elaborated for each internode. The form factor for bamboo culm was the ratio between TCV and the cylinder volume of
3.1. Results and Discussion

3.1. Yellow Ample Bamboo Sample Characteristics

The samples of yellow ample bamboo (*Bambusa vulgaris* var. *striata*) have shown varied characteristics of diameter at breast height, total length, and the number of internodes. Statistically, the sample diameter ranged from 2.7 to 8.2 cm, the total length of the culm was 3.1 – 11.9 m, and the number of internodes for each culm was 14 - 49. The diameter of *Bambusa vulgaris* in Indonesia mostly ranges from 5 to 10 cm [5]. Variations in diameter at breast height and in total length have also affected the number of internodes and the length of internodes for each sample. In this study, the length of the internodes ranged from 1.7 to 79.6 cm (Table 1). The smallest internode length occurred in the top area of the bamboo culm. The top area of the bamboo culms also showed the smallest diameter of internodes and wall thickness. The WT can reach 0 cm (there is no hollow area) before the top areas. Internode characteristics among bamboo culms were varied, either in the number of internodes or the length of internodes [15].

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3.2. Internode Morphometric of *Bambusa vulgaris*’ Culm

3.2.1. Two-Dimensional Internode Morphometric

The two-dimensional areas were affected by the value of diameter. This study revealed that the diameter of *Bambusa vulgaris* var. *striata* decreased along the culm. Furthermore, the basal area of the culm also follows the diameter distribution pattern. However, the wall thickness affected the hollow area where the wall thickness in the middle area was bigger than the base area and then decreased until the top area. The mathematical expression of the hollow area is different from the mathematical expression of the cross-sectional area and ring area (Figure 3).
The pattern of CSA, HA, and RA of *Bambusa vulgaris* var. *striata* was similar to the research findings of Cheng et al. [3] *Pseudosasa amabilis* ((McClure) Keng f.) bamboo culm. The CSA and RA formed two-order functions, while the HA showed third-order functions.

![Graphs showing CSA, HA, and RA patterns](image)

**Fig. 3. Two-dimensional Bambusa vulgaris culm morphology**

### 3.2.2. Three-Dimensional Internode Morphometric

The three-dimensional internode morphometric (RCV, HCV, and TCV) of *Bambusa vulgaris* var. *striata* showed a four-order function (Figure 4). These findings were also similar to HCV and TCV of *Pseudosasa amabilis* [3]. Whether *Bambusa vulgaris* var. *striata* or *Pseudosasa amabilis*, the volume increases from the base side until the middle culm and decreases until the top side of the culm.

The diameter of *Bambusa vulgaris* var. *striata* decreases from the base area to the top area. The consequence was that the basal area of the internode in the base area was bigger than the top area (Figure 3). However, the internode length has shown a different pattern: the top area was dense rather than the base area. It affected the volume calculation, where the volume multiplies between basal area and length of culm (Equations 5-7).

Bamboo internode characteristics of *Phyllostachys* spp. were sparse in the middle culm and dense at the top [19]. Sparsity in the mid areas indicates that
internode length was longer than on the top side. A similar result also showed in *Pseudosasa amabilis* where the longest internodes were the segment ten and decreased until the top areas [3]. With the internode length in the middle being longer than the top areas, the volume of the internodes in the mid becomes larger.

Moreover, the diameter of the culm of *Pseudosasa amabilis* showed a different pattern on internodes 5-6 due to rapid growth [3].

![Three dimensional of Bambusa vulgaris culm morphology](image)

**Fig. 4. Three dimensional of Bambusa vulgaris culm morphology**

The similar pattern among total cylinder volume, hollow cylinder volume, and ring cylinder volume can be explained by Equations (5) and (6), in which volume was composed of the internode length and the internodes' basal area. Internode length decreases from the longest in the middle toward the top of the culm [2], [35].

3.2.3. **Four-Dimensional Culm Morphometric**

Bamboo is also similar to the other woody species in that the biomass can be estimated from the volume using equation 8. The wood density value for *Bambusa vulgaris* from the base to the top areas is around 0.69 gr/cm³ [1]. Moreover, the pattern of internode biomass is relatively
similar to the pattern of ring cylinder volume (Figure 5). The biggest biomass in the middle internode is also found in Bambusa multiplex ((Lour.) Raeusch. ex Schult.f.) in China and then decreasing until the top of culm [37].

Mathematically, the relationship between internode number and weight has formed a four-order function, where the x was the internode number, and y was the internode weight. The internode weight increased from the base until the internode number 7 and then decreased until the last internode number. This pattern is also found in Pseudosasa amabilis where the biggest weight occurred in the 7th internode [3].

![Graph showing internode biomass vs. internode number with a fitted line and equation](image)

**Fig. 5. Four-dimensional Bambusa vulgaris culm morphology**

### 3.2.4. Non-Dimensional Culm Morphometric

The bamboo form factor is the ratio between actual bamboo volume and cylindrical bamboo volume [12], [18], [33]. The bamboo form factor is also expressed as culm volume reduction [41] and culm form [12]. In this research, the value of the internode form factor along the culm was less than 1.0 (Figure 6). The volume on the base side was bigger than the volume on the top side. Furthermore, this form factor was strong evidence that the bamboo culm forms a tapered shape. The taper model of bamboo has been studied by Sanquetta et al. [33] for bamboo plantations (Bambusa vulgaris Schrad. ex J.C. Wendl and Bambusa oldhamii Munro) in Brazil.
The minimum, average, and maximum values of Bambusa vulgaris var. striata culm factor were 0.768, 0.816, and 0.876, respectively. Nevertheless, the culm form factor of five bamboo species in Japan (Phyllostachys pubescens, Phyllostachys bambusoides, Phyllostachys nigra var. henonis, Phyllostachys nigra Munro, Phyllostachys bambusoides f. castillonis) ranged from the lowest 0.307 ± 0.035 to the highest 0.614 ± 0.067 [18].

Moreover, the detailed measurement for Phyllostachys pubescens (Moso) has shown that the culm factor was 0.629 [12]. Every bamboo species has its culm characteristics; hence the value of the culm factor can be varied. Furthermore, the research on culm morphometrics of bamboo should be applied to other bamboo species.

### 3.3. Relationship between Relative Internode Number and Internode Length

The culm morphology has varied in the previous section due to the culm size. Eliminating this problem, Higuchi [9] proposed a culm relativized method, either the internode number or internode length. Furthermore, Inoue et al. [15], [19] have elaborated more details on node distribution using relative value, namely RIN, RIL, and RCIL. Moreover, Cheng [4] proposed another approach to understanding the bamboo biosystem using log (internode number) and log (accumulated internode length).

RCIL of Bambusa vulgaris var. striata can be estimated from the value of RIN using the equation RCIL = -1.556RIN^3 + 1.8261RIN^2 + 0.7468RIN - 0.033 (R^2 = 0.999; P>0.001) (Figure 7). The relationship between RIN and RCIL has formed a third-order function [9]. For
instance, the third-order function can be found in the relationship between RIN and RCIL in *Phyllostachys pubescens* in Japan [15].

The relationship pattern between RIN and RCIL can be found in some bamboo species in Japan. For instance, *Sasa senaensis*, *Sasa kurilensis*, *Pleioblastus chinensis*, *Pleioblastus simonii* (Madake), *Phyllostachys bambusoides* (Siebold & Zucc.), and *Phyllostachys pubescens* (Moso) have formed a third-order equation [9], [15]. Although Cheng [4] used log (internode number) and log (accumulated internode length), the relationship between log (internode number) and log (accumulated internode length) also showed a third-order equation. In this study, the species *Bambusa vulgaris* var. *striata* has also formed a similar pattern in the relationship between RIN and RCIL.

The relationship between RIN and RIL has also shown a third-order function. In this study the third-order function is expressed by $RIL = 3.8312RIN^3 - 8.926RIN^2 + 5.2101RIN - 0.0062 \ (R^2 = 0.970; \ P>0.001)$ (Figure 8). The peak point of RIL in *Bambusa vulgaris* var. *striata* was reached when the value of RIN was around 0.4. A similar pattern was also found in *Phyllostachys pubescens* in Japan, where the maximum RIL occurred when RIL was 0.410.
4. Conclusions

In conclusion, the mathematical expression for two-dimensional culm morphometrics was a two and three-order function, while for three and four-dimensional culm morphometrics, it was a four-order function. Moreover, for non-dimensional culm morphometric, there was no relationship between internode number and form factor. Relationship relative internode number with relative internode length and relative cumulated internode length have shown the third-order function.

We realized that this finding only fits Bambusa vulgaris var. striata from Indonesia. Moreover, the random sampling on choosing bamboo culm samples has made a great variance. Further research to elaborate on the culm morphometrics with other bamboo species and combined with stratification sampling will be helpful to get more understanding of the mathematical expression of culm morphometrics.

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References


