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A MATHEMATICAL MODEL FOR PRODUCING YIELD TABLES ECOLOGICALLY DIFFERENTIATED BY REGION

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Abstract: *The paper presents the mathematical expression of a new structural model, defined by the tree number distribution per hectare. This structural model should ensure maximal efficiency in exercising the forest functions, essentially aiming at continuously obtaining the greatest production or protection effects. From a practical point of view it helps improving the technique for the elaboration of the site specific yield tables. Another expected benefit is optimizing the solutions for controlling the structural and functional organisation and management of the forests.*

Key words: *forest management planning, biometrics, forest mensuration, yield table, forest ecosystem.*

1. Introduction

In the conditions of an increasingly intensive forestry practices, the yield tables generally applicable throughout the country – although fully bringing their efficient contribution to the success of current silviculture – cannot correctly reflect the typical course of the stand development. As, within the same production class, especially in the case of the general yield tables, there may be framed ecologically non-homogeneous stands, consequently auxologically developed on various sites with different productive potential, imprinting special dynamics on the growth.

Therefore it is necessary to draft regional or local modern site specific yield tables for the most homogeneous ecological spaces that should reflect both the particularities of the site types and the auxological particularities of the stands, on the assumption of low or moderate-intensity felling, whereby the aim is to maintain the stands as close to the natural density as possible, which is deemed to be a valuation of the optimal density in terms of wood production, in relation to the species, site and age of the stand [2]. Nevertheless, in order to elaborate yield tables on ecological basis, it is justified to adapt several variants of intensities for the secondary felling in relation to the maximization of the wood production or of the protective effects upon the stands,

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ensuring greater flexibility for the yield tables as cultural models and programs with positive effects on the stand growth and production.

In the regular high forest, the basic criterion for verifying the wood production continuity currently is the surface occupied by different age classes. The continuity is admittedly provided when the age classes are equally represented on the surface. Certainly, by accepting such a structural condition, the continuity is easily verified; however, if the non-homogeneity of the stands is considered, one will notice that this procedure is not the most adequate; it does not comply with the current need to determine the production of a forest on species, yield classes and mean diameter categories corresponding to the stands at exploitability, too.

Thus, by analogy with the distribution function of the multi-headed trees in the forest regime, such a function may be considered for regular stands run in this mode. It expresses the true distribution of the number of trees in diameter classes for the entire production facility.

This distribution provides, in the case of the regular high forest, a better image on the difference between the real and the normal growing stock in terms of wood production continuity. Furthermore, the expression of the growing stock per number of trees has the advantage of providing a more relaxed perspective regarding freedom of action in forestry. In the current conditions, the inventory of the growing stock has been enforced not only separately, to species, but also on assortments, so that in the case of the regular-type forest, the continuity in exercising the economic or socio-ecological functions might be also pursued in relation to the growing stock structure on diameter categories (classes).

In doing so, the production may be seen both in terms of quantity, as a result of the growing stock increase, and in terms of quality, as an effect of managing the growing stock through the normal structure.

However, to maximize this effect, the growing stock must fulfil some structural conditions, in the sense that a well determined relation must exist between stands of various dimensions.

In other words, the growing stock must be constituted to permanently provide the greatest growth, and at the same time, in return for this growth, to allow for harvesting the most valuable trees.

2. Methods

In a forest where the normal growing stock has been achieved, the continuity is admittedly provided if a quantity of wood material equal to its growth is extracted out of it. Nevertheless, this growth cannot be harvested as such; some entire trees that make up the growing stock are extracted instead of it.

This presupposes, in addition to a growing stock of a certain structure, the introduction of criteria as objective as possible, in order to separate the production from the growing stock, which consider both the self-thinning speed and the idea of achievement, respectively the maintenance of the normal structure of the growing stock.

For the growing stock in regular high forest to fulfil such a condition, its structure can be expressed using the tree number variation per hectare, in relation to the mean tree diameter. This means that the growing stock structure for regular high forest may be defined by a distribution function that expresses, *through the number of trees per hectare on* *mean diameter categories, the structural model of the normal growing stock* within an management unit [1], [10].

Knowing the parameters of such a function allows determining quite easily the number of trees extracted per species, production classes and mean diameter categories within a management unit. These parameters can be determined by adjusting a curve to the tree number variation per hectare, in relation to the *mean diameter of the basal area* ***(d)***.

However, in order to obtain a general equation as regards the correlation between the number of trees per hectare and the mean stand diameter, the relative index system was adopted, resulting in homogeneous unit series, allowing for comparison [6-9].

To this end, *the number of trees per hectare was expressed in relative units as compared to the absolute values corresponding to the mean diameter* of the basal area ***d=20 cm***, according to the relation:

$$N_{rj} = \frac{N_j}{N_{20}} \quad (1)$$

$\boldsymbol{N_{rj}}$ is the number of trees per hectare expressed in relative values, of a default mean diameter category $\boldsymbol{j}$; $\boldsymbol{N_j}$ – number of trees per hectare in absolute values, of a default mean diameter category $\boldsymbol{j}$, given by the yield tables; $\boldsymbol{N_{20}}$ – number of trees per hectare corresponding to the mean diameter ***d=20 cm***.

The variation coefficient ***s%*** of the values $\boldsymbol{N_r}$ per mean diameter categories is of only ***1-10%***, which shows that mean values $\boldsymbol{N_r}$ have high stability in relation to the species and site quality. Future researches will bring new specifications on the stability degree of the mean values $\boldsymbol{N_r}$ in relation to the species [3], [4].

Knowing the value stability of this indicator, we analytically expressed the relations between the number of trees per hectare, in relative units, and the mean diameter of the basal area ***(d)***, regardless of the species (spruce, fir, beech) and production class, by a sixth-degree polynomial:

$$N_r = a_0 + a_1 d + a_2 d^2 + a_3 d^3 + \\ + a_4 d^4 + a_5 d^5 + a_6 d^6 \quad (2)$$

The parameters in function (2) are experimentally determined by the least-square method.

This way, for the regular high forest stands of fir, spruce and beech, under Romanian yield tables [4], [5], by solving the normal equation system, the values coefficients $\boldsymbol{a_0}$, $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{a_3}$, $\boldsymbol{a_4}$, $\boldsymbol{a_5}$, $\boldsymbol{a_6}$ were obtained, in relation to the mean diameter values of the basal area ***d=20 cm*** (Table 1).

The coefficients $\boldsymbol{a_0}$, $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{a_3}$, $\boldsymbol{a_4}$, $\boldsymbol{a_5}$, $\boldsymbol{a_6}$ were used to calculate the number of trees per hectare, expressed in relative values.

The fact that these parameters are independent of both site quality and species (spruce, fir, beech) is a great advantage, facilitating the analytical expression of the tree number variation per hectare in regular high forest.

Table 1

Experimental values of the coefficients $\boldsymbol{a_0}$, $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{a_3}$, $\boldsymbol{a_4}$, $\boldsymbol{a_5}$, $\boldsymbol{a_6}$, *of the relation (6)*

Coefficient	**Value**
$\mathbf{a_0}$	14,844
$\mathbf{a_1}$	$- 831{,}62 \cdot 100^{-1}$
$\mathbf{a_2}$	$1574{,}4 \cdot 100^{-2}$
$\mathbf{a_3}$	$- 5304{,}1 \cdot 100^{-3}$
$\mathbf{a_4}$	$8257{,}4 \cdot 100^{-4}$
$\mathbf{a_5}$	$- 3495{,}0 \cdot 100^{-5}$
$\mathbf{a_6}$	$- 2394{,}8 \cdot 100^{-6}$

Based on the expressions (1) and (2), the general regression equation for the tree number variation per hectare as a function of mean diameter of the stands becomes:

$$N_j = \left(a_0 + a_1 d_j + a_2 d_j^2 + a_3 d_j^3 + a_4 d_j^4 + a_5 d_j^5 + a_6 d_j^6\right) N_{20} \tag{3}$$

This relation must be stochastically understood. It presents, in addition to the scientific interest in finding out the biometric laws of the regular high forest special importance for determining the number of trees per hectare and elaborating a structural model of the normal (optimal) growing stock.

In relation to the tree number determination per hectare, $\boldsymbol{N_{20}}$ of the relation (3), we specify that, for certain management units, the **20 cm**-diameter category cannot always be sufficiently represented per species and production classes, facing some difficulties in the tree number determination per hectare for this mean diameter category. It is therefore more appropriate, for the calculations, to determine the number of trees per hectare ***(N_D)*** from other mean diameter types ***(D)***, which are well represented in the management unit.

In this case, considering the relation (1), it may be written:

$$N_{20} = \frac{N_D}{N_r} \tag{4}$$

If the value given by relation (4) is introduced in the expression (3) and the equation (2) is also considered, wherein, this time, the diameter ***(D)*** is introduced, for which the number of trees per hectare ***(N_D)*** is determined, there is obtained:

$$N_j = \frac{\left(a_0 + a_1 d_j + a_2 d_j^2 + a_3 d_j^3 + a_4 d_j^4 + a_5 d_j^5 + a_6 d_j^6\right)}{\left(a_0 + a_1 D + a_2 D^2 + a_3 D^3 + a_4 D^4 + a_5 D^5 + a_6 D^6\right)} N_D \tag{5}$$

or

$$N_j = N_{rD} \cdot N_D \tag{6}$$

Using equation (5) for $\boldsymbol{N_D}$**=1**, Table 2 was drafted, which enables the determination of the number of trees per hectare for categories of mean diameters across an entire management unit, if the number of trees per hectare $\boldsymbol{N_D}$, corresponding to the mean diameter ***D*** of the well represented stands across the management unit is known.

Further, the distribution for mean diameter categories of the tree number throughout the management unit is easily obtained, depending on the distribution of the growing stock on mean diameter categories.

Viewed from a practical standpoint, equation (5) allows reducing the number of measurements to be taken in order to determine the number of trees per hectare for the even-aged stands within a management unit.

Furthermore, this equation contributes to improving the technique for determining the production per number of trees in a regular high forest [9].

Table 2

Relative values N_{rD} *of the number of trees per hectare, for* $N_D=1$*, of the relation (6)*

Species: spruce, fir, beech									
D \ ***d***	**26**	**28**	**30**	**32**	...	**44**	**46**	**48**	**50**
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
16	0,4988	0,4426	0,3934	0,3519	...	-	-	-	-
18	0,5900	0,5235	0,4653	0,4163	...	-	-	-	-
20	0,6827	0,6058	0,5385	0,4817	...	0,2913	-	-	-
22	0,7747	0,6875	0,6111	0,5466	...	0,3306	0,3088	0,2992	-
24	0,8841	0,7845	0,6974	0,6238	...	0,3772	0,3524	0,3300	-
26	1,0000	0,8874	0,7888	0,7056	...	0,4267	0,3986	0,3732	0,3507
28	1,1269	1,0000	0,8889	0,7951	...	0,4809	0,4492	0,4206	0,3952
30	1,2678	1,1250	1,0000	0,8945	...	0,5409	0,5053	0,4732	0,4446
32	1,4173	1,2576	1,1179	1,0000	...	0,6047	0,5649	0,5290	0,4970
34	1,5709	1,3939	1,2391	1,1084	...	0,6703	0,6261	0,5863	0,5509
36	1,7231	1,5290	1,3592	1,2158	...	0,7352	0,6868	0,6431	0,6042
38	1,8684	1,6579	1,4737	1,3183	...	0,7972	0,7447	0,6973	0,6552
40	2,0288	1,8003	1,6003	1,4315	...	0,8657	0,8086	0,7572	0,7114
42	2,1846	1,9386	1,7232	1,5414	...	0,9322	0,8707	0,8154	0,7661
44	2,3436	2,0796	1,8486	1,6536	...	1,0000	0,9341	0,8747	0,8218
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

3. Discussion and conclusions

Knowledge of the relationship between the growing stock structure, growth and production allows for consistently guiding the planning and regulation of wood harvesting in order to achieve the structural conditions set by the management plan [8].

By such actions, scientific and practical information of the utmost importance are obtained and the management plan has the tool to ensure the continuity of forest functions and to indicate the strategy to follow in order to enhance the effect of these functions.

The research brings a theoretical and practical contribution to the action of optimizing the solutions controlling the structural and functional organisation and management of the forests, whose management, as essence of the working activity, presupposes determining an optimal structural model of the future forest, defined by certain conditions whose specification and then adaptation to the new socio -ecological or economic functions is by definition a matter of control [1].

The proposed *structural model* is meant to ensure maximal efficiency in exercising the forest functions, aiming to obtain continuously the greatest production or protection effects. At the same time it helps producing yield tables that should reflect the growth and development characteristics of the stands across the area for which such tables of zonal or regional interest are elaborated.

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