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MATHEMATICAL MODELLING OF DISCS FOR BEET EXTRACTION WORKING PROCESS

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Abstract: *This paper makes analysis and mathematical modeling of disc displacement type organs of beet harvesters machinery workflow in order to find the optimal structural and functional thereof parameters. Writing to disc motion equations are used to determine the correct operating conditions in different cases of use. The highlight opportunities for improving the working of the machine by improving construction discs. Although sugar beet acreage in Romania decreased lately appeared priority programs that encourage farmers to cultivate its.*

Key words: *beet, extraction, disc type organs*

1. Introduction

The beet harvest process supposed as main operations leaves cutting and deployment-extraction. Finding the optimal values of the structural and functional parameters bodies running these operations directly determines the quality of work performed [3].

Of the many forms of organ displacement beet roots (fork, curved coulters, rosettes, disks), are preferred lately the disc, due to the advantages it presents: high harvest rate, reduced injury due to effort smaller roots movement in the direction of advance of the machine, choppers intensive soil layer and therefore better clearance roots and training of more than soil while these organs can work well in any conditions.

Currently mechanical harvesting beet is generalized worldwide so harvester equipment with the best assemblies, including equipment deployment – extraction is the crucial element in market competition [4].

2. Mathematical modelling of discs kinematics

Disc – type displacement organs consist of two conical or spherical discs, less flat, arranged witch the plane of rotation inclined to the soil surface as and to the direction of advancement (Figure 1) [1].

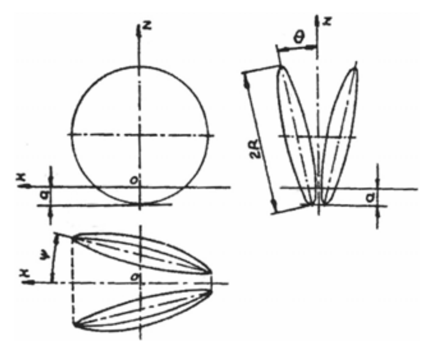

Fig. 1. *Discs beet dislocated geometry*

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Displacement disc may be passive, case whare rotational movement is caused of the contact with the ground to advance the machine and fixed, when the rotary motion received from the motion transmission organs of the machine. In both cases the movement of the disc is the result of two motions: translational motion with the machine and rotation.

In the general case, the motion of a point on the disc is a curve in space (because of the inclinations of the plates) projecting in the plane - perpendicular to the direction of advance in the form of an ellipse, and the horizontal and vertical planes parallel to the direction of forward in the form of trohoide [2].

The equation of motion of a point M (x, y, z), found on disc dislocation - extracting considered plan (Figure 2) were expressed in parametric form:

a) in the vertical plane of the direction of advancement (*xoz*):

$$x = R \cdot \cos\omega t \cdot \cos\varphi + v_m \cdot t ;\quad y = 0 ;\quad z = R \cdot \sin\omega\, t \cdot \cos\theta . \tag{1}$$

b) in the vertical - perpendicular plane to the direction of advancement (*yoz*):

$$x = 0 ;\quad y = R \cdot \cos\omega\, t \cdot \sin\varphi ;\quad z = R \cdot \sin\omega\, t \cdot \cos\theta . \tag{2}$$

where R is the radius of the disc,

ω – rotational speed of the disc,

φ - angle of the plane *xoz*,

θ – angle to the plane *yoz*,

v_m – forward speed of the machine,

t – time.

Eliminating the time from the equations (1) gives the expression:

$$x = \cos(\arcsin\frac{z}{R \cdot \cos\theta}) \cdot \cos\varphi + \frac{R}{\lambda}\arcsin\frac{R}{\cos\theta} , \tag{3}$$

which is the equation of a trohoide, wherein $\lambda = \dfrac{\omega \cdot R}{v_m}$.

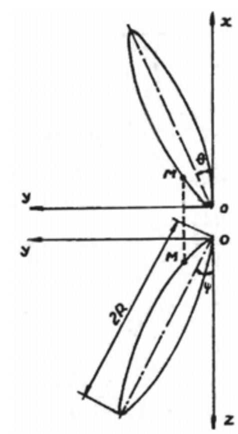

Fig. 2. *Projection of point M in plans xoz and yoz*

From the system of equations (2) we obtain:

$$\frac{y^2}{2 \cdot R^2\left(1 - \sin^2 \omega t\right) \cdot \sin^2 \psi} + \frac{z^2}{2 \cdot R^2 \cdot \sin^2 \omega t \cdot \cos^2 \theta} = 1 \tag{4}$$

which represented an ellipse ecuation.

So, a point on the in motion disc describes in the vertical plane of forward direction a trohoide and in the vertical plane perpendicular to advance an ellipse.

To analyze of the working process of the disc, the projection on the horizontal plane motion (*xoy*) no interest.

For active disc case, if the machine gets moving through independent power outlet,

ω is a constant value $\omega = \frac{\pi \cdot n}{30}$ where n is the wheel speed, in *rot/ min.*

For passive discs rotation speed ω can be determined by applying the principle of conservation of energy in the form:

$$m' \cdot v_m = P \tag{5}$$

where m' is the mass per unit time, P – soil resistance force opposing the advance of the disc.

The disc penetrates in the soil to a depth S and mobilize a layer wich section:

$$A = S' \cdot \sin\psi \tag{6}$$

wherein S' is the vertical projection of the surface S of the ground penetrating segment of a circle and is calculated by the formula:

$$S' = S \cdot \cos\theta = \frac{4}{3} \cdot h \cdot \sqrt{\frac{2 \cdot R}{h} - 1} \cdot \cos\theta, \tag{7}$$

where the H is the arrow the circle sector $h = \frac{a}{\cos\theta}$, and a is the depth working. Mass m' will be:

$$m' = \frac{\gamma}{g} \cdot A \cdot v_m,$$

where γ is the specific weight of the soil and g – gravity acceleration. The force P is given by relation:

$$P = m' \cdot v_m = \frac{\gamma}{g} \cdot \frac{4}{3} \cdot h^2 \cdot \sqrt{\frac{2R}{h} - 1} \cdot \sin\varphi \cdot \cos\theta \cdot v_m^2 \cdot \tag{8}$$

The force P acting on the weight center of the section, which is located at a distance R' from the disc center. This distance can be expressed by the relationship:

$$R' = \frac{s^3}{12 \cdot S}$$

wherein s is the length of the chord of a circle segment. Thus:

$$R' = \frac{1}{2} \cdot (2R - h) = R - \frac{h}{2} \tag{9}$$

The force P acting through its components tangential: $P_t = P \cdot \cos\psi$ and perpendicular $P_n = P \cdot \sin\psi$.

The perpendicular force will cause a friction force $F_f = P \cdot \sin\psi \cdot tg\varphi$, where φ is the friction angle with the ground.

The force that causes rotation of the disc will be:

$$F = \frac{\gamma}{g} \cdot \frac{4}{3} \cdot h^{2\cdot} \sqrt{\frac{2 \cdot R}{h} - 1} \cdot \sin\psi \cdot \cos\phi \cdot v_m^2 \cdot (\cos\psi - \sin\psi \cdot tg\varphi) \tag{10}$$

or:

$$F = P_t - F_f = P \cdot (\cos\psi - \sin\psi \cdot tg\varphi).$$

Taking into account the energy conservation law in the disc plane can write:

$$P \cdot \cos\psi \cdot v_m \cdot \cos\psi = F \cdot R' \cdot \omega \tag{11}$$

in which:

$$\omega = \frac{P \cdot v_m \cdot \cos^2\psi}{F \cdot R'} = \frac{v_m \cdot \cos^2\psi}{\left(R - \frac{h}{2}\right) \cdot (\cos\psi - \sin\psi \cdot tg\varphi)} \tag{12}$$

3. Mathematical modeling of the angular speed and workflow

Examination of the relationship (12) allows to highlight the factors that depend on the angular velocity and namely:
- the more the car speed increases, the angular velocity increases;
- in heavy soil, whose friction coefficient wich disc material is bigger and the angular velocity of the discs is greater;
- at working depths *a*, large ($a = h \cdot \cos\theta$), the angular velocity decreases;
- for big inclination angle ψ , the angular velocity also decreases.

Given of the working conditions variability, namely unevenness working depth, moisture condition that alters the friction coefficient and so on, we can say that as in the case of passive discs we deal with a variable angular velocity *ω*.

This situation have – as will be shown – influence on workflow.

For active discs, angular velocity can find constructive set to forward speed of the machine in the following reports of determination (dependency) [1]:

$$a.\ \omega = \frac{v_m}{R_1 \cdot \cos\psi},\quad b.\ \omega < \frac{v_m}{R_1 \cdot \cos\psi},\quad c.\ \omega > \frac{v_m}{R_1 \cdot \cos\psi}, \tag{13}$$

In the case *a* the discs worked without slippage; in the case *b* the discs are pushed by the machine framework, then there is a negative slip, and in the case *c* that the discs skated.

From the point of view of the working process, where the *b* case is the disadvantaged, because during deployment and extraction of the roots of sugar beet clamped between the discs are pushed forward and, as such, can occur more damages.

If active pair discs have the same angular velocity, extraction of ground beets is without twisting them around their axis and therefore no risk of injury, even if wavy discs.

For active discs cases that would different rotation speeds and sometimes in passive discs cases, when one disc the other work under different conditions in the beet root extract can be twisted. This situation can be two aspects: if the discs are corrugated beets injuries can occur, and if the disc is smooth or fine corrugations can be improved cleaning of soil thereof.

After extraction, beets are dumped onto conveyor roots. Disposal is the resultant direction and velocity tangential speed transmission and is due to centrifugal force and lift force due to the wedge effect caused by tilted arrangement of the two discs.

Centrifugal force is given by:

$$F_c = m \cdot \omega^2 \cdot r = m \cdot \frac{v^2}{r} \tag{14}$$

where: *m* is the mass of the root; *v* – velocity of the center of gravity of the beet; *r* – radius of the center of gravity layout beet.

Mass *m* of different beets various high limits (0.2...4 *kg*), is proportional to the cube of the diameter of the beet, as shown in the following relation [1]:

$$m = \frac{\gamma}{g} \cdot \frac{\pi \cdot d^2}{4} \cdot \frac{h}{3} = \frac{\gamma}{g} \cdot \frac{\pi \cdot d^3\,(2...3)}{4 \cdot 3}, \tag{15}$$

where: $\frac{\gamma}{g}$ it is beet density; *d* – beet diameter; *h* – height beet; was considered as the beet height *h*=(2...3)*d* and assimilate beet form with a cone.

High beets are caught between the lower-ray disc, and the smaller the radii greater (Figure 3) according to the equation:

$$r = R - \frac{d}{2 \cdot tg\psi}, \tag{16}$$

wherein r has the meaning given in the figure.

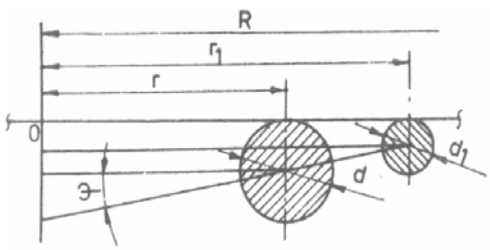

Fig. 3. *Beets position to the discs of different extraction positions*

Lifting force F resulting from the condition of support and push beet discs (Figure 4) and has the value:

$$F = \frac{m \cdot g}{2} \cdot \sin\theta \cdot tg\psi . \tag{17}$$

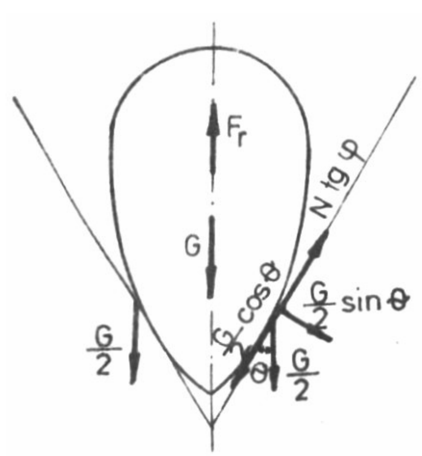

Fig. 4. *Beet position to record the time displacement*

Determining the point C of tangency between the two discs (Figure 5) can be done by the angle α formed by the beam passing through this horizontal diameter.

Figure 5 follows:

$$K = 2 \cdot R \cdot (1 - \sin\alpha) \cdot \sin\frac{\xi}{2}, \tag{18}$$

in which:

$$\alpha = \arcsin\left(\frac{K}{2 \cdot R \cdot \sin\frac{\xi}{2}} - 1\right) \tag{19}$$

where: k=d+s, d is the minimum diameter of beet and s – the width of the trench produced the disc, given by:

$$s = 2 \cdot \sqrt{2 \cdot R \cdot h - h^2} \cdot \sin\psi , \tag{20}$$

and ξ – the angle between the discs in a plane containing point C and the centers of discs ($\xi \approx 2\Psi$).

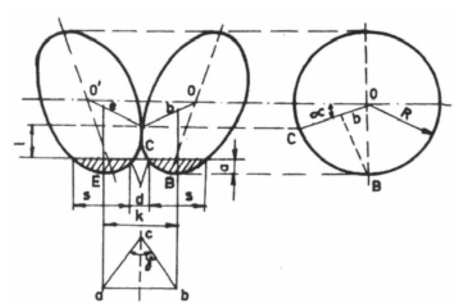

Fig. 5. *The geometry of the extraction process by the disc beet*

The position of point of tangency C to the soil surface can be determined from Figure 5, wich the relation:

$$t \approx R \cdot (1 - \sin\alpha) - a . \tag{21}$$

4. Conclusions

1. Passive extraction disks not always ensure proper disposal of beet root transporters.
2. Active discs whose angular velocity relationship (13) checks performed largely due to the wedge effect work with a behavior similar to a fork dislocation bodies.
3. Disposing of the carrier beets depends largely on its diameter and the angular velocity of the disc. For proper disposal would be required beets developed as uniform.
4. For active disks whose angular velocity relationship (13) check, throwing is better, cleaning is better because these discs work slip.
5. The point of tangency of the disks is important from the point of view of extraction (angle ξ difficult than extracting value), and the disposal. Point of tangency should be at such a height as not to allow the passage of roots through the disc.
6. Using activ discs wich angular speed which to satisfy the relationship (13) and at the same time to vary by up to 10 % among them contribute to better cleaning of the ground roots.

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