

# ABOUT SUPERCAPACITORS PARAMETERS DETERMINATION

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**Abstract:** *The parameters determination of the supercapacitors is a required stage in the identification process of the supercapacitors with the applications in which they are integrated. After a short overview on the supercapacitors models, an RC model for a pack of supercapacitors is presented. Between the measurement methods for the supercapacitors parameters, the constant charge method and the capacitance-time domain conversion method are presented. Based on the RC model proposed, parameters determination is made, in a case study that proves the viability of the constant charge method determination.*

**Key words:** *supercapacitors, parameters, models, measurement methods.*

## 1. Introduction

Supercapacitors are the storage devices offering a very high capacity in a small size. There are several models in the literature that permit to determine the supercapacitors parameters [1], [3].

Between the aims of the parameters determination of the supercapacitors the following are listed [2]:

- technologies improvement;
- proposed models verification and validation;
- electrical design of power circuits containing supercapacitors.

In this paper a simple model for a pack of supercapacitors is proposed. Based on it, the parameters determination is done, using and presenting two methods. A case study is following, to exemplify the constant current charge method. In this way, the supercapacitors proposed model is validated and the parameters determination method is tested.

## 2. Models of Supercapacitors

Modeling the supercapacitors aims to estimate their behavior in different conditions and systems in which they are integrated [4]. For this, it is important to know the characteristics that define the parameters evolution of the supercapacitors, grouped according to specific models. Between the existing models, the theoretical simple one can be described by a simple RC series circuit. This model is taking into account the supercapacitors datasheets, being easily used in practice [1]. The disadvantage is that it does not describe the complexity of the phenomena associated to energy storage.

In Figure 1 the simple RC series model [1] is presented. There are authors who rely on energy considerations in order to determine the supercapacitor model [12].

In Figure 2, the supercapacitor schema with two branches is presented. The model is often used due to its simplicity, but in this model the nonlinearity phenomena of

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the electric double layer is not shown.

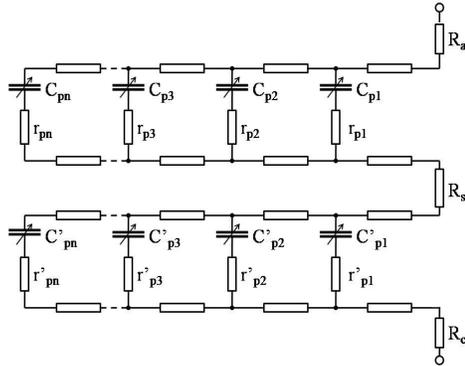


Fig. 1. The RC series model of a supercapacitor

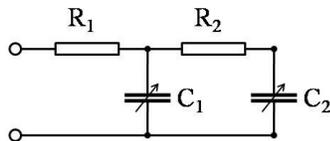


Fig. 2. The two branches model of Bonert and Zubieta for a supercapacitor

In the transportation area, a single cell supercapacitor is not sufficient to achieve the performance required by applications, due to the low terminal voltage. For this reason, the packs of supercapacitors are used, in series and with balanced construction [6], [12].

The presented research uses the two branches model for a pack of supercapacitors. Because the model is extrapolated to a pack of supercapacitors, the parameters define the whole pack.

In Figure 3 the electrical equivalent schema of the pack is presented.

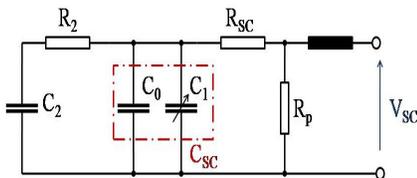


Fig. 3. RC model of supercapacitors

The circuit  $R_2C_2$  is the slow branch that describes the phenomena after the charge/discharge. This branch can be neglected because the charge/discharge processes are very fast. The inductance  $L$  achieves rapid variations of the current circuit and in many applications it is neglected too. The resistance  $R_p$ , which describes the supercapacitor self-discharge is the losses resistance. The  $R_{sc}C_0-C_1$  branch describes the energy evolution during the charge/discharge cycles.

The pack capacity consists of variable part:  $C_1$  that varies with the terminal and the fixed  $C_0$  initial capacity, according to the formula:

$$C_{sc} = C_0 + C_1, \tag{1}$$

where the variable part respects the relation:

$$C_1 = k V_c, \tag{2}$$

and  $V_c$  is the pack capacity voltage.

The  $R_{sc}$  represents internal series resistance of the supercapacitor, with values up to 1 mΩ for powerful packs.

Without the slow branch, reducing the supercapacitor model is like in Figure 4.

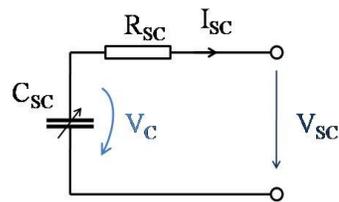


Fig. 4. Supercapacitors model reduced to the mainline (generator convention)

### 3. Measurement Methods of Supercapacitors Parameters

There are several methods for the parameters determination of supercapacitors, like: charge/discharge at constant voltage,

impedance spectrometry etc. [7-8], [10-11]. This paragraph presents two of these methods.

### 3.1. Constant Current Charge Method

The constant current charge method is most useful, due to their simplicity and correctness [9].

In Figure 5 the general schema of this method is presented.

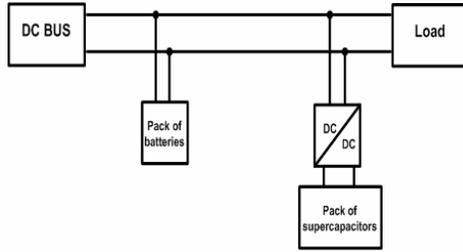


Fig. 5. General schema used for constant current charge method

The energy source of the schema is the pack of batteries that ensures the constant DC bus voltage. The bidirectional DC/DC converter allows the current flow between the pack of supercapacitors and the DC bus, during the charge and discharge supercapacitors processes. The Load is composed by the resistances used for fast discharge of supercapacitors. To measure the supercapacitors' characteristics voltage and current sensors are used.

The work stages according to this method are:

- a constant current charge is applied at the supercapacitors terminals;
- during the charge process, the voltage across supercapacitors terminals is measured;
- data is processed to obtain the experimental response: curves  $I_{sc}(t)$ ,  $U_{sc}(t)$ ;
- model parameters are extracted from the experimental response  $U_{sc}(t)$ ; are identified: the internal resistance and the equivalent capacity of the pack

The energetic responses are computed:

the delivered power  $P_{sc}(t)$  and the delivered energy  $W_{sc}(t)$  of the supercapacitors pack.

### 3.2. Capacitance Time-Domain Conversion Method

Converting the capacitance into a time interval is another method that can be used to determine the supercapacitors parameters.

The work stages according to this method are:

- a constant current charge is applied;
- a potential difference is created, preselected as function of supercapacitor's working voltage;
- measurement of time period needed to create the voltage variation;
- data processing and computing in order to obtain the supercapacitor capacity.

The voltage measured at the terminals of a capacitor  $u_c$  is given by the following relation:

$$u_c = \frac{1}{C} \int_0^{\tau} i \cdot dt. \quad (3)$$

In the case of supercapacitor charge/discharge with a constant value of the current ( $i = I_0 = const.$ ), during a  $\tau$  period of time, the voltage  $u_c$  can be written as follows:

$$u_c = \frac{I_0}{C} \cdot \tau. \quad (4)$$

The capacity  $C_x$  to be measured results from relation (4) by considering a certain voltage variation,  $u_c = \Delta V$ :

$$C = \frac{I_0}{\Delta V} \cdot \tau. \quad (5)$$

In other words, relation (5) shows that the value of the capacitance  $C$  can be obtained as function of time  $\tau$  needed to create the voltage variation  $u_c = \Delta V$ , in

conditions of a constant charge/discharge current  $I_0$ .

The electric circuit of the capacitance measurement, based on proposed capacity-time domain converting is presented in Figure 6.

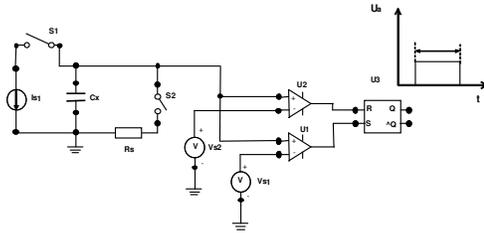


Fig. 6. The capacitance-time domain conversion method

The current value from constant current source  $I_{s1}$  can be selected by the user from one or more predetermined values.

The voltage difference  $\Delta V$  is selected as function of the working voltage of the tested capacitor.

For example, the first voltage point  $V_{s1}$  can be twenty percent of working voltage and the second voltage  $V_{s2}$  point can be eighty percent of the working voltage. In case the working voltage of the tested capacitor is 1.5 V, the voltage difference is  $\Delta V = 0.9$  V.

The capacitor  $C_x$  is supposed to be not charged at starting time  $t_0$  of the measurement. The electric circuit contains a constant current generator  $I_{s1}$  that starts charging the capacitor  $C_x$  at  $t_0$  through switch  $S_1$ .

Two comparators  $U_1$  and  $U_2$  are used, whose reference levels are fixed to  $V_1$  respectively  $V_2$  with voltage reference sources, in order to establish the voltage variation  $\Delta V = V_2 - V_1$ . A flip-flop  $U_3$  switches its output value when the capacitor is charged.

The measurement process involves the following steps:

a) at  $U_C = V_1$ , the output of comparator  $U_1$  is '1' and the output  $Q$  of flip-flop  $U_3$

switches to 'high' state (logical '1');

b) at  $U_C = V_2$ , the output of comparator  $U_2$  is '1' and the output  $Q$  of flip-flop  $U_3$  switches to 'low' value (logical '0'). At the output  $Q$  of circuit  $U_3$  a time impulse  $\tau$  will be obtained, that is directly proportional to the capacitance  $C_x$ , according to relation (3);

c) turning off the switch  $S_1$  (stopping the charging process);

d) turning on the switch  $S_2$  (discharging the capacitor);

e) watching the value  $U_C < V_1$ , after which the measuring process can be restarted;

f) determining the time value  $\tau$ , that will give the size of capacitance  $C_x$ .

The determination of  $\tau$  is realized by means of a time base generator with a frequency  $f_0 = 10^3 \dots 10^5$  Hz and a gate circuit (AND), that will be subjected to impulses (I.G. - Pulses generator) of frequency  $f_0$  during the time  $\tau$  (Figure 7).

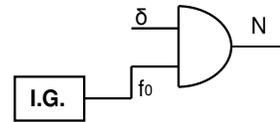


Fig. 7. Measuring principle of the system

The number of pulses  $N$  is given by the relation:

$$N = f_0 \cdot \tau. \quad (6)$$

By replacing the value  $\tau$  from (5) in (6), will be obtained:

$$N = f_0 \cdot \frac{\Delta U}{I_0} \cdot C_x. \quad (7)$$

The capacitance  $C_x$  results as:

$$C_x = K \cdot N, \quad (8)$$

where:

$$k = \frac{I_0}{\Delta U \cdot f_0}. \quad (9)$$

The error of measurement due to converter characteristics is basically determined by:

- precision of the current generator;
- precision in fixing the limits  $V_1$ ,  $V_2$  and respectively  $\Delta U$ .

#### 4. Supercapacitors Parameters Determination: Case Study Using Constant Current Charge Method

The pack of supercapacitors used is BOSTCAP BMOD0050 E015 B1 (5.8 F, 150 V, 10 modules  $\times$  10 cells in series).

The working test bench to determinate the supercapacitors parameters contain:

- the pack of batteries, 23 cells in series, 12 V/100 Ah each;
- the bidirectional current converter, that allows current with values between 10 and 400 A. Reference current of converter - by a low-pass filter strategy of a DC bus current;
- resistances (Load);
- voltage/current sensors;
- 280 V DC bus.

The used method to characterize the pack of supercapacitors is the charge at constant current which permit to see the temporal answers: current and voltage according to the time.

In order to identify the supercapacitors parameters, the following stages were passed:

- a charge at constant current is performed (50 A);
- the voltage at the terminals of the supercapacitors is measured;
- the charge current intensity is measured;
- model parameters are extracted from the experimental response obtained: voltage depending on time.

Then, by data processing, are identified the internal resistance and the equivalent capacity composed by a constant capacity and specific capacity (the fast branch) of

the pack.

The amount of supercapacitors stored electrical charge at a moment is given by:

$$Q_{sc}(t) = C_{sc} V_c(t). \quad (10)$$

Electrical characteristics of current, during a supercapacitor's constant current charge is expressed by:

$$I_{sc}(t) = \frac{dQ_{sc}(t)}{dt} = C_{sc} \frac{dV_c(t)}{dt}. \quad (11)$$

From the experimental response it can be observed that at the beginning and at the end of charge it is a voltage drop due to the pack of supercapacitors  $R_{sc}$  internal resistance. From the graph, the potential difference  $\Delta V$  is determined, between the points marking the beginning of charge and the first voltage step. The internal resistance  $R_{sc}$  is determined with [3], [5]:

$$R_{sc} = \frac{\Delta V_0}{I_{sc}}. \quad (12)$$

From relations (1-2), (10-11) the current expression  $I_{sc}$  results:

$$I_{sc} = C_0 \frac{dV_c}{dt} + 2k V_c \frac{dV_c}{dt}. \quad (13)$$

It approximates  $\frac{dV_c}{dt} \approx \frac{\Delta V_c}{\Delta t} = p$ , where  $\Delta t$  represents charge time (while  $I_{sc}$  has a constant value), and  $\Delta V_c$  is the difference potential between the terminals of the charge line [3].

The report is the charge line pant, noted with  $p$  and (13) becomes:

$$I_{sc} = p (C_0 + 2k V_c). \quad (14)$$

On the other hand, the total electrical charge accumulated by the pack of supercapacitors during the charge is:

$$Q_{sc} = \int I_{sc} dt = I_{sc} \Delta t. \quad (15)$$

From (10), written for time interval  $\Delta t$  and (15) results:

$$C_{sc} \Delta V = I_{sc} \Delta t, \quad (16)$$

where  $\Delta V$  is the voltage variation  $V_c$  during the charge time  $\Delta t$ .

The relation (16) becomes:

$$C_0 \Delta V + k \Delta V^2 = I_{sc} \Delta t. \quad (17)$$

From (14) and (17) results:

$$k = \left( \frac{1}{p} - \frac{\Delta t}{\Delta V} \right) \frac{I_{sc}}{2 V_1 - \Delta V}, \quad (18)$$

$$C_0 = \frac{1}{p} - \left( \frac{1}{p} - \frac{\Delta t}{\Delta V} \right) \frac{2 V_1}{2 V_1 - \Delta V} I_{sc}. \quad (19)$$

With these relations and using the experimental graphics, the parameters of supercapacitors pack can be determined.

In Table 1 the extracted values from the graphs in Figure 8 are shown, for the parameters involved in relations (18), (19).

In Table 2 are presented:

- the identified values according to parameters identified in Table 1 and the relations (12), (18), (19);

- values from datasheets.

In Table 3 the principals' parameters, resulted from experiments and computations are presented.

Starting by the equations resulted from the mathematical model of the electrical schema, simulations were realized. To implement the pack of supercapacitors model the Matlab Simulink software was used. The parameter initialization was made using an "M" file and the equations describing the electrical current and voltage variations during the charge were written using Fcn functions.

Table 1

*Parameters extracted from the graphics*

$\Delta t$ , [s]	15.48
$\Delta V$ , [V]	148.88
$V_1$ , [V]	10.321
$\Delta V_0$ , [V]	9.7
$p$	8.83

Table 2

*Supercapacitors pack BPAK0058 E015 B1 parameters*

	Identified	From datasheets
$R_{sc}$ , [ $\Omega$ ]	0.19	0.19
$C_0$ , [F]	5.6616	-
$k$ , [F/V]	0.0016	-
$C_{sc}$ , [F]	5.65	5.8

Table 3

*Supercapacitors pack BPAK0058 E015 B1 parameters*

<b>Time for charge</b> , [s]	15.48
<b>Power</b> , [W]	65250
<b>Energy</b> , [kJ]	1010.07
<b>Internal resistance</b> , [ $\Omega$ ]	0.19

The current and the voltage obtained experimentally and by simulations at a constant current charge (50 A) are presented in Figures 8, respectively 9, for the pack of supercapacitors: 10 cells of BPAK0058 E015 B1.

It can be observed that the results identified by the experiments and calculations using graphics-method presented are very close to the datasheets.

Analyzing the experimental and the simulated results, it can be observed that the simplified model proposed in Figure 4 accurately expresses the supercapacitors functionality during the charge.

## 5. Conclusions

Several equivalent models of the supercapacitors were presented, after that a series model was proposed and verified.

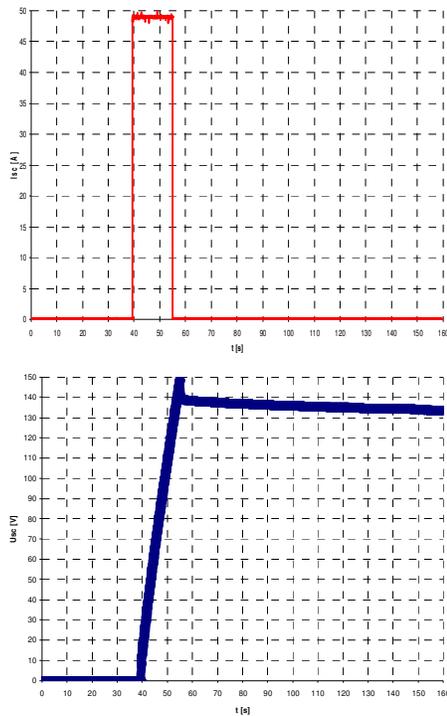


Fig. 8. Experimental characteristics obtained by the constant current (50 A) charge method for BPAK0058 E015 B1 module

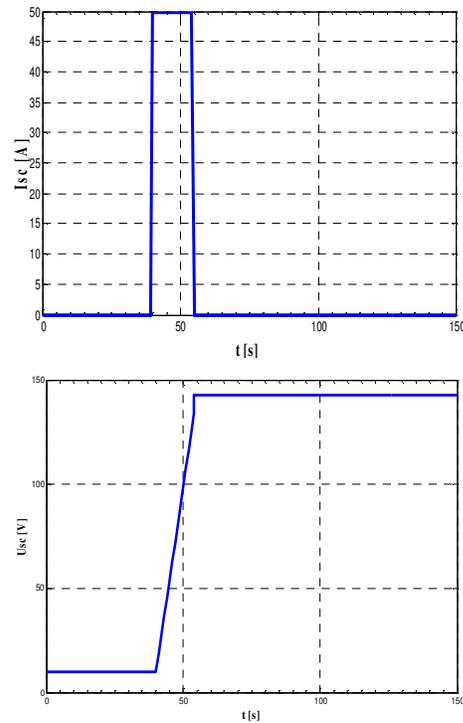


Fig. 9. Simulations characteristics obtained by the constant current (50 A) charge method for BPAK0058 E015 B1 module

Two methods for supercapacitors parameters determinations were detailed.

In a case study, a type of supercapacitors was selected and the constant current charge method was used to establish the experimental curves of current and voltage. The same types of curves obtained by simulations were carried out.

The experimental parameters of the pack of supercapacitors were identified. The results obtained were validated by tests and by comparison to the theoretical results after the simulations.

The experiments done permit to evaluate the internal parameters of supercapacitors: internal resistance  $R_{sc}$  and capacity  $C_{sc}$ .

It can be seen that the  $R$ - $C$  proposed model is close to the standard model found in datasheets. The time constant values are

lower than at the complex models that represent an advantage in the data acquisition. Finally, due to the small errors between the results analyzed, identification parameters method presented in case study is correct and quite accurate.

### Acknowledgements

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