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GEARED LINKAGE WITH LARGE ANGULAR STROKE USED IN TRACKED PV SYSTEMS

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Abstract: *The photovoltaic platforms deliver a higher energetic result when are provided with tracking mechanisms. In this paper there is presented a new type of geared linkage composed of a triangular linkage fitted with a planetary gear pair. This geared linkage, as basic mechanism for azimuth tracking, was analytically modeled in two cases: when the linkage base is larger than the rocker and when the base is smaller than the rocker. This new tracking system has been comparatively analyzed, by numerical simulations. The results of the numerical simulations highlight that: a) the azimuth angular stroke are amplified only by the geared linkage with the base larger than the rocker and b) this stroke increases with the planetary gear internal ratio* $|i_0|$.

Key words: *tracking geared linkage, angular stroke, planetary gear internal ratio.*

1. Introduction

Photovoltaic systems convert solar radiation into electric energy. To improve their efficiency, by increasing the received solar radiation falling perpendicularly [2] on the photovoltaic surface, a tracking system is required.

Nowadays a significant number of tracking systems types are used, some of which function with the help of rotating actuators. The advantage of the rotating actuators is that they can accomplish large angular strokes. Their disadvantage is the large over all size and the high priced production process. Other tracking systems use linear actuators, which are cheaper but are not able to perform the necessary large angular strokes due to the transmission angle that becomes too small.

One existent solution, to perform high angular strokes using a linear mechanism, is the two-contour linkage [1-5] in which the second contour accomplishes the movement amplification delivered by the first one. This solution from [1], [4], [5] consists of a deformable triangle mechanism, serially connected to a quadrilateral mechanism of 4 *R* type (*R* = revolute joint); herein the second linkage amplifies the angular stroke of the first linkage. These mechanism's angles do not pass too much the value of 180° due to the transmission angles.

This paper approaches the development of a new azimuth tracking mechanism to obtain increased azimuth angular strokes

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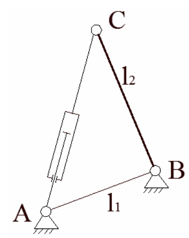
using a *geared linkage, as basic mechanism.* In this new tracking system, the angular stroke of the deformable triangle linkage (Figure 1) is amplified by a planetary gear pair (Figure 2).



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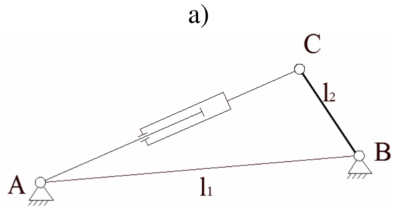


b)

Fig. 1. *Two types of the triangle linkage driven by a linear actuator: a)* $l_1 < l_2$*; b)* $l_1 > l_2$

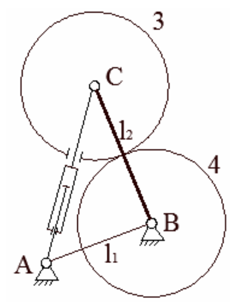
The objectives of this paper are: 1) to attain the conceptual solutions and the kinematical modeling for the new proposed geared linkage; 2) to do the correspondent numerically simulations and to analyze the results; 3. the synthesis on a specific case.

2. New Conceptual Solutions of Geared Linkage with for Large Angular Strokes

Starting from a triangle typed linkage (Figure 1a, b) and adding a planetary gear pair to it, a new mechanism is obtained (Figure 2a, b).

The geared linkage components are (Figure 2): a frame (length = l_1), a rocker (length = l_2) and gears 3 and 4. Gear 3 is fixed connected to an element of a linear actuator and gear 4 is fixed connected to an azimuth pole.

The study begins with the kinematical modeling of the triangle linkage, considering two cases: $l_1 > l_2$ (Figure 2a) and $l_1 < l_2$ (Figure 2b). To avoid the jamming of the mechanism, in its initial or final position, a minimum value for the pressure angle (γ) is chosen.



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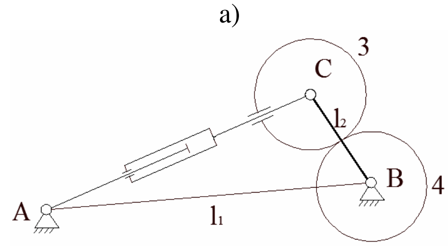


b)

Fig. 2. *Two types of the geared linkage: a)* $l_1 < l_2$*; b)* $l_1 > l_2$

Based on Sinus Theorem, applied in Figure 3a, for ABC_1 and ABC_2 triangles, the following expressions can be written:

$$\frac{l_1}{\sin\gamma} = \frac{l_2}{\sin(C_2AB)} = \frac{AC_2}{\sin(ABC_2)}, \tag{1}$$

$$C_2AB = \arcsin\big((l_2/l_1)\cdot\sin\gamma\big) \le 90°, \tag{2}$$

$$\frac{l_1}{\sin\gamma} = \frac{l_2}{\sin(C_1AB)} = \frac{AC_1}{\sin(ABC_1)}, \tag{3}$$

$$C_1AB = 180° - \arcsin\big((l_2/l_1)\cdot\sin\gamma\big). \tag{4}$$

According to previous relations, the rocker's angular stroke can be calculated (see Figure 3a):

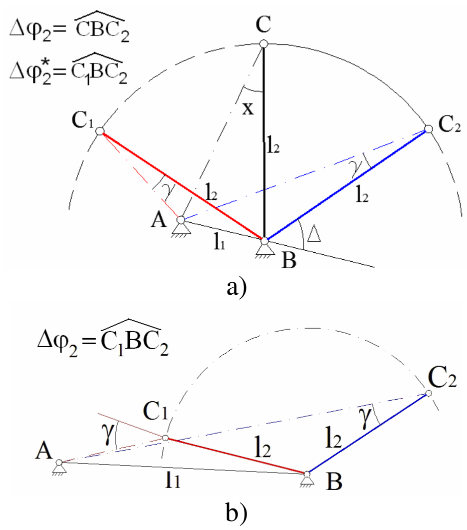


Fig. 3. *a) Initial and final position of the linkage for minimum admitted* γ *and intermediary position for maximum* γ ($\gamma_{max} = x$) *when* $l_1 < l_2$*; b) Initial and final position of the linkage when* $l_1 > l_2$

$$\Delta\varphi_2^* = C_2BC_1 = 180° - 2\arcsin((\sin\gamma)/k), \quad (5)$$

$$k = l_1 / l_2 . \quad (6)$$

Applying the same theorem in Figure 3b, following correlations are obtained:

$$\frac{l_1}{\sin\gamma} = \frac{l_2}{\sin(C_2AB)} = \frac{AC_2}{\sin(ABC_2)}, \quad (7)$$

$$C_2AB = \arcsin((l_2 / l_1)\cdot\sin\gamma), \quad (8)$$

$$\frac{l_1}{\sin\gamma} = \frac{l_2}{\sin(C_1AB)} = \frac{AC_1}{\sin(ABC_1)}, \quad (9)$$

$$C_1AB = \arcsin((l_2 / l_1)\cdot\sin\gamma). \quad (10)$$

According to relations (8) and (10) it can be concluded that C_1, A and C_2 are collinear (see Figure 3b). Therefore, the rocker's angular stroke C_2BC_1 and the stroke *s* of the linear actuator become (Figure 3b):

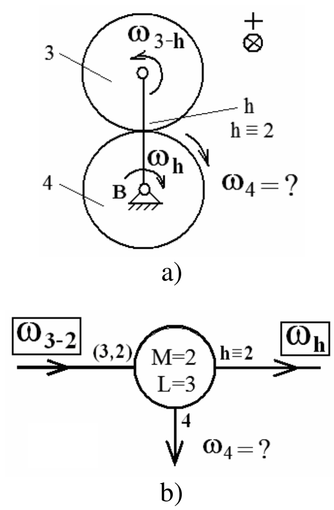


Fig. 4. *a) Differential planetary gear pair and its b) black box scheme*

$$\Delta\varphi_2 = C_2BC_1 = 180° - 2\gamma, \quad (11)$$

$$s = C_2C_1 = 2l_2\cos\gamma . \quad (12)$$

A gear pair (a differential planetary gear pair) is attached to both linkages from Figure 3 to amplify their angular stroke. The internal speed ratio of the differential planetary mechanism is (Figure 4a):

$$i_0 = i_{4-3}^h = \frac{\omega_{4-h}}{\omega_{3-h}} = -\frac{z_3}{z_4}. \quad (13)$$

Considering both the triangle linkages (Figure 3a and b) and the differential planetary gear pair (Figure 4), the following analytical model for the geared linkage, as basic mechanism, can be obtained:

$$\Delta\varphi_4 = i_0\cdot\Delta\varphi_{3-2} + \Delta\varphi_2 . \quad (14)$$

a) For $l_1 < l_2$:

$$\Delta\varphi_{3-2} = -|x-\gamma| = \gamma - \arcsin k < 0, \quad (15)$$

$$\Delta\varphi_2 = C_2BC = 90° - \arcsin((\sin\gamma)/k) > 0, \quad (16)$$

$$\Delta\varphi_4 = 90° + i_0(\gamma - \arcsin k) - \\ - \arcsin((\sin\gamma)/k); \quad (17)$$

b) For $l_1 > l_2$:

$$\Delta\varphi_{3-2} = 2\gamma - 180° < 0, \quad (18)$$

$$\Delta\varphi_2 = C_1BC_2 = 180° - 2\gamma > 0, \quad (19)$$

$$\Delta\varphi_4 = i_0(2\gamma - 180°) + 180° - 2\gamma. \quad (20)$$

Numerical simulations are done based on the above analytical model. Consequently, a geared linkage practical employment is evaluated through a comparative analysis.

3. Numerical Simulation, Comparative Analysis and Results

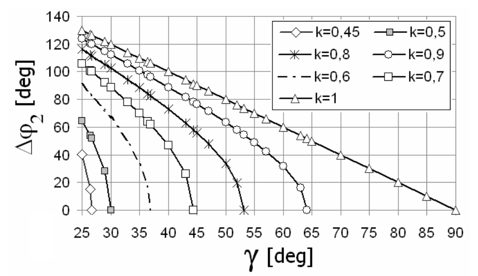
Numerical simulations are done considering the above described geometrical models for both the linkages and the geared linkages.

In Figure 5 are illustrated the results for the first case: $l_1 < l_2$. For each value $k = l_1/l_2$, in the range of 0.45…1, a corresponding $\varphi(\gamma)$ curve is exemplified: *a*) for the alone linkage (without gears, Figure 4a, b) for the geared mechanism with inverted slider-crank as basic mechanism having $i_0 = -1$ (Figure 4b, c) for the geared mechanism with inverted slider-crank as basic mechanism having $i_0 = -1.4$ (Figure 4c). Moreover Figure 4 highlights that this geared mechanism with inverted slider-crank as basic mechanism (with $l_1 < l_2$) brings no significant advantage as compared to the alone linkage.

Unlike the case with $l_1 < l_2$ (Figure 2a), the case with $l_1 > l_2$ (Figures 2b and 3b) presents important advantages: the angular stroke of the alone linkage dose not depend on the *k* value (Figure 6a), and the angle φ_4 of the geared mechanism with inverted slider-crank as basic mechanism can reach values higher than 180º, especially for $i_0 \leq -1$ (Figure 6b). Moreover, the relative dimension l_1/l_2 of this geared mechanism with inverted slider-crank as basic mechanism is computed established as against the ratio $k_a = AC_1/l_2$ (Figure 3b):

$$AB^2 = AC_1^{\,2} + BC_1^{\,2} + 2AC_1 \cdot BC_1 \cdot \cos\gamma \Rightarrow$$

$$(l_1/l_2)^2 = 1 + k_a^{\,2} + 2k_a\cos\gamma. \quad (21)$$



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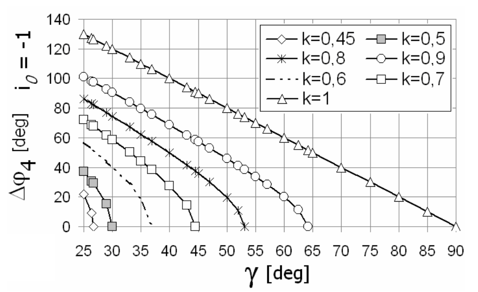

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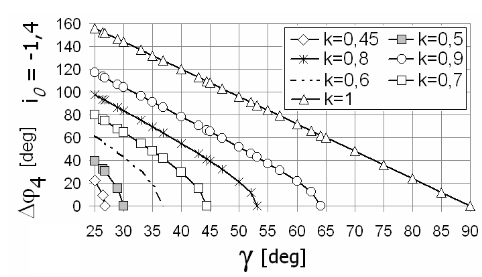


c)

Fig. 5. *Angular stroke variation* ($\Delta\varphi_2$ *and* $\Delta\varphi_4$) *when* $l_1 < l_2$ *for: a) the linkage* (*Figure 3a*); *b) and c) the geared linkage* (*Figure 2a*) *with internal ratio* $i_0 = -1$ *and respectively* $i_0 = -1.4$

Some graphical results, obtained by numerical simulations using (21), are ilustrated in Figure 7.

According to Figures 6 and 7 the relative dimensions for the geared linkage (Figure 2b) can be determined.

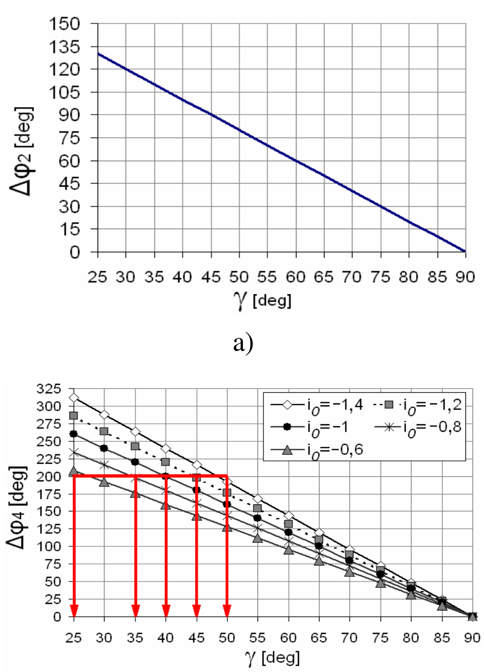
To exemplify the method for a hypothetical in-practice demand, the following requirements are considered:

a) the tracking angular stroke: $\varphi_4 = 200°$;

b) the minimum admited value for the transmission angle (to avoid the blocking of the mechanism) is: $\gamma \geq 30°$;

c) the rocker length $l_2 = 300$ mm (resulted form the system wind load considerations);

d) a linear actuator is used, having an adjustable length (Figure 3b) $AC_1 = 180$ mm (i.e. $k_a = AC_1/l_2 = 0.6$) and a linear stroke $s = C_1C_2 = 500$ mm.



a)

b)

Fig. 6. *Angular stroke variations when $l_1 > l_2$ for: a) the linkage (Figure 3b); b) the geared linkage (Figure 2b) with different ratio i_0*

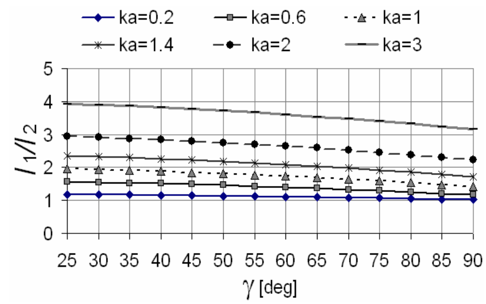


Fig. 7. *Variations of the ratio l_1/l_2 as against the ratio $k_a = AC_1/l_2$ (s. Figure 3b)*

Calculus example Table 1

Input data (see Figs. 3b, 4, 6b and 6c):

$\Delta\varphi_4 = 200°$; $\gamma_{min.ad.} = 30°$;
$l_2 = 300$ mm; $k_a = AC_1/l_2 = 0.6$;
$s = AC_{max} - AC_{min} = 500$ mm

i_0	γ	k_a	l_1/l_2	l_1	s
-	[°]	-	-	[mm]	
–0.8	34.44	0.2	1.17	351.12	494.83
		0.6	1.53	459.85	494.83
		1	1.91	573.10	494.83
		1.4	2.29	688.64	494.83
		2	2.88	864.23	494.83
		3	3.86	1159.89	494.83
–1	40	0.2	1.16	348.10	459.62
		0.6	1.50	452.91	459.62
		1	1.87	563.81	459.62
		1.4	2.25	677.82	459.62
		2	2.83	851.92	459.62
		3	3.82	1146.15	459.62
–1.2	44.54	0.2	1.15	345.33	427.65
		0.6	1.48	446.51	427.65
		1	1.85	555.24	427.65
		1.4	2.22	667.84	427.65
		2	2.80	840.59	427.65
		3	3.77	1133.53	427.65
–1.4	48.33	0.2	1.14	342.83	398.90
		0.6	1.46	440.68	398.90
		1	1.82	547.42	398.90
		1.4	2.19	658.74	398.90
		2	2.76	830.26	398.90
		3	3.74	1122.06	398.90

For the imposed value of the angular stroke several values for $\gamma(i_0) > 30°$ are resulted (see Figure 6b and Table 1); moreover, the ratio l_2/l_1 (depending on the ratio k_a) can be determined from Figure 7 and Table 1, for each previous obtained pair of values i_0 and γ.

For a simplified view, in Table 1, are presented only 24 of the possible solutions, while the rest of solutions can be determined through interpolation.

According to Table 1, the most adequate stroke (as against the imposed value of 500 mm) is s = 459.62 mm; therefore, the required geared linkage with $l_1 > l_2$ (Figure 2b) works on the following parameters: $\gamma_{min} = 40°$; $i_0 = -1$; $k_a = 0.6$; $l_2 = 300$ mm and $l_1 = 452.91$ mm.

4. Conclusions

Between the two triangle linkages presented in this paper (with $l_1 < l_2$ and $l_1 > l_2$), only the second case can be used in the synthesis of tracking geared linkage with large angular stroke.

In comparison with double-contour linkages, the geared linkage can reach angular strokes much higher than 180º, when the basis has a superior length to the rocker's one; the angular stroke increases according to the internal gear ratio $|i_0|$.

Based on the analytical model and on the numerical simulations, developed for the second case ($l_1 > l_2$), a kinematical synthesis algorithm was elaborated. This algorithm was exemplified for a representative application according to the PV systems' tracking domain.

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