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ANALYTICAL MODEL OF THE CUTTING PROCESS WITH SCISSORS-ROBOT FOR HAPTIC SIMULATION

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Abstract: *In this paper, an analytical model for cutting process of a plate of material with a pair of scissors is presented. The cutting model predicts that, during task, force/torque responses of the scissors depend on the toughness and geometry of the plate. A force/position control law for a pair of scissors in contact with a partially known environment is proposed. The environment is a rigid plate of material of known geometry but of unknown pose. A strategy for online estimation of the object pose is adopted, based on visual data provided by direct observation as well as on forces measured during the interaction with the environment. This information is used by a hybrid force/position control strategy of the scissors haptic system.*

Key words: *cutting process, robot hybrid-control, haptic rendering.*

1. Introduction

Physical human-robot interaction is an interesting topic where a user may need to share the workspace with a small robot for robotics' services such haptic systems. In the haptic technique the human arm interact with a small robot and a force exchange occurring between the robot and its user. An energy-to-power ratio may be transferred by the robot to user. During interaction with robot result a contact force, detectable by means of sensitive perception of the human.

In order to increase the human-robot haptic system capability, is the proper use of the two main "senses": vision and touch. Vision and force based control for physical interaction include control of close interaction between human operator and robot-tool, which, both may lead to improving available haptic system' performance, without necessarily considering a novel architecture design.

The improvements for physical interaction anticipating and reacting can be achieved through the carefulness of combinations of external/internal robot sensing in the electronic hardware. Software procedures intelligently supervise and control the virtual scissors in the virtual environment.

On the other hand, a more complete set of external sensory devices can be used to monitor task execution and reduce the risks of unexpected behaviors.

Modern actuation strategies, as well as hybrid force/position control strategy, seem to be anyway crucial in human-robot haptic interaction. However the most robust architecture is endangered by system unpredictable behaviors and human faults.

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2. Related Work

There are several approaches to address the problem of the analytical model for cutting process with a pair of scissors.

For example Mahvash [3] propose a method for modeling of cutting process based on the concepts of mechanics tool contact and fracture mechanics.

A second approach consists in the introducing of an analytical model to simulate cutting with scissors whose blades have a particular shape proposed by Mahvash [4].

Also several approaches were used to model scissor cutting for haptic simulation. Okamura et al. used force data recording and replaying for haptic simulation of scissor cutting [5]. Only the information obtained during force recording could be replayed during simulation.

At the same time, the state-of-the-art in modelling of the cutting forces is such that the cutting process with scissors-robot is not capable of carrying out complex cutting tasks involving the special shape of the scissors' blades. In this paper, we extend the analytical model to include scissors with arbitrary curves. We also perform experimental tests to verify the physical model using her virtual prototype. The cutting model is used for the haptic system synthesis for the interaction between a virtual pair of scissors with a virtual plate via the physical scissors, handled by human user.

3. Analytical Model of the Cutting Process with a Pair of Scissors

This section presents an analytical model based on the concepts of contact mechanics and fracture mechanics to calculate forces applied to scissors during cutting of a plate of material.

The model considers the process of cutting as a sequence of deformation and fracture phases. During deformation phases, forces applied to the scissors are calculated from a torque-angle response model synthesized from measurement data multiplied by a ratio that depends on the position of the cutting crack edge and the curve of the blades.

We will present in following the cutting process of a plate of material with a pair of scissors. We will analyze two cases: only the upper blade of the scissors has angular motion during interaction, in the first case and second case, when twice blades have angular motion.

Figure 1 shows cutting of a plate of material when only the upper blade of the scissors has angular motion during interaction. The plate rests along the edge of the lower blade of the scissors and it does not move during cutting.

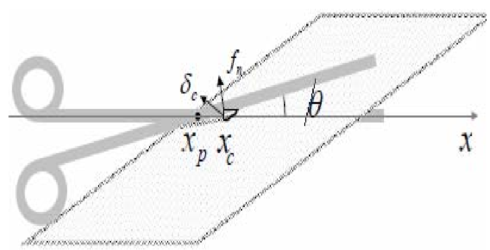

Fig. 1. *Explanation for a scissors with the upper blade angular motion*

Only the pivot of the scissors can move on the surface of the plate along a straight line parallel to an axis denoted by x. The lower blade of the scissors does not have any angular motion [3].

The coordinate x_p defines the location of the pivot and θ defines the opening angle of the scissors. The component of force applied to the upper blade in the direction normal to the blade edge is needed for haptic applications. This component is defined as f_n, and it is calculated during two distinct regimes by a contact model.

3.1. Contact model

The static state of deformation in an elastic object under load generally depends on the forces that are applied to the boundary of the object [3].

When a rigid tool contacts a deformable object, the position and orientation of the tool with respect to the object uniquely defines the forces applied to the object. Therefore, the resultant force applied by the tool can be expressed by a nonlinear function of the position and orientation of the tool.

When we cut an object with a pair of scissors, the force that we feel between our fingers includes two main components: friction forces of the contact of the blades, and the forces of cutting of the object. In this approach we do not model the friction forces.

However, friction forces can be measured by opening or closing empty of the scissors, and be summed into the model.

Figure 2 shows the interaction between a pair of scissors and a thin plate with thickness h at the time t when the plate is locally deformed [3].

A Cartesian frame is defined at the pivot of the scissors such that the x axis is along the symmetry line of the scissors. We assume that the pivot of the scissors does not change orientation during cutting.

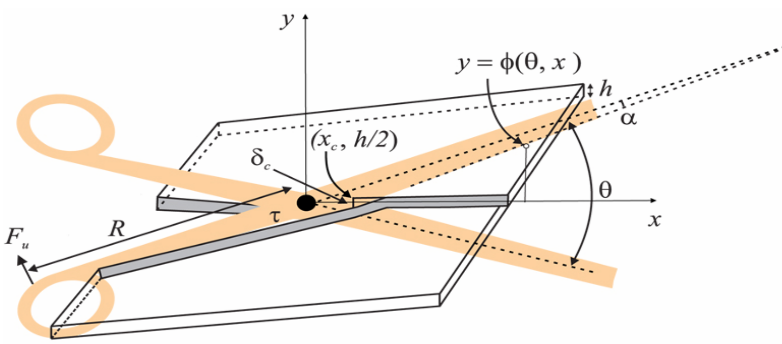

Fig. 2. *The states of scissors and a plate during the cutting process*

We also assume that the pivot of the scissors does not move. The twice blades have angular motion. The opening angle of the scissors is defined by θ, and the position of the edge of the crack made by the scissors is defined by the point x_c, with respect to the pivot point of the scissors. The blades locally deform an area of the plate around the crack edge. The deformation can take various forms including twisting, stretching, compression, or a combination there of.

At time t, the plate is locally deformed. During time period dt, a small area of the plate, $h\, dx_c$, is cut. During deformation, the upper edge of the rupture point is displaced from $(x_c, h/2)$ to $(x_c, h/2 - \delta)$, where δ is a displacement length (Figure 2).

In response to deformation of the plate, the force f_n is applied to the upper blade along the normal to the blade's edge at point $(x_c, h/2 - \delta)$.

The displacement length δ is determined on the base of the curve of the edge of the upper blade. The curve of the blade edge has a significant effect on the torque response of the scissors. Here, we define the curve of the edge of the upper blade in the Cartesian frame as:

$$y = \varphi\,(x, \theta), \tag{1}$$

where, (x, y) is a point on the edge of the blade and $\phi(.)$ is a nonlinear function.

We obtain $\phi(.)$ by fitting an analytical curve to the edge of the upper blade. We extract the blade edge from a real image of the blade.

Considering (1), the displacement length δ caused by a blade with curve ϕ(.) is obtained by:

$$h/2-\delta=\phi(x_c,\theta),\quad \delta=h/2-\phi(x_c,\theta).\tag{2}$$

The normal force, $\boldsymbol{f}_n$ is calculated by a nonlinear function:

$$f_n=g(\delta),\tag{3}$$

where, $g(\cdot)$ is a nonlinear function of the point displacement, obtained by measurement of material properties.

The torque τ caused by f_n at the shaft-pivot is calculated by:

$$\boldsymbol{\tau}=x_c f_n\cos(\alpha),\tag{4}$$

where α is the angle between the blade's edge and the centerline of the blade. The angle α is not zero because scissors' blades are slightly conical as shown in Figure 2.

The force felt by the user at the handle is calculated by:

$$F_u=\frac{x_c}{R}f_n\cos(\alpha),\tag{5}$$

where R is the distance between the pivot and the handle. The analytical model accurately predicts the normal applied force.

4. Force/Position Control with Visual Observation and Force Feedback

A pair of scissors is essentially a small robot with two Degrees of Freedom (DOF): rotational (cutting) DOF and translational DOF. Degrees of Freedom can refer both to how a device keeps track of position, and how a device outputs forces (Figure 3).

A haptic scissors system is a 2 DOF device with force feedback.

A haptic scissors system keeps track of position with her translational DOF and rotates in the forward-backward by her rotational DOF and gives forces in those same DOF.

Consider a scissors-robot in contact with a rigid object. During the interaction, in the contact point on the object surface, the measure of the force at the scissors-robot blades can be used to compute the feed-back torque at the fingers of the user.

The forces of contact between the blade of the scissors and the plate are expressed by a nonlinear function of deflection at the crack front multiplied by the length of the contact region between the blade and the plate.

Fig. 3. *Two-degree-of-freedom haptic scissors*

On the other hand, the force can be expressed as a function of the object pose and of the robot position.

The position and orientation of a frame $O_0\{x_0, y_0, z_0\}$ attached to a rigid object with respect to a base coordinate frame $O\{x,y,z\}$ can be expressed in terms of the coordinate vector of the origin O_0 and of the rotation matrix $\boldsymbol{R}_0(\varphi_0)$, where φ_0 is a $(p \times 1)$ vector corresponding to a suitable parameterization of the orientation.

In the case that a minimal representation of the orientation is adopted, e.g., Euler angles, it is $p = 3$, while it is $p = 4$ if unit quaternion are used.

Hence, the $(m \times 1)$ vector $\boldsymbol{X}_0 = [\boldsymbol{O}^T{}_0\ \varphi^T{}_0]^T$ defines a representation of the object pose with respect to the base frame in terms of $m = 3 + p$ parameters.

The homogeneous coordinate vector $\tilde{p}$ = $[\boldsymbol{p}^T\ 1]^T$ of a point P of the object with respect to the base frame can be computed as $\tilde{p} = \boldsymbol{H}_0(x_0)\ {}^0\tilde{p}$, where ${}^0\tilde{p}$ is the homogeneous coordinate vector of P with respect to the object frame and $\boldsymbol{H}_0$ is the homogeneous transformation matrix representing the pose of the object frame referred to the base frame:

$$H_0(x_0) = \begin{bmatrix} R_0(\phi_0) & O_0 \\ \mathbf{0}^T & 1 \end{bmatrix}, \tag{6}$$

where, $\mathbf{0}$ is the (3×1) null vector.

It is assumed that the geometry of the object is known and that the interaction involves a portion of the external surface which satisfies a twice differentiable scalar equation $\phi({}^0p) = 0$.

When in contact, the tip point P_q of the elementary scissors robot instantaneously coincides with a point P of the object, so that the tip position ${}^0\boldsymbol{p}_q$ satisfies the constraint equation:

$$\phi({}^0\boldsymbol{p}_q) = 0\,, \tag{7}$$

The unit vector normal ${}^0\boldsymbol{n}$ to the surface at the point P is expressed in the object frame. The (3×1) contact force vector, ${}^0\boldsymbol{f}$ is aligned to the normal unit vector ${}^0\boldsymbol{n}$ to the surface of the object.

4.1. Control Strategy

This work focuses on techniques for augmenting the performances of haptic scissors system by means of control architecture.

Haptic scissors capabilities in close interaction with human operator can be considered as mimicking sensing and actuation of human. This leads to consider fully integrated human vision and force feed-back based control.

Thanks to the visual perception, the user integrated into a robotic system may achieve global information on the surrounding environment that can be used for task planning and accidents avoidance. Human vision and force/torque feed-back are two complementary sensing capabilities that can be exploited in a synergic way to enhance the precision in the control of a scissors-robot during the interaction with the environment.

On the other hand, the perception of the force applied to the robot allows adjusting the motion so that the local constraints imposed by the environment during the interaction are satisfied.

The precision of the accomplished task and dependability of a robotic system are strictly connected to the availability of sensing information on the external environment [1].

Moreover, human vision may substitute the complex infrastructure needed for "intelligent environments" to detect the interaction in the operational space.

4.2. Interaction Control

When the scissors-robot moves in free workspace, the unknown object pose and the position of the hand of a human user can be estimated online by using the data provided by the human user eyes.

When the scissors-robot is in contact to the object, also the force measurements and the joint position measurements are used. Joint values are used for evaluating the position of the scissors for the interaction configuration with the object.

The Cartesian desired velocity (or force) for the contact (control) points, is transformed in the corresponding joint trajectory via proper inverse kinematics. Any point on the object can be considered as a control point.

Consider a scissors-robot in contact with an object in the scissors-to-hand configuration. In the following, some modeling assumption concerning the environment and the scissors-robot are illustrated.

The proposed algorithm can be used to estimate online the pose of an object in the workspace; this allows compute the surface equation with respect to the base frame in the form:

$$\phi(R_0^T(p_q - o_0)) = \phi(q,t) = 0\,, \quad (8)$$

where, the last equality holds in view of the direct kinematics equation of the robot. In the following, it is assumed that the object does not move; the general case of moving object is more complex but can be analyzed in a similar way.

Hence, the constraint equation is $\phi(q,t) = 0$; moreover $J_\phi(q)\dot{q} = 0$, where $J_\phi(q) = \partial\phi / \partial q$ is a Jacobean matrix.

The dynamic model of the manipulator in contact with the object is:

$$D(q)\ddot{q} + n(q,\dot{q}) = \tau - J_\phi^T(q)\lambda\,, \quad (9)$$

where $\boldsymbol{D}$ is the (n x n) symmetric and positive definite inertia matrix, $n(q,\dot{q})$ is the (n x 1) vector taking into account Coriolis, centrifugal, friction and gravity torques, τ is the (n x 1) vector of the joint torques, and λ, is the lagrangean multiplier associated to the constraint. In the case of scissors robot we can assume that Coriolis, centrifugal, friction and gravity torques are less important and can be neglected.

The on-line computation of the constraint equations can be suitably exploited for interaction control. In the following, the case of the hybrid force/position control is considered.

4.3. Hybrid Force/Position Control of the Scissors-Robot

An inverse dynamics control law can be adopted, by choosing the control torque τ as [2]:

$$\tau = D(q)\alpha_r + n(q,\dot{q}) + J_\phi^T(q)h_\lambda\,. \quad (10)$$

According to the hybrid force/position control strategy, it is useful to apply the change of coordinates with intermediate variables: α_r, a_r, a_p, r_p, h_λ defined as:

$$\begin{aligned} \alpha_r &= J_r^{-1}(q)(a_r - \dot{J}_r(q)\dot{q})\,, \\ a_r &= \begin{bmatrix} 0 & a_P^T \end{bmatrix}^T, \\ a_P &= \ddot{r}_{Pd} + \mathbf{K}_{Dr}(\dot{r}_{Pd} - \dot{r}_P) + \\ &\quad + \mathbf{K}_{\mathrm{Pr}}(r_{Pd} - r_P)\,, \end{aligned} \quad (11)$$

where $\boldsymbol{K}_{\mathrm{Dr}}$ and $\boldsymbol{K}_{\mathrm{Pr}}$ are suitable feedback gains.

The vector $\boldsymbol{r}_{\mathrm{p}}$ allows to specify any scissors-robot position and orientation which satisfies the constraint. The force vector $\boldsymbol{F}$, in the new coordinate $\boldsymbol{r}$ is: $F = [1\ \ 0]^T$.

The scalar λ is computed from the measured contact force. Velocity vector $\dot{\boldsymbol{r}}$ and force vector $\boldsymbol{F}$ are orthogonal, i.e., $\boldsymbol{F}^T\dot{\boldsymbol{r}} = 0$ for any $\dot{\boldsymbol{r}}$ that satisfy the constraint (such that $\dot{\boldsymbol{r}}_F = 0$).

A force/position control task can be assigned by specifying the desired force

$F_d = \lambda_d(t)$. The intermediate variable h_λ is calculated as:

$$h_\lambda = \lambda_d + k_{I\lambda} \int_0^t (\lambda_d(\tau) - \lambda(\tau)) \mathrm{d}\tau, \quad (12)$$

with $k_{I\lambda}$ suitable feedback gain.

On the base of the above relations one can configured an inner/outer structure. The inner loop implements hybrid control whereas the outer loop computes the estimation of the object pose as well as the desired force and motion trajectories.

The cutting model is computationally efficient, so it can be used for real-time computations of the corresponding equations.

5. Virtual Prototype of the Scissors-Robot

The performance and validity of this system are evaluated using a virtual prototyping. The virtual scissors enable people to experience a realistic sense of touch using their computer.

The human arm is connected to the device's handle. Using 2D haptic interface device, the computer users may feel 2D objects, feel their shapes and textures, feel the dynamic properties of objects, and feel many other effects. The virtual scissors gives force feedback through interchangeable scissors handles that a user holds on to.

Using simulators, we may build experimental environments. Complexity, specificity can be gradually increased to a level where virtual systems can lead to real challenges of the physical world.

We can investigate, design, visualize and finally see the performances of a scissors in the virtual environment before it is used on real object. It is possible that our solutions may fail or even blow up, but only in simulating on the virtual prototype.

The simulation can be used to perform analysis and design studies on any cutting device, which can be modeled as a set of rigid bodies interconnected by joints, influenced by forces, driven by prescribed motions, and restricted by constraints [5].

In this paper the authors propose the Delphi environment, for the simulation of the haptic systems, using visual programming.

The simulator, created in the Delphi environment, was used to test the performances of the haptic scissors in their integrated environment.

The simulation methodology uses a number of possible scenarios and involves the presence of the user to create some virtual objects in the virtual environment.

6. Physical Prototype of the Scissors-Robot

In the physical world the static state of deformation in a plastic object under load generally depends on the forces that are applied to the boundary of the object [1]. When a rigid tool contacts a deformable object, the position and orientation of the tool with respect to the object uniquely defines the forces applied to the object. A physical prototype of a scissors haptic device consists of a scissors-robot with two DOF and an electrical DC-motor. Force/torque feedback is created through the electrical motor.

Figure 4 is showing the system overview. Experimental haptic system for cutting samples of paper confirm the model, and is rendered in a haptic virtual environment.

Fig. 4. *Scissors device* (*bottom-side of the picture*) *and virtual scissors prototype* (*up-side of the picture*)

The whole system may achieve global information on the environment using vision of the user.

7. Conclusions

The integration of force and visual control to achieve human-robot interaction has been discussed.

A position-force technique based on visual control made by the user and the force feedback control has been presented, employing a pose estimation algorithm on the basis of visual direct observation, force and joint position data. A 2-DOF hybrid force/position control technique was proposed in this paper. A pose estimation algorithm is adopted, based on force and positions data.

Simulation results have demonstrated the superior performance of the proposed hybrid control technique with respect to an algorithm using only human's visual perception for the estimation of the object pose. This haptic rendering technique is computationally efficient and can be applied to real-time cutting process.

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