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THE APPLICATION OF TAGUCHI'S "QUALITY LOSS" CONCEPT TO DIMENSIONAL PRECISION AND ISO FITS

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Abstract: *This article is an attempt at highlighting several "critical points" and "discrepancies" found in some speciality literature papers with respect to the „Quality Loss" function (QL). Alongside the special importance it plays in the field of product quality, this function also features some inconsistencies and application constraints, mainly in the area of tolerances and ISO fits.*

Key words: *quality loss function, target value, fit, clearance, interference.*

1. Introduction

Taguchi's "Quality Loss" concept (QL), also called "loss to society", is based on this loss assessment by using the square deviations of the quality characteristics with respect to its nominal value considered as "target" value [1], [4].

This statement regarding the "target" value being equal to the nominal value is not applicable to dimensional tolerance and standardized fits in machine building. Based on this finding and other QL related critical elements, we have undertaken first to propose, in the present article, a brief presentation of Taguchi's LQ concept. This will be done with respect to deviations from the „target" value as treated in some specialist literature references [1], [4], a.s.o., by highlighting the elements that constrain the correct application of this concept in the field of dimensional accuracy and ISO fits. That is why we consider that it is beneficial and appropriate to propose some measures for the correct application of Taguchi's QL concept to the above mentioned field.

2. Taguchi's "Quality Loss" Concept and Determination of Its Scope of Application

Dr. Genichi Taguchi based his quality quantification on two main instruments i.e. [4]:

a) the Signal/Noise ratio (*S/N*), establishing the intrinsic quality of a parameter considered as a signal (product, process) with respect to disturbing factors called noise factors in specialist literature [1], [4] a.s.o.;

b) the „Quality Loss Function QLF" caused by deviations of the quality characteristics with respect to their nominal value considered as "target" value. With respect to this latter statement made in papers [1], [4], we consider that the nominal value cannot be considered a "target" value in all cases.

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Such is the case, for instance, of the permissible dimensions of fits with guaranteed clearance or guaranteed interference. In these cases, the nominal dimensions lie beyond the tolerance zone of the parts surfaces (of shafts and holes) and hence the nominal dimension cannot be considered as "target" value.

From a psychological point of view, the QL function is Taguchi's cornerstone, the access way to a new approach of the quality concept. Taguchi has defined quality as a characteristic (a means) that avoids losses to society throughout the product life, from the moment it reaches the customer until going out of service [2], [5].

Taguchi's QL has for its aim the quantification, in terms of loss - both to the manufacturer as well as to the customer - of cases when products feature quality loss. The loss starts at the moment when the product is delivered and is quantified via the deviation value of the characteristic versus its nominal value considered as "target" [1], [4] etc. We believe that this statement is accurate if only the "target" value has been correctly chosen. Otherwise, as we are going to show in the present article, there will be negative effects upon the quantification of performance (production) quality.

Taguchi's hypothesis, confirmed by experience, resulting from manufacturing practice and customer dissatisfaction with product quality failure, has led to the conclusion that quality loss $L(y)$ is proportional to the square of the characteristic deviation from its nominal (from the "target") value [4].

$L(y)$ is a quadratic function resulting from a reduction operation of a Taylor series development of elements defining quality loss. Function $L(y)$ may be shown graphically as in Figure 1 and may be written:

$$L(y) = k(y-m)^2, \tag{1}$$

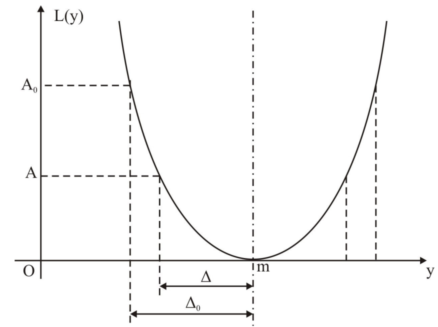

Fig. 1. *Diagram of Taguchi's "Quality Loss" function*

where: y is the characteristic quality variable that may be: the permissible dimension or dimensional tolerance, the tolerance of the micro-geometric form, position, orientation and surface, run-out tolerance, (front, radial run-out) etc.; m - nominal value of magnitude y, considered as "target" value [4]; $y - m$ - deviation of quality characteristic with respect to its nominal value; k - proportionality constant whose value depends on the economic impact of the quality criterion [4] and can be evaluated with relation:

$$k = \frac{A_0}{\Delta_0^2}, \tag{2}$$

where: Δ_0 is the functional tolerance, called customer tolerance; A_0 - customer loss caused by exceeding functional tolerance (loss A_0 may be represented by repair or replacement costs, when the functional tolerance has been exceeded. In A_0 one may integrate other customer incurred costs, such as for example: in cars, consumption and higher fuel and lubricant cost, respectively, because of larger clearance; costs associated with downtime during repair and re-assembling operations; continuing financial expenses to be paid during downtime, for example leasing location).

According to the presentation in Figure 1, Δ_0 represents 1/2 from the functional tolerance, $2\Delta_0$ noted in the technical specifications, or in the context of Taguchi's QL concept, Δ_0 represents the deviation of the characteristic taken under examination with respect to its "target" value.

In Figure 2, considering the value of the quality characteristic *y* at the tolerance limit, also called functionality limit, in the context of Taguchi's QL concept, it follows that:

- for case (1) in Figure 2:

$$\Delta_{0(-)} = y_1 - m\,; \qquad (3)$$

- and for case (2), respectively:

$$\Delta_{0(+)} = y_2 - m\,, \qquad (4)$$

where: y_1 and y_2 represent the value of the quality characteristic, the lower one (case 1) and the upper one (case 2) of functional tolerance $2\Delta_0$ (Figure 2).

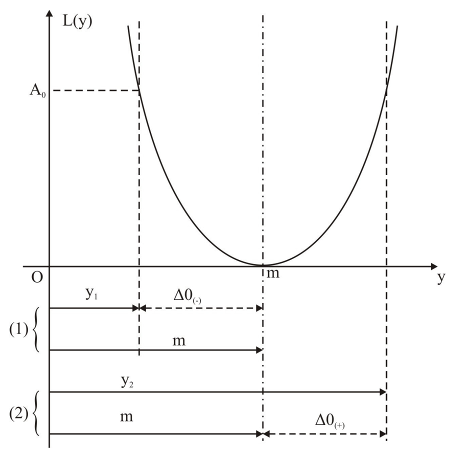

Fig. 2. *Diagram of Taguchi's function at functionality limit*

According to relations (3) and (4), it follows that the deviation of the quality characteristic with respect to target value "*m*" can be determined with relation:

$$y - m = \pm\Delta_0\,. \qquad (5)$$

Engineering practice resorts to production tolerance $2\Delta < 2\Delta_0$ (Figure 1) on account of the random influence of measuring errors upon the accurate assessment of the real value of the variable under consideration, as well as the need to prevent acceptance at the limit of prescribed tolerance and of values exceeding this tolerance. At the production tolerance limits (Figure 1) there occurs loss *A* smaller than loss A_0 (at functional tolerance limit $2\Delta_0$).

The value of half of production tolerance Δ is determined by starting from the equation of the loss to society function, relation (1), written as:

$$L(y) = A = k(y - m)^2\,, \qquad (6)$$

where, through similarity with relations (3), (4) and (5) and Figure 2 one can write $y - m = \pm\Delta_0$. According to relation (2), by writing, $k = \dfrac{A_0}{\Delta_0^2}$, equation (6) becomes:

$$L(y) = A = k(y - m)^2 = \frac{A_0}{\Delta_0^2}\Delta^2\,. \qquad (7)$$

In relations (6) and (7), *A* is the average cost (expenses) for the manufacturer when exceeding production deviation Δ; A_0 - average cost to the customer when exceeding functional deviation Δ_0; *m* - "target" value. The values for costs A_0 and *A* are determined on the basis of statistical data processing obtained in production practice and product use.

Functional deviation Δ_0 is determined in the conception stage (research-design) by calculation or it may be taken experimentally so that the operational conditions might be ensured from a technical and economic point of view. The production deviation Δ is determined based on relation (7):

$$\Delta = \sqrt{\frac{A}{A_0/\Delta_0^2}} = \Delta_0\sqrt{\frac{A}{A_0}}\,. \qquad (8)$$

In the case of symmetrical deviations to the nominal value (Figure 1), Δ_0 and Δ represent 1/2 of the functional tolerance and production tolerance, respectively.

This means that in the case of symmetrical tolerance with respect to the nominal value of the quality characteristic,

Taguchi's QL concept, as presented in specialist literature, has been correctly expressed.

Another deficiency one can find is the confusion between the notion of tolerance and that of deviation. That happens because according to the representation in Figure 1, Δ_0 and Δ represent, as we have claimed, deviations with respect to the target value (*m*), equal to half of the functional tolerances (2 Δ_0) and production tolerances (2 Δ), respectively.

In order to avoid the above mentioned (found) deficiencies, we propose that the target value should be considered equal to the prescribed average value corresponding to half of the tolerance zone (Figure 3) [3].

The limit values of the permissible deviations prescribed for holes (Figure 3) are determined with the general relation:

$$E_{pr.\lim} = V_{pr.\lim} - N \,, \tag{9}$$

where: $E_{pr.\lim}$ is the permissible limit deviation; $V_{pr.\lim}$ - permissible limit value of the variable under consideration, prescribed in the constructive-technological documentation; *N* - nominal value.

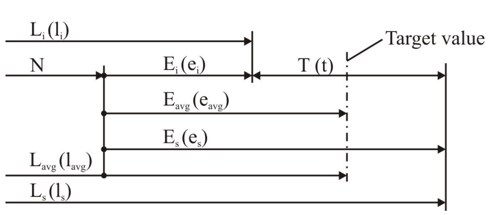

Fig. 3. *Representation of sizes, deviations, tolerance and "target" values in holes* (*shafts*)

Depending on the lower (L_i) and upper (L_s) limit values of the prescribed tolerance *T*, in the case of holes (Figure 3), we can discriminate between:

- Lower E_i and upper E_s deviations and L_i and L_s limits written as follows:

$$E_i = L_i - N; \;\; E_s = L_s - N. \tag{10}$$

From relation (10) we get:

$$L_i = N + E_i; \; L_s = N + E_s. \tag{11}$$

- Tolerance (*T*) of the variable under consideration is determined by using known relations, depending on the permissible deviation limits E_s and E_i (Figure 3):

$$\begin{aligned} T &= L_s - L_i = N + E_s - (N + E_i) \\ &= E_s - E_i. \end{aligned} \tag{12}$$

In the case of shafts (Figure 3), relations (9)...(12) will be the same, written this time with small letters, except for the nominal dimension which will be written with the same notation *N*.

Thus, in the case of shafts (Figure 3), the following relations will result:

$$e_i = l_i - N; \;\; e_s = l_s - N \,, \tag{13}$$

$$l_i = N + e_i; \; l_s = N + e_s \,, \tag{14}$$

$$T = l_s - l_i = N + e_s - (N + e_i) = e_s - e_i \,. \tag{15}$$

3. Identification of QL Concept Limited Application to Dimensional Accuracy and ISO Fits

For each quality characteristic one can "define a specific relation between the economic loss and the deviation of this characteristic with respect to its nominal value" [4]. In the case of dimensional tolerances and standardized ISO fits [9],

[10] taking the nominal values for "target" is valid only in isolated exceptional cases (for example only with permissible dimensions featuring fundamental *js* deviations for shafts and *JS* for holes). For example, we consider permissible values Φ65*js7* and Φ65*JS7*, that according to [8-10] may be written as Φ65*js7* = Φ65*JS7* = Φ65 ± 0.015 mm. One notes that these dimensions feature the tolerances and deviations, respectively, that are symmetrical to the nominal dimension N = Φ65 mm, which accounts for the fact that the "target" value in these cases can be considered as fairly equal to the nominal value N. However, for the other fundamental deviations regulated by present standards [9], [10], taking the nominal value N as "target" value is a wrong hypothesis. That can be noticed, for example, both in the case of permissible dimensions for a fit with guaranteed interference as well as in the case of fits with guaranteed clearance in a single hole-basis system as well as in a shaft-basis-system.

For example:

I. Fit with guaranteed interference:

- in hole-basis systems:

$$\Phi 65\frac{H7\begin{pmatrix}+0{,}03\\0\end{pmatrix}}{s6\begin{pmatrix}+0{,}072\\+0{,}053\end{pmatrix}}\text{ mm;}$$

- in shaft-basis systems:

$$\Phi 65\frac{S7\begin{pmatrix}-0{,}042\\-0{,}072\end{pmatrix}}{h6\begin{pmatrix}0\\-0{,}019\end{pmatrix}}\text{ mm.}$$

II. Fit with guaranteed clearance:

- in hole-basis systems:

$$\Phi 65\frac{H7\begin{pmatrix}+0{,}03\\0\end{pmatrix}}{g6\begin{pmatrix}-0{,}010\\-0{,}029\end{pmatrix}}\text{ mm;}$$

- in shaft-basis systems:

$$\Phi 65\frac{G7\begin{pmatrix}+0{,}04\\+0{,}01\end{pmatrix}}{h6\begin{pmatrix}0\\-0{,}019\end{pmatrix}}\text{ mm.}$$

In the two examples I and II, considered in the two systems of hole basis and shaft-basis, the tolerances and deviations, respectively, being non-symmetrical with respect to the nominal dimension N = Φ65 mm, as well as in many other cases, the "target" value cannot be equal to the nominal dimension.

At the same time, we agree that the production tolerance, as noted in Figure 1, is equal to 2Δ, while the functional tolerance is equal to $2\Delta_0$. That means that Δ is the production deviation and Δ_0 is the functional deviation, respectively. Both deviations are symmetrical with respect to "target" value m.

In order to eliminate most of the customer costs caused by quality loss, the manufacturer should put in a greater effort to establish and use production tolerance 2Δ which is less than functional tolerance $2\Delta_0$ (Figure 1). The production tolerance founded on customer made complaints will ensure a steady equilibrium point, in terms of A_0 and A (Figure 1) loss to the customer and the manufacturer, respectively. The manufacturer shall carry out supplementary work for making products within the production tolerance value, which will amount on the average to cost A (Figure 1). That will prevent cost loss $A_0 > A$ (Figure 1).

In order to make it easier to express the implementation of Taguchi's concept QL to the domain of permissible dimensions and fits in general, we propose the use of notations made in Figure 3, with capitals for holes and small letters for shafts. As a "target" value, we suggest that one should consider as an average value of the permissible dimension (L_{avg}, and l_{avg} respectively), or the average deviation (E_{avg} and e_{avg}, respectively). That is accounted for by the fact that, in the cases under consideration from Figure 3, the "target" value equal to the nominal value (N) is wrong. This means that the parts made in compliance with target value N = Φ65 mm will be scrapped, because they do not comply with the prescribed permissible limits L_i and L_s (l_i and l_s) and the permissible prescribed deviations either, E_i and E_s (e_i and e_s). At the same time, upon assembly, provided the "target" value is equal to N, all the assemblies will feature the clearance j and the fit S, respectively, equal to zero, given by relations:

$$j = N_D - N_d = N - N = 0\,, \qquad (16)$$

and respectively,

$$S = N_d - N_D = N - N = 0\,, \qquad (17)$$

where: N_d and N_D (equal to N) represent the nominal value of shafts (d) and holes (D), respectively, from the fit considered. As they are equal, the result will be $j = S = 0$.

That is the reason why the result according to relations (16) and (17) will be unsuitable because clearance and interference S will be equal to zero. The respective assemblies will not fulfil the prescribed function and consequently will be scrapped. We suggest that this drawback should be eliminated by considering the "target" equal to the average prescribed value L_{avg} (l_{avg}) or to the average deviation E_{avg} (e_{avg}) (Figure 3). In this case, the assembling will yield the correct values for the maximum, minimum and average clearance and interference given by known relations [8]:

- With clearance fits:

$$j_{\max} = L_s - l_i = E_s - e_i\,, \qquad (18)$$

$$j_{\min} = L_i - l_s = E_i - e_s\,, \qquad (19)$$

$$j_{\text{med}} = 0.5(j_{\max} + j_{\min}) = L_{\text{med}} - l_{\text{med}}\,; \quad (20)$$

- With interference fits:

$$S_{\max} = -j_{\min} = l_s - L_i = e_s - E_i\,, \qquad (21)$$

$$S_{\min} = -j_{\max} = l_i - L_s = e_i - E_s\,, \qquad (22)$$

$$S_{\text{med}} = 0.5(S_{\max} + S_{\min}) = l_{\text{med}} - L_{\text{med}} = -j_{\text{med}}. \qquad (23)$$

In conclusion, when manufacturing the parts (holes and shafts), the "target" value will be the average size or average deviation (L_{avg} or E_{avg} in holes and l_{avg} or e_{avg} in shafts, respectively). In assembling these parts, the "target" value will be equal to j_{avg} in clearance fits and equal to S_{avg} in interference fits, respectively. Thus, one will eliminate the drawbacks shown with respect to relations (16) and (17).

To be more specific, we have drawn up Table 1 with the permissible dimensions and limit values for sizes, deviations, as well as with the correct and incorrect "target" values in machining the respective parts.

From the presentation made in Table 1, one notices that the correct target values are different from the nominal dimension N = Φ65 mm.

In case the above considered fit holes and shafts were made with dimensions equal to nominal dimensions N = Φ65 mm, the respective parts would be scrapped as they cannot comply with the prescribed permissible limit values and consequently, assembling will not yield the prescribed interference and clearances values, respectively.

Table 2 shows the limit values of clearance and interference characteristics of hole - basis system fits. At the same time, one also presents the "target" value equal to j_{avg} for the clearance fit and S_{avg} for the interference fit. These values are the same, irrespective of the fitting system (hole-basis system or shaft-basis system).

Table 1

Limit and "target" values for permissible sizes in fits Φ65H7/s6, Φ65S7/h6, Φ65H7/g6 and Φ65G7/h6

No.	Permissible dimension, [mm]	Dimens. N, [mm]	Prescribed limit dimensions, [mm]		Target value [mm]	
			L_i, l_i	L_s, l_s	Correct	Incorrect
1	$\Phi 65H7\left(^{+0,03}_{0}\right)$	Φ65	65	65.03	65.015	$N = 65$
2	$\Phi 65s6\left(^{+0,072}_{+0,053}\right)$		65.053	65.072	65.0625	
3	$\Phi 65S7\left(^{-0,042}_{-0,072}\right)$		64.928	64.958	64.943	
4	$\Phi 65h6\left(^{0}_{-0,019}\right)$		64.981	65	64.9905	
5	$\Phi 65g6\left(^{-0,010}_{-0,029}\right)$		64.971	64.990	64.9805	
6	$\Phi 65G7\left(^{+0,040}_{+0,010}\right)$		65.010	65.040	65.025	

Table 2

Clearance and interference values for fits featuring Φ65H7/g6 and Φ65H7/s6

No.	Prescribed fit	Limit values of clearance and fits, [mm]	Target values, [mm] Correct	Incorrect
1	$\Phi 65\dfrac{H7\left(^{+0,03}_{0}\right)}{g6\left(^{-0,010}_{-0,029}\right)}$	$j_{max} = E_s - e_i = 0.059$	$j_{avg} = 0.0345$	$j = N_D - N_d = 0$
		$j_{min} = E_i - e_s = 0.010$		
2	$\Phi 65\dfrac{H7\left(^{+0,03}_{0}\right)}{s6\left(^{+0,072}_{+0,053}\right)}$	$S_{max} = e_s - E_i = 0.072$	$S_{avg} = 0.0475$	$S = N_d - N_D = 0$
		$S_{min} = e_i - E_s = 0.023$		

The aim of Taguchi's PQ concept is quantification, in terms of product quality loss to both manufacturer and customer. The loss starts from the moment when the product is delivered and it is quantified through the magnitude of the deviation of the characteristic achieved as compared to its nominal value considered as "target" value [7].

Therefore, a product, no matter how good, should be continuously improved, at the level of market requirements and customer purchasing potential. That is actually the key to manufacturing success and to obtaining profit on a competitive market [6].

4. Conclusions

Based on the above, it follows that, within Taguchi's QL concept shown in [4], there exists, as a first „critical" element, a functional tolerance considered to be equal to Δ_0 and a production tolerance considered to be equal to Δ instead of $2\Delta_0$ and, 2Δ, respectively.

A second "critical element" is the nominal value, considered to be an ideal value ("target" value) for a parameter under analysis, which, in the case of dimensional accuracy and ISO fits, is correct only for symmetrical deviations and tolerances and incorrect in the case of non symmetrical deviations and tolerances with respect to the nominal value (for example the case of clearance and interference fits, irrespective of the shaft-basis or hole-basis systems).

With respect to the first "critical element" likely to be improved, i.e. the confusion regarding the notions of deviation and tolerance, we propose that these notions should be considered as distinct and should not be confused. At the same time, the notions of deviation and manufacturing (machining) error, should be considered as distinct and should not be confused. In the case of non symmetrical deviations and tolerances with respect to the nominal value, it would be fair to consider that the quality loss is proportional to the squared manufacturing error (machining) with respect to the correctly established "target" value, equal to the prescribed average value, but not with respect to the nominal value.

In the case of clearance, interference and transition fits, the nominal dimension cannot be considered as "target". In this case, the QL concept may be correctly adapted so that the "target" value might be considered equal to the prescribed permissible average value, that is, equal to the prescribed average clearance value (j_{avg}), equal to the prescribed average interference (S_{avg}), respectively. Thus, the implementation of the prescribed function will be effectively carried out from an economic and technical point of view.

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