Bulletin of the *Transilvania* University of Braşov
Series I: Engineering Sciences • Vol. 6 (55) No. 2 - 2013

# MODEL REFERENCE ADAPTIVE CONTROL FOR A DC ELECTRICAL DRIVE

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***Abstract:*** *The Model Reference Adaptive System (MRAS) is used to obtain system's performance specifications in terms of model reference. A model reference describes the desired response for a DC electrical drive. In MRAS the adjustment mechanism can be obtained by applying two methods: by using a gradient method (the MIT rule) or by using a stability theory (Lyapunov method). Therefore, to control a DC electrical drive, the paper's authors have been studied both methods. The appropriate design and experimental results are all done in Matlab/Simulink.*

***Key words:*** *MRAS, model reference, adaptive control, MIT rule, Lyapunov method.*

## 1. Introduction

The Model Reference Adaptive System (MRAS) is one of the main approaches to adaptive control.

Adaptive control is a technique that provides an automatic adjustment of a controller in real time. The automatic adjustment is performed in order to maintain the controller's system performances in case the parameters of the process are (*i*) unknown or (*ii*) changing in time [1].

Both cases are detailed further on. In case (*i*) of unknown but constant parameters, the adaptive control technique is designed to provide in the closed loop an automatic tuning procedure; this procedure will be applied to all the unknown but constant parameters. In case (*ii*) when the parameters are changing unpredictably in time, to maintain the system performances, the control system must use an adaptive control [1], [4], [5].

The original scheme for MRAS proposed by Whitaker in 1958 was introduced for the flight control. For the current study, MRAS was applied to control a DC electrical drive (Figure 1) [1].

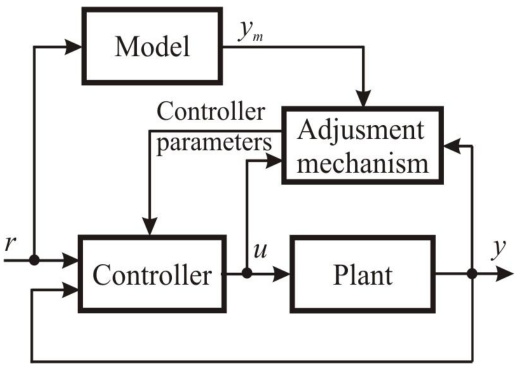


Fig. 1. *General block diagram of MRAS*

Both, the gradient method and the Lyapunov method, mentioned above were used to design and simulate 2 distinctive MRAS systems.

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## 2. Model Reference Adaptive Control (MRAC)

In case of using a model reference, the desired behaviour of a process can be described. This is the case of MRAC, and the process is represented by a linear time-invariant (LTI) system driven by the input reference and its associated transfer function $G_m(s)$.

MRAS has 2 loops. The first one (inner loop) includes the process itself and the classical feedback. The second one (outer loop) is used to adjust controller parameters. As a first step in the designing process of the transfer function for the model reference ($G_m(s)$), the reference input signal $r(t)$ shall be considered. Further on, the process $y(t)$ must follow the output signal $y_m(t)$ which represents the system's desired response.

As a consequence, the error signal $e(t)$ represents the difference between the system output and the model reference, and it has to be very small. A decision on how small the error can be is influenced by the model reference, the process, and the command signal. Only in the case of reducing the error signal to 0 (zero) for all the command signals, a perfect model is achieved [2], [5]. In case of MRAC, the parameters can be adjusted in 2 ways: (*i*) by using a gradient method, or (*ii*) by applying a stability theory.

### 2.1. Design of the MRAS by Using a Gradient Method

The gradient method, or MIT rule, was developed by the Instrumentation laboratory at Massachusetts Institute of Technology (MIT).

#### 2.1.1. The MIT rule

It shall be considered that in the closed loop system, the controller has one adjustable parameter θ. The parameter $e$ represents the error between the output of the process ($y(t)$) and the output of the model reference ($y_m(t)$). The goal is to adjust parameter θ in order to minimize loss function $J(\theta)=\frac{1}{2}e$ [4], [6]. Function $J$ has to be small, therefore is reasonable to change the parameters in the direction of negative gradient for $J$:

$$\frac{d\theta}{dt}=-\gamma\frac{\partial J}{\partial\theta}=-\gamma e\frac{\partial e}{\partial\theta}, \quad (1)$$

where: θ is the controller parameter, $e$ is the error between the process and the model outputs; $y$ is the adaptation gain; $\frac{\partial e}{\partial\theta}$ is the sensitivity derivative of the system.

The first equation is called the MIT rule. The choice of loss function is arbitrary. If the loss function is $J(\theta)=|e|$, then the adjustment rule becomes:

$$\frac{d\theta}{dt}=-\gamma\frac{\partial e}{\partial\theta}\operatorname{sign} e, \quad (2)$$

where sign is the signum function.

#### 2.1.2. MIT Rule for the First Order System

The process is described by:

$$\frac{dy(t)}{dt}=-ay(t)+bu(t), \quad (3)$$

where $u$ is the control variable and $y$ is the measurement output.

The model reference is described by:

$$\frac{dy_m(t)}{dt}=-ay_m(t)+br(t). \quad (4)$$

A perfect following of the model reference is achieved with the controller low [3]:

$$u(t) = t_0 r(t) - s_0 y(t) . \tag{5}$$

The error of the system is:

$$e(t) = y(t) - y_m(t) . \tag{6}$$

To apply the MIT rule, Equation (5) is introduced in Equation (3) as below:

$$\begin{aligned} &\frac{\mathrm{d}y(t)}{\mathrm{d}t} = -ay(t) + bu(t) \\ &\quad \Leftrightarrow py(t) = -ay(t) + bu(t) \\ &\quad \Rightarrow y(t) = \frac{bt_0}{p + a + bs_0} r(t), \end{aligned} \tag{7}$$

where $p$ is the differential operator.

So, the error will be:

$$e(t) = \left( \frac{bt_0}{p + a + bs_0} - \frac{b_m}{p + a_m} \right) r(t). \tag{8}$$

The sensitivity derivatives are obtained by taking the partial derivatives of the error with respect to the controller parameters:

$$\begin{aligned} \frac{\partial e(t)}{\partial t_0} &= \frac{b}{p + a + bs_0} r(t), \\ \frac{\partial e(t)}{\partial s_0} &= -\frac{b}{p + a + bs_0} y(t). \end{aligned} \tag{9}$$

All these equations cannot be used because of the unknown process parameters. To solve this impediment, the following approximation can be done:

$$p + a + bs_0 = p + a_m . \tag{10}$$

After the approximation is done, the adjustment of the controller parameters can be obtained:

$$\begin{aligned} \frac{\mathrm{d}t_0(t)}{\mathrm{d}t} &= -\gamma \left( \frac{1}{p + a_m} r(t) \right) e(t), \\ \frac{\mathrm{d}s_0(t)}{\mathrm{d}t} &= \gamma \left( \frac{1}{p + a_m} y(t) \right) e(t), \end{aligned} \tag{11}$$

where the parameter $b$ is introduced in the adaptation gain $\gamma$.

The MIT rule can perform well, if the adaptation gain is small. Unfortunately, by applying the MIT rule, the system is not stable in closed loop.

### 2.2. Design of the MRAS Using the Stability Theory

The adjustment rules can also be obtained by using stability theory. Even in case the adaption gain does not have small values, MRAS reaches the mandatory target of converging to zero of the error signal (Eq. 12) [4], [6]:

$$\lim_{t \to \infty} e(t) = \lim_{t \to \infty} (y(t) - y_m(t)) = 0 . \tag{12}$$

The same MRAS general schema that was used for the MIT method is applicable. But the emphasis will be on how the adapting mechanism is implemented, as this is specific for this case. It shall be assumed, further on, that the given differential equation is characteristic for the closed loop system:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t), f(0)) = 0 , \tag{13}$$

where $x$ is the state vector of the system.

An acceptable supposition is that the given Eq. (13) may define the whole system dynamics. For a system to be stable, a Lyapunov function (noted with $V$) which reacts to all the system parameters variations and validates all the conditions described in Eq. (14) needs to be found:

$$\begin{aligned} &1.\ V(x) > 0,\ x \neq 0;\ V(0) = 0; \\ &2.\ V\ \textit{can be differentiable}; \\ &3.\ \dot{V}(t) = \frac{\partial V}{\partial x} \dot{x}(t) \leq 0, \end{aligned} \tag{14}$$

Thus, the closed loop system is stable only in case the $V$ function is positive definite and the function's derivative is negative semi-definite. In conclusion, the problem is in finding of such an appropriate function.

### 2.2.1. Lyapunov Theory for First Order System

The process model, model reference, and controller low are described by the same equations as those used in describing the MIT rule ((Eq. (3), (4), (5)). Here, the goal is to minimize the error $e(t) = y(t) - y_m(t)$ ) [2], [3], so:

$$\frac{\mathrm{d}e(t)}{\mathrm{d}t} = \frac{\mathrm{d}y(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_m(t)}{\mathrm{d}t} = -ay(t) + bu(t) + a_m y_m(t) - b_m r(t). \quad (15)$$

As a next step, Eq. (6) is introduced in Eq. (12) as follows:

$$\begin{aligned}\frac{\mathrm{d}e(t)}{\mathrm{d}t} &= \frac{\mathrm{d}y(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_m(t)}{\mathrm{d}t} \\ &= -ay(t) + b(t_0 r(t) - s_0 y(t)) + a_m y_m(t) - b_m r(t) \\ &= -ay(t) + bt_0 r(t) - bs_0 y(t) + a_m y_m(t) - b_m r(t).\end{aligned} \quad (16)$$

Furthermore, term $a_m y_m(t)$ is added to and subtracted from Eq. (13):

$$\begin{aligned}\frac{\mathrm{d}e(t)}{\mathrm{d}t} &= \frac{\mathrm{d}y(t)}{\mathrm{d}t} - \frac{\mathrm{d}y_m(t)}{\mathrm{d}t} \\ &= -ay(t) + b(t_0 r(t) - s_0 y(t)) + a_m y_m(t) - b_m r(t) = \cdots \\ &= -a_m e(t) - (bs_0 + a - a_m) y(t) + (bt_0 - b_m) r(t).\end{aligned} \quad (17)$$

The error goes to zero if the parameters of the controller are:

$$t_0 = \frac{b_m}{b}$$

and (18)

$$s_0 = \frac{a_m - a}{b}.$$

To drive the parameters $t_0$ and $s_0$ to their desired values, a parameters adjustment mechanism needs to be built. In this purpose, Lyapunov function will be:

$$V(e, t_0, s_0) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(bs_0 + a - a_m)^2 + \frac{1}{b\gamma}(bt_0 - b_m)^2\right). \quad (19)$$

This function is zero only when the error is zero and the controller parameters have correct values. The derivative of $V$ is:

$$\begin{aligned}\frac{\mathrm{d}V(t)}{\mathrm{d}t} &= e\frac{\mathrm{d}e(t)}{\mathrm{d}t} + \frac{1}{\gamma}(bs_0 + a - a_m)\frac{\mathrm{d}s_0(t)}{\mathrm{d}t} \\ &+ \frac{1}{\gamma}(bt_0 - b_m)\frac{\mathrm{d}t_0(t)}{\mathrm{d}t} = \cdots = -a_m e^2(t) \\ &+ \frac{1}{\gamma}(bs_0 + a - a_m)\left(\frac{\mathrm{d}s_0(t)}{\mathrm{d}t} - \gamma y(t)e(t)\right) \\ &+ \frac{1}{\gamma}(bt_0 - b_m)\left(\frac{\mathrm{d}t_0(t)}{\mathrm{d}t} + \gamma r(t)e(t)\right).\end{aligned} \quad (20)$$

If the parameters are updated as:

$$\begin{aligned}\frac{\mathrm{d}t_0(t)}{\mathrm{d}t} &= -\gamma r(t)e(t), \\ \frac{\mathrm{d}s_0(t)}{\mathrm{d}t} &= \gamma y(t)e(t),\end{aligned} \quad (21)$$

the result is:

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = -a_m e^2(t). \quad (22)$$

The adjustment rule obtained from the Lyapunov theory is simple because it does not require filtering of the signals. This rule (Eq. 21) is similar to adjustment law obtained with the MIT rule.

## 3. Dynamic Model for DC Electrical Drive

### 3.1. General Description

All the DC electrical drive parameters are unknown. The adaptive control must be performed on all the controller parameters as it is described in the next step-by-step procedure: 5 V reference signal is applied to the DC electrical drive input; arithmetical average is done for 3 different measurements; 4th order Butterworth low pass filter is applied to the resulted signal; 1st order element is selected to match recorded data.

The DC electrical drive model is a 1st order element and its transfer function is:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b}{s+a} = \frac{0.79}{9.3s+1}.$$

A model reference with the below depicted transfer function is selected:

$$G_m(s) = \frac{Y_m(s)}{R(s)} = \frac{b_m}{s+a_m} = \frac{1}{4s+1}.$$

The resulted DC electrical drive model is represented below (Figure 2):

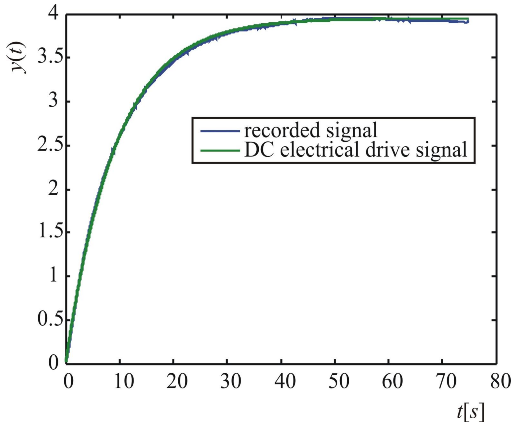


Fig. 2. *Representation of the two signals: the recorded signal and the DC electrical drive signal*

### 3.2. Controller Parameters Adjustment by Using the Gradient Method

For the 2 unknown parameters of the process, an adaptive law with 2 adjusting parameters is selected (Eq. 5). To find proper values for the adaptive law's adjusting parameters, the same adjusting mechanism that was detailed in paragraph (2.1.1) of the current study, is going to be used.

A step input was applied for a period of 100 seconds to the simulated system represented in Figure 3.

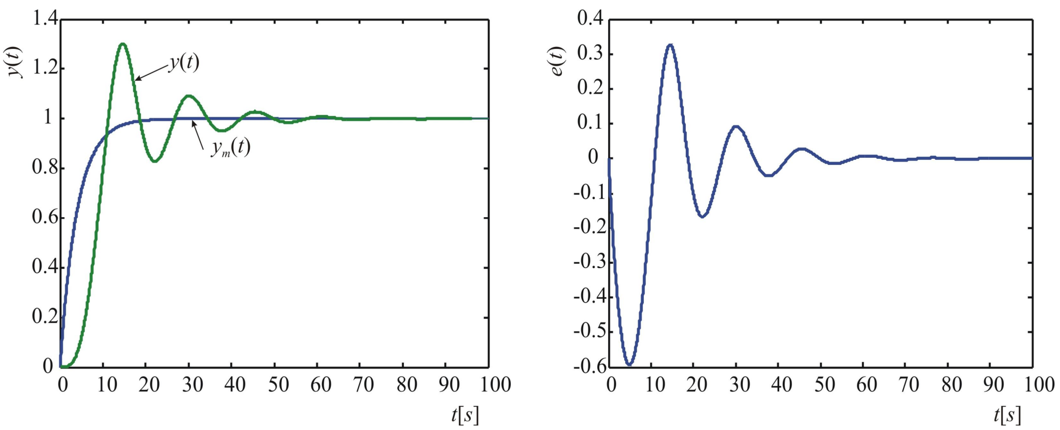


Fig. 3. *Gradient method.*
*The output signals ( $y(t)$, $y_m(t)$ ) and adjustment error signal (e)(t)) progression*

A short analyse of the same figure reveals that for an adaption gain equal to 1, the progression of the 3 signals (output, model reference, and error adjustment for the whole system) demonstrates the system's possibility to be adjustable.

### 3.3. Controller parameters adjustment by using the Lyapunov method

To make the process output signal to follow the process model reference, a P controlling law with two adjustment parameters is used (Figure 4). The same adjustment mechanism already detailed in paragraph (2.2.1) is used to determine the controller parameter values.

## 4. Conclusions

Both methods can be applied on any system type. By increasing the adaptation gain, the system is adapting faster; in this way, system stability is kept.

The current study, which was performed on both adaptive control applications, reveals that the Lyapunov method is better than the gradient method.

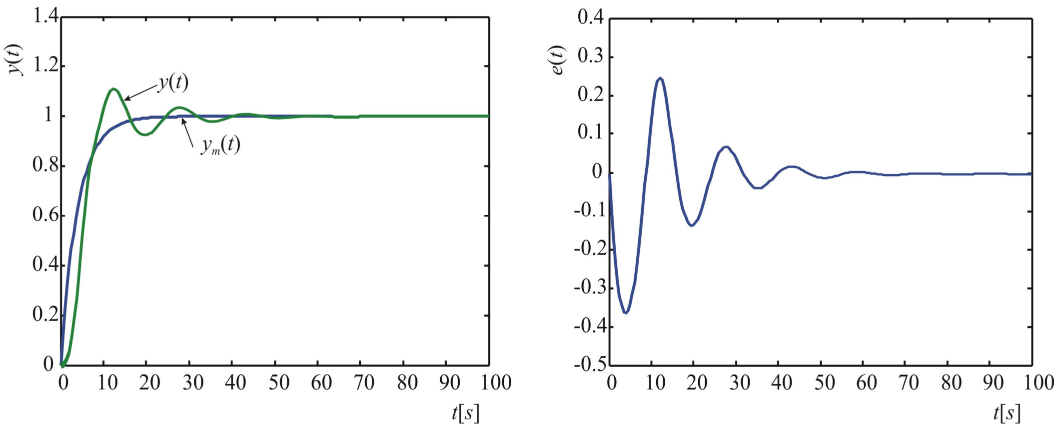


Fig. 4. *Lyapunov method.*
*The output signals ( $y(t)$, $y_m(t)$ ) and adjustment error signal (e)(t)) progression*

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